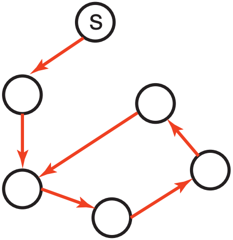
Summary of Search Strategies

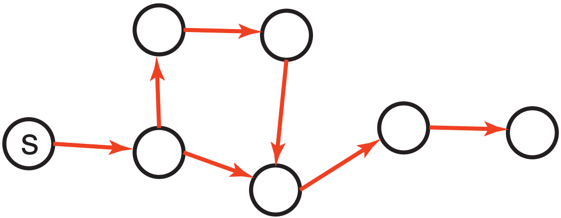
Strategy	Frontier Selection	Halts?	Space
Depth-first	Last node added	No	Linear
Breadth-first	First node added	Yes	Exp
Heuristic depth-first	Local min $h(n)$	No	Linear
Best-first	Global min $h(n)$	No	Exp
Lowest-cost-first	Minimal $cost(n)$	Yes	Exp
A^*	Minimal $f(n)$	Yes	Exp

Cycle Checking



- A searcher can prune a path that ends in a node already on the path, without removing an optimal solution.
- Using depth-first methods, with the graph explicitly stored, this can be done in constant time.
- For other methods, the cost is linear in path length.

Multiple-Path Pruning



- Multiple path pruning: prune a path to node *n* that the searcher has already found a path to.
- Multiple-path pruning subsumes a cycle check.
- This entails storing all nodes it has found paths to.
- Want to guarantee that an optimal solution can still be found.

Multiple-Path Pruning & Optimal Solutions

Problem: what if a subsequent path to n is shorter than the first path to n?

- remove all paths from the frontier that use the longer path.
- change the initial segment of the paths on the frontier to use the shorter path.
- ensure this doesn't happen. Make sure that the shortest path to a node is found first.

Multiple-Path Pruning & A^*

- Suppose path p to n was selected, but there is a shorter path to n. Suppose this shorter path is via path p' on the frontier.
- Suppose path p' ends at node n'.
- $cost(p) + h(n) \leq cost(p') + h(n')$ because p was selected before p'.
- $cost(p') + cost(n', n) < cost(p)$ because the path to n via p' is shorter.

$$cost(n', n) < cost(p) - cost(p') \leq h(n') - h(n).$$

You can ensure this doesn't occur if
$|h(n') - h(n)| \leq cost(n', n)$.

Monotone Restriction

- Heuristic function h satisfies the monotone restriction if $|h(m) - h(n)| \leq cost(m, n)$ for every arc $\langle m, n \rangle$.
- If h satisfies the monotone restriction, A^* with multiple path pruning always finds the shortest path to a goal.

Iterative Deepening

- So far all search strategies that are guaranteed to halt use exponential space.
- Idea: let's recompute elements of the frontier rather than saving them.
- Look for paths of depth 0, then 1, then 2, then 3, etc.
- You need a depth-bounded depth-first searcher.
- If a path cannot be found at depth B, look for a path at depth $B + 1$. Increase the depth-bound when the search fails unnaturally (depth-bound was reached).

Iterative-deepening search

```
Boolean natural_failure;
Procedure dbsearch(⟨n_0, ..., n_k⟩ : path, bound : int):
    if goal(n_k) and bound = 0 report path ⟨n_0, ..., n_k⟩;
    if bound > 0
       for each neighbor n of n_k
                 dbsearch(⟨n_0, ..., n_k, n⟩ , bound − 1);
    else if n_k has a neighbor then natural_failure := false;
end procedure dbsearch;
Procedure idsearch(S : node):
    Integer bound := 0;
    repeat
       natural_failure := true;
       dbsearch(⟨s⟩ , bound);
       bound := bound + 1;
    until natural_failure;
end procedure idsearch
```

Iterative Deepening Complexity

Complexity with solution at depth k & branching factor b:

level	breadth-first	iterative deepening	# nodes
1	1	k	b
2	1	$k-1$	b^2
$k-1$	1	2	b^{k-1}
k	1	1	b^k
	$\geq b^k$	$\leq b^k \left(\frac{b}{b-1}\right)^2$	

Depth-first Branch-and-Bound

- Way to combine depth-first search with heuristic information.
- Finds optimal solution.
- Most useful when there are multiple solutions, and we want an optimal one.
- Uses the space of depth-first search.

Depth-first Branch-and-Bound

- Idea: maintain the cost of the lowest-cost path found to a goal so far, call this *bound*.
- If the search encounters a path p such that $cost(p) + h(p) \geq bound$, path p can be pruned.
- If a non-pruned path to a goal is found, it must be better than the previous best path. This new solution is remembered and *bound* is set to its cost.
- The search can be a depth-first search to save space.

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- The search can be a depth-first search to save space.
- How should the bound be initialized?

Depth-first Branch-and-Bound: Initializing Bound

- The bound can be initialized to ∞.
- The bound can be set to an estimate of the optimal path cost. After depth-first search terminates either:
  - A solution