The word *cause* is not in the vocabulary of standard probability theory. It is an embarrassing yet inescapable fact that probability theory, the official mathematical language of many empirical sciences, does not permit us to express sentences such as "Mud does not cause rain"; all we can say are that the two events are mutually correlated, or dependent – meaning that if we find one, we can expect to encounter the other. Scientists seeking causal explanations for complex phenomenon or rationales for policy decisions must therefore supplement the language of probability with a vocabulary for causality, one in which the symbolic representation for "Mud does not cause rain" is distinct from the symbolic representation for "Mud is independent of rain". Oddly, such distinctions have yet to be incorporated into standard scientific analysis.

– Judea Pearl, *Causality*, p 134.

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- Assume no causal cycles; apparent cycles, e.g., $poverty \rightarrow sickness$ and $sickness \rightarrow poverty$, are modeled using time.

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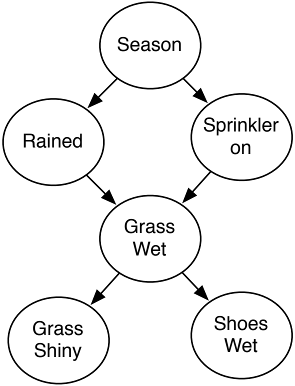
- A structural causal model defines a causal mechanism for each modeled variable.

Sprinkler Example

Variables:

- Season dry or wet
- Rained last night
- Sprinkler was on last night
- Grass wet
- Grass shiny and appears to be wet
- Shoes wet after walking on grass



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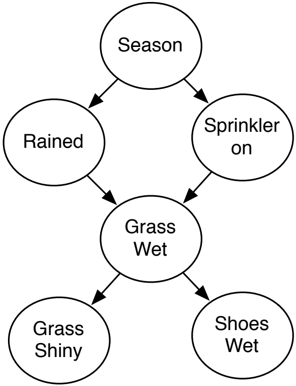

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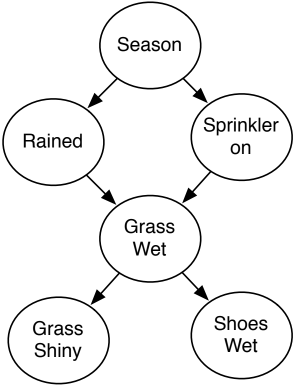

- Which probabilities change if you observe sprinkler on?
- Which probabilities change if you turn the sprinkler on?

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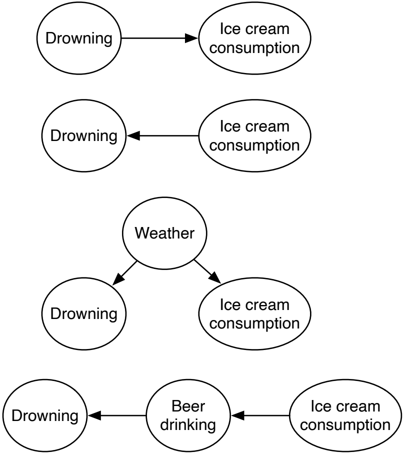

- Ice cream consumption and drowning are correlated.
- The top two can be made to fit the data
- Which is a better causal model?
- What experiments could be used to test the models?

Clicker Question

Which of the following is not true:

A All belief networks are causal networks

B All causal networks are belief networks

C A causal network predicts the effect of an intervention

D An intervention changes the value of a variable by some mechanism external to the model

E Intervening on a variable only affects the descendents of the variable

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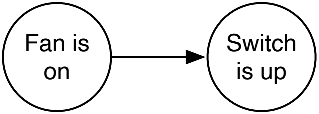
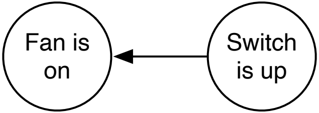
Which is a reasonable causal model?

A fan is connected to a switch...

A) 

B) 

C) both

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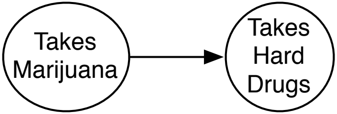

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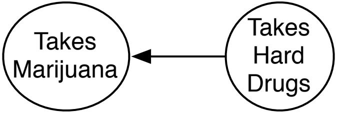

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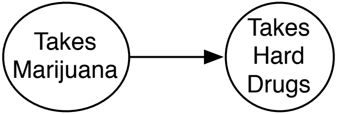

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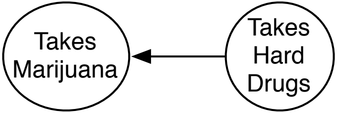

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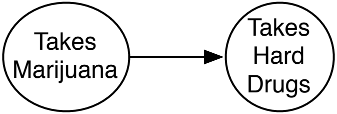

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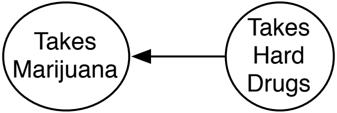

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Try: `drugs` example in `probDo.py` of `AIPython`.

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  - $do(X{=}v)$ becomes $ForceX{=}v$

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- Conjecture: causal belief networks are more natural and more concise than non-causal networks.
- Conjecture: causal model are more stable to changing circumstances (transportability)

