Integrity Constraints

- In the electrical domain, what if we predict that a light should be on, but observe that it isn't?
  What can we conclude?
- We will expand the definite clause language to include integrity constraints which are rules that imply *false*, where *false* is an atom that is false in all interpretations.
- This will allow us to make conclusions from a contradiction.
- A definite clause knowledge base is always consistent.
  This won't be true with the rules that imply *false*.

Horn clauses

- An integrity constraint is a clause of the form

  $$false \leftarrow a_1 \wedge \ldots \wedge a_k$$

  where the a_i are atoms and *false* is a special atom that is false in all interpretations.
- A Horn clause is either a definite clause or an integrity constraint.

Negative Conclusions

- Negations can follow from a Horn clause KB.
- The negation of α, written $\neg\alpha$ is a formula that
  - is true in interpretation I if α is false in I, and
  - is false in interpretation I if α is true in I.
- Example:

$$KB = \left\{ \begin{array}{l} false \leftarrow a \wedge b. \\ a \leftarrow c. \\ b \leftarrow c. \end{array} \right\} \qquad KB \models \neg c.$$

Disjunctive Conclusions

- Disjunctions can follow from a Horn clause KB.
- The disjunction of α and β, written $\alpha \vee \beta$, is
  - true in interpretation I if α is true in I or β is true in I (or both are true in I).
  - false in interpretation I if α and β are both false in I.
- Example:

$$KB = \left\{ \begin{array}{l} \mathit{false} \leftarrow a \wedge b. \\ a \leftarrow c. \\ b \leftarrow d. \end{array} \right\} \qquad KB \models \neg c \vee \neg d.$$

Questions and Answers in Horn KBs

- An **assumable** is an atom whose negation you are prepared to accept as part of a (disjunctive) answer.
- A **conflict** of *KB* is a set of assumables that, given *KB* imply *false*.
- A **minimal conflict** is a conflict such that no strict subset is also a conflict.

Conflict Example

Example: If $\{c, d, e, f, g, h\}$ are the assumables

$$KB = \left\{ \begin{array}{l} false \leftarrow a \wedge b. \\ a \leftarrow c. \\ b \leftarrow d. \\ b \leftarrow e. \end{array} \right\}$$

- $\{c, d\}$ is a conflict
- $\{c, e\}$ is a conflict
- $\{c, d, e, h\}$ is a conflict

Using Conflicts for Diagnosis

- Assume that the user is able to observe whether a light is lit or dark and whether a power outlet is dead or live.
- A light can't be both lit and dark. An outlet can't be both live and dead:

$$false \leftarrow dark\_l_1 \;\&\; lit\_l_1.$$

$$false \leftarrow dark\_l_2 \;\&\; lit\_l_2.$$

$$false \leftarrow dead\_p_1 \;\&\; live\_p_2.$$

- Assume the individual components are working correctly:

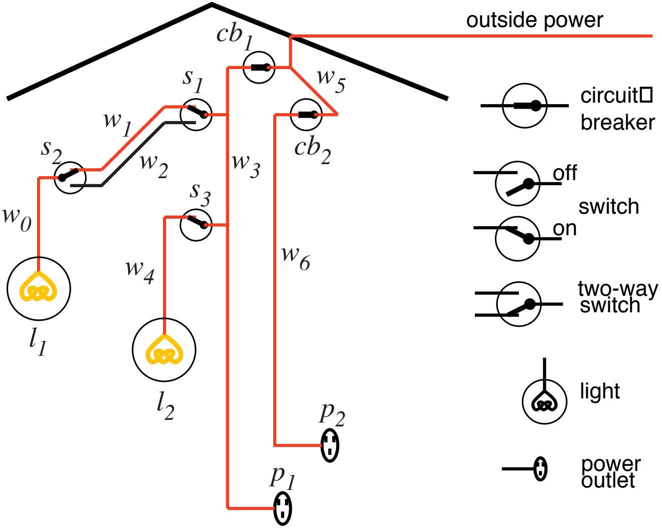
$$assumable\ ok\_l_1.$$

$$assumable\ ok\_s_2.$$

$$\ldots$$

- Suppose switches s_1, s_2, and s_3 are all up:
  $up\_s_1.\ up\_s_2.\ up\_s_3.$

