Group Decision Making

- How should a group make a decision? Vote!

- Suppose there are three people, Alice, Bob and Cory, who have to choose a holiday destination.
  There are three possible destinations: X, Y, and Z.
  Individual preferences:
  - ▶ Alice: $X \succ Y \succ Z$
  - ▶ Bob: $Y \succ Z \succ X$
  - ▶ Cory: $Z \succ X \succ Y$

- What wins in pairwise votes?
  Condorcet paradox

- Surely this is just an outlier!!

Arrow's impossibility theorem

Desirable properties of preference of the group

- the group preference is complete and transitive
- individuals can have any complete and transitive preferences they like
- if every individual prefers outcome o_1 to o_2, the group prefers o_1 to o_2.
- the group preference between outcomes o_1 and o_2 depends only on the individual preferences on o_1 and o_2 and not on the individual preferences on other outcomes.
- no individual gets to unilaterally decide the outcome (non-dictatorship).

Arrow's Theorem: If there are three or more outcomes, these properties cannot simultaneously hold for any social preference function.

Mechanism Design

A *mechanism* specifies the actions available to each agent and the outcomes of each action profile.

Agents have utilities over outcomes.

Desirable properties of mechanisms:

- A mechanism should be easy for agents to use. It it should be easy for an agent to determine what to do based on their preferences.

  A *dominant strategy* is one that is best for the agent, no matter what other agents do.

  If there is a mechanism with dominant strategies there is a equivalent mechanism where each player's dominant strategy is to be *truthful* (revelation principle).

- A mechanism should give the best outcome aggregated over all of the agents. A mechanism is *economically efficient* if the outcome chosen is one that maximizes the sum of the utilities of the agents.

Example: meeting scheduler

Two mechanisms for scheduling meetings:

- the users to specify when they are available or not, and for the scheduler to select the time that has the most people available.
- users to specify their utility for the various times, and the scheduler chooses the time that maximizes the sum of the utilities.

Are these dominant-strategy truthful?

Alice, Bob, and Cory have to decide whether to meet on Monday, Tuesday, or Wednesday, with the following utilities for the meeting days:

	Monday	Tuesday	Wednesday
Alice	0	8	10
Bob	3	4	0
Cory	11	7	6

Should Alice be honest?

Gibbard–Satterthwaite theorem

Gibbard–Satterthwaite theorem as long as there are three or more outcomes, the only mechanisms with dominant strategies have a dictator: an agent whose preferences determine the outcome.

Vickrey–Clarke–Groves mechanism

- Introduce money, so that, for any two outcomes o_1 and o_2, for each agent there is some (possibly negative) amount d such that the agent is indifferent between the outcomes o_1 and $o_2 + d$.

- VCG mechanism: Agents pay according to how much their participation affects the outcome.

- Agent i pays the sum of the value for the *other* agents if i had not participated minus the sum of the values for the other agents if i had participated.

The VCG mechanism is both economically efficient and dominant-strategy truthful, assuming that agents only care about their utility and not about other agents' utilities or other agents' payments.

Example: meeting scheduler

VCG Mechanism:

	Monday	Tuesday	Wednesday	Payment	Net Value
Alice	0	8	10	3	5
Bob	3	4	0	1	3
Cory	11	7	6	0	7
Total	14	19	16		

Tuesday is chosen as the meeting day.

- What happens without payments?
- What should the payments be?
- What happens with payments?

Auctions

- A common way to sell things is by an auction.
- The outcome that maximizes the payoffs is to give the item to the person who had the highest bid.
- According to the VCG mechanism, the top bidder should get the item and pay the value of the second-highest bid.
  This is known as a second-price auction.
- This is equivalent (up to bidding increments) to having an ascending auction, where
  people specify how much they want to pay as a proxy bid;
  an agent converts proxy bids into real bids.