

Conditional independence

Random variable X is **independent** of random variable Y **given** random variable(s) Z if,

$$P(X | Y, Z) = P(X | Z)$$

i.e. for all $x_i \in \text{domain}(X)$, $y_j \in \text{domain}(Y)$, $y_k \in \text{domain}(Y)$ and $z_m \in \text{domain}(Z)$,

$$\begin{aligned} P(X = x_i | Y = y_j \wedge Z = z_m) \\ &= P(X = x_i | Y = y_k \wedge Z = z_m) \\ &= P(X = x_i | Z = z_m). \end{aligned}$$

That is, knowledge of Y 's value doesn't affect the belief in the value of X , given a value of Z .

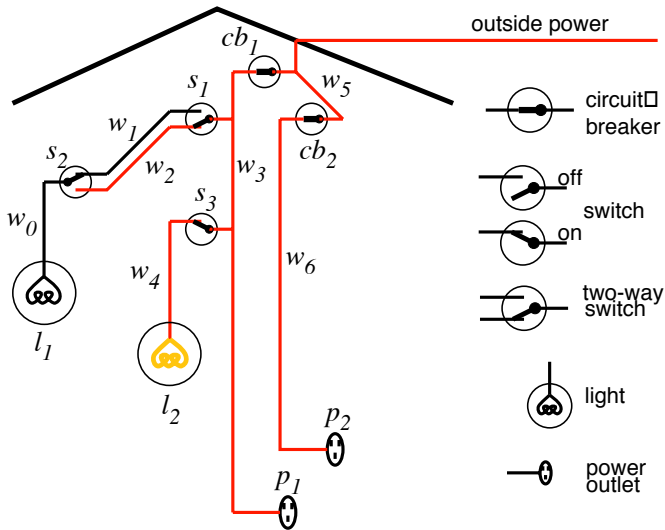
Example

Consider a student writing an exam.

What are reasonable independences among the following?

- Whether the student works hard (W)
- Whether the student is intelligent (I)
- The student's answers on the exam (A)
- The student's mark on an exam (M)

Example domain (diagnostic assistant)

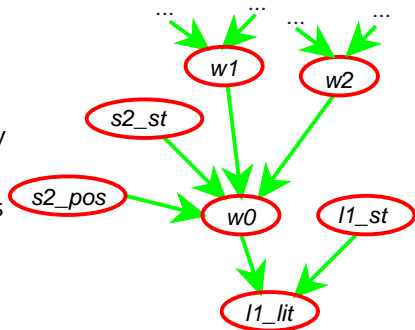


Examples of conditional independence?

- The party forming government of Canada is dependent or independent of whether light l_1 is lit given whether there is outside power?
- Whether there is someone in a room is independent of whether a light l_2 is lit given the position of switch s_3 .
- Whether light l_1 is lit is independent of the position of light switch s_2 given whether there is power in wire w_0 .
- Every other variable may be independent of whether light l_1 is lit given whether there is power in wire w_0 and the status of light l_1 (if it's *ok*, or if not, how it's broken).

Idea of belief networks

- I_1 is lit ($L1_lit$) depends only on the status of the light ($L1_st$) and whether there is power in wire $w0$.
- In a belief network, $W0$ and $L1_st$ are **parents** of $L1_lit$.
- $W0$ depends only on whether there is power in $w1$, whether there is power in $w2$, the position of switch $s2$ ($S2_pos$), and the status of switch $s2$ ($S2_st$).



- Totally order the variables of interest: X_1, \dots, X_n
- Theorem of probability theory (chain rule):
$$P(X_1, \dots, X_n) = \prod_{i=1}^n P(X_i \mid X_1, \dots, X_{i-1})$$
- The **parents** of X_i , $parents(X_i)$, are those predecessors of X_i that render X_i independent of the other predecessors. That is,

$$parents(X_i) \subseteq X_1, \dots, X_{i-1}$$

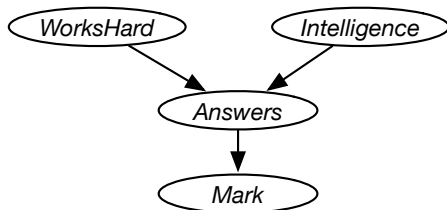
$$P(X_i \mid parents(X_i)) = P(X_i \mid X_1, \dots, X_{i-1})$$

- So $P(X_1, \dots, X_n) = \prod_{i=1}^n P(X_i \mid parents(X_i))$
- A **belief network** is a graph: the nodes are random variables; there is an arc from the parents of each node into that node.

Student Writing an Exam Example

Give a belief network for the variables in order:

- *WorksHard*: Whether the student works hard
- *Intelligent*: Whether the student is intelligent
- *Answers*: The student's answers on the exam
- *Mark*: The student's mark on an exam

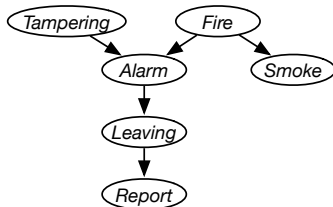


What if the variables were in the opposite order?