Electrical Environment



Representing the Electrical Environment

$light\_l_1.$

$light\_l_2.$

$up\_s_1.$

$up\_s_2.$

$up\_s_3.$

$live\_outside.$

$lit\_l_1 \leftarrow live\_w_0 \land ok\_l_1.$

$live\_w_0 \leftarrow live\_w_1 \land up\_s_2 \land ok\_s_2.$

$live\_w_0 \leftarrow live\_w_2 \land down\_s_2 \land ok\_s_2.$

$live\_w_1 \leftarrow live\_w_3 \land up\_s_1 \land ok\_s_1.$

$live\_w_2 \leftarrow live\_w_3 \land down\_s_1 \land ok\_s_1.$

$lit\_l_2 \leftarrow live\_w_4 \land ok\_l_2.$

$live\_w_4 \leftarrow live\_w_3 \land up\_s_3 \land ok\_s_3.$

$live\_p_1 \leftarrow live\_w_3.$

$live\_w_3 \leftarrow live\_w_5 \land ok\_cb_1.$

$live\_p_2 \leftarrow live\_w_6.$

$live\_w_6 \leftarrow live\_w_5 \land ok\_cb_2.$

$live\_w_5 \leftarrow live\_outside.$

- If the user has observed l_1 and l_2 are both dark:

$$dark\_l_1.\ dark\_l_2.$$

- There are two minimal conflicts:

  $\{ok\_cb_1, ok\_s_1, ok\_s_2, ok\_l_1\}$ and

  $\{ok\_cb_1, ok\_s_3, ok\_l_2\}$.

- You can derive:

  $\neg ok\_cb_1 \vee \neg ok\_s_1 \vee \neg ok\_s_2 \vee \neg ok\_l_1$

  $\neg ok\_cb_1 \vee \neg ok\_s_3 \vee \neg ok\_l_2$.

- Either cb_1 is broken or there is one of six double faults.

Diagnoses

- A consistency-based diagnosis is a set of assumables that has at least one element in each conflict.
- A minimal diagnosis is a diagnosis such that no subset is also a diagnosis.
- Intuitively, one of the minimal diagnoses must hold. A diagnosis holds if all of its elements are false.
- Example: For the proceeding example there are seven minimal diagnoses: $\{ok\_cb_1\}$, $\{ok\_s_1, ok\_s_3\}$, $\{ok\_s_1, ok\_l_2\}$, $\{ok\_s_2, ok\_s_3\}, \ldots$

Recall: top-down consequence finding

To solve the query $?q_1 \wedge \ldots \wedge q_k$:

> ac := "$yes \leftarrow q_1 \wedge \ldots \wedge q_k$"
> **repeat**
>
> > **select** atom a_i from the body of ac;
> > **choose** clause C from KB with a_i as head;
> > replace a_i in the body of ac by the body of C
>
> **until** ac is an answer.

Implementing conflict finding: top down

- Query is *false*.
- Don't select an atom that is assumable.
- Stop when all of the atoms in the body of the generalised query are assumable:
  - this is a conflict

Example

$false \leftarrow a.$

$a \leftarrow b \,\&\, c.$

$b \leftarrow d.$

$b \leftarrow e.$

$c \leftarrow f.$

$c \leftarrow g.$

$e \leftarrow h \,\&\, w.$

$e \leftarrow g.$

$w \leftarrow f.$

assumable $d, f, g, h.$

Bottom-up Conflict Finding

- Conclusions are pairs $\langle a, A \rangle$, where a is an atom and A is a set of assumables that imply a.
- Initially, conclusion set $C = \{\langle a, \{a\} \rangle : a \text{ is assumable}\}$.
- If there is a rule $h \leftarrow b_1 \wedge \ldots \wedge b_m$ such that for each b_i there is some A_i such that $\langle b_i, A_i \rangle \in C$, then $\langle h, A_1 \cup \ldots \cup A_m \rangle$ can be added to C.
- If $\langle a, A_1 \rangle$ and $\langle a, A_2 \rangle$ are in C, where $A_1 \subset A_2$, then $\langle a, A_2 \rangle$ can be removed from C.
- If $\langle \mathit{false}, A_1 \rangle$ and $\langle a, A_2 \rangle$ are in C, where $A_1 \subseteq A_2$, then $\langle a, A_2 \rangle$ can be removed from C.

Bottom-up Conflict Finding Code

$C := \{\langle a, \{a\}\rangle : a \text{ is assumable }\};$

repeat

    select clause "$h \leftarrow b_1 \wedge \ldots \wedge b_m$" in T such that

        $\langle b_i, A_i\rangle \in C$ for all i and

        there is no $\langle h, A'\rangle \in C$ or $\langle \mathit{false}, A'\rangle \in C$

            such that $A' \subseteq A$ where $A = A_1 \cup \ldots \cup A_m$;

    $C := C \cup \{\langle h, A\rangle\}$

    Remove any elements of C that can now be pruned;

until no more selections are possible