

- A **proof** is a mechanically derivable demonstration that a formula logically follows from a knowledge base.
- Given a proof procedure, $KB \vdash g$ means g can be derived from knowledge base KB .
- Recall $KB \models g$ means g is true in all models of KB .
- A proof procedure is **sound** if $KB \vdash g$ implies $KB \models g$.
- A proof procedure is **complete** if $KB \models g$ implies $KB \vdash g$.

One **rule of derivation**, a generalized form of *modus ponens*:

If " $h \leftarrow b_1 \wedge \dots \wedge b_m$ " is a clause in the knowledge base, and each b_i has been derived, then h can be derived.

This is **forward chaining** on this clause.

(This rule also covers the case when $m = 0$.)

Bottom-up proof procedure

$KB \vdash g$ if $g \in C$ at the end of this procedure:

$C := \{\}$;

repeat

select clause " $h \leftarrow b_1 \wedge \dots \wedge b_m$ " in KB such that

$b_i \in C$ for all i , and

$h \notin C$;

$C := C \cup \{h\}$

until no more clauses can be selected.

Example

$a \leftarrow b \wedge c.$

$a \leftarrow e \wedge f.$

$b \leftarrow f \wedge k.$

$c \leftarrow e.$

$d \leftarrow k.$

$e.$

$f \leftarrow j \wedge e.$

$f \leftarrow c.$

$j \leftarrow c.$

Soundness of bottom-up proof procedure

If $KB \vdash g$ then $KB \models g$.

- Suppose there is a g such that $KB \vdash g$ and $KB \not\models g$.
- Then there must be a first atom added to C that isn't true in every model of KB . Call it h . Suppose h isn't true in model I of KB .
- There must be a clause in KB of form

$$h \leftarrow b_1 \wedge \dots \wedge b_m$$

Each b_i is true in I . h is false in I . So this clause is false in I . Therefore I isn't a model of KB .

- Contradiction.

- The C generated at the end of the bottom-up algorithm is called a **fixed point**.
- Let I be the interpretation in which every element of the fixed point is true and every other atom is false.
- I is a model of KB .
Proof: suppose $h \leftarrow b_1 \wedge \dots \wedge b_m$ in KB is false in I . Then h is false and each b_j is true in I . Thus h can be added to C .
Contradiction to C being the fixed point.
- I is called a **Minimal Model**.

If $KB \models g$ then $KB \vdash g$.

- Suppose $KB \models g$. Then g is true in all models of KB .
- Thus g is true in the minimal model.
- Thus g is in the fixed point.
- Thus g is generated by the bottom up algorithm.
- Thus $KB \vdash g$.