Example: fire alarm belief network

Variables:

- **Fire**: there is a fire in the building
- **Tampering**: someone has been tampering with the fire alarm
- **Smoke**: what appears to be smoke is coming from a window
- **Alarm**: the fire alarm goes off
- **Leaving**: people are leaving the building *en masse*.
- **Report**: told people are leaving the building *en masse*.



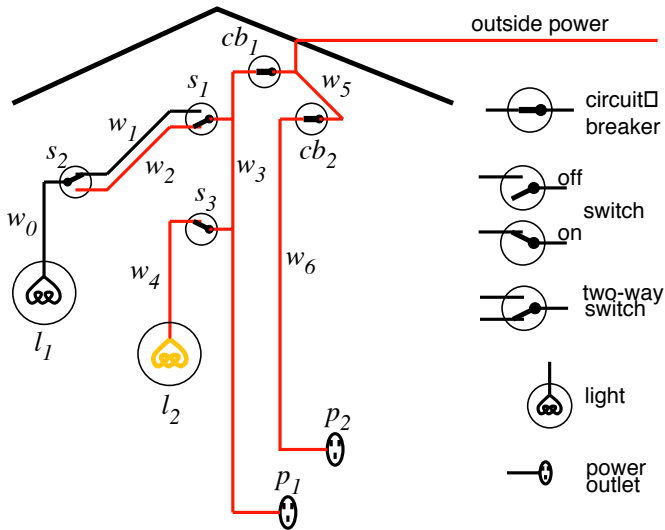
AIPython.org: `bn_report` in `probGraphicalModels.py`

Components of a belief network

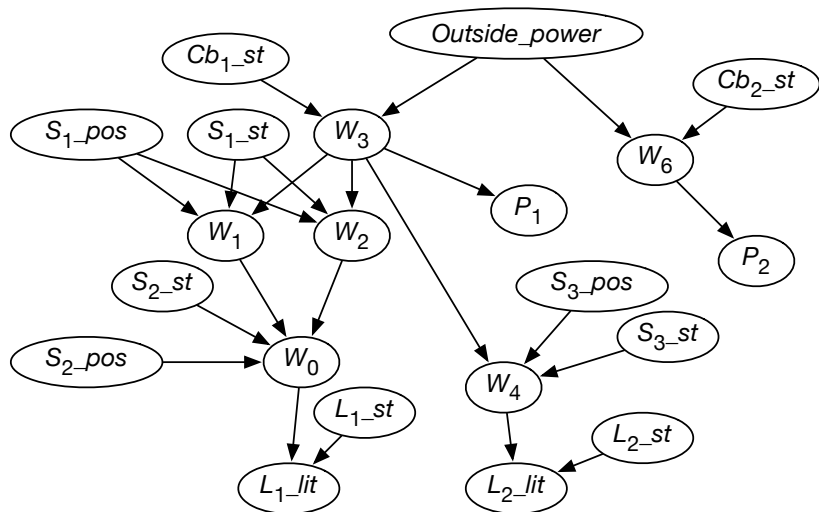
A belief network consists of:

- a directed acyclic graph with nodes labeled with random variables
- a domain for each random variable
- a set of conditional probabilities, one for each variable given its parents (including prior probabilities for nodes with no parents).

Task Domain: Electrical Environment



Example belief network



Example belief network (continued)

The belief network also specifies:

- The domain of the variables:
 W_0, \dots, W_6 have domain $\{live, dead\}$
 $S_1_pos, S_2_pos,$ and S_3_pos have domain $\{up, down\}$
 S_1_st has $\{ok, upside_down, short, intermittent, broken\}$.
- Conditional probabilities, including:
 $P(W_1 = live \mid s_1_pos = up \wedge S_1_st = ok \wedge W_3 = live)$
 $P(W_1 = live \mid s_1_pos = up \wedge S_1_st = ok \wedge W_3 = dead)$
 $P(S_1_pos = up)$
 $P(S_1_st = upside_down)$

Belief network summary

- A belief network is a directed acyclic graph (DAG) where nodes are random variables.
- The **parents** of a node N are those variables on which N directly depends.
- A belief network is automatically acyclic by construction.
- A belief network is a graphical representation of dependence and independence:
 - ▶ A variable is independent of its non-descendants given its parents.

Constructing belief networks

To represent a domain in a belief network, you need to consider:

- What are the relevant variables?
 - ▶ What will you observe?
 - ▶ What would you like to find out (query)?
 - ▶ What other features make the model simpler?
- What values should these variables take?
- What is the relationship between the variables?
This is in terms of a directed graph, representing conditional dependence.
- How does the value of each variable depend on its parents?
This is expressed in terms of the conditional probabilities.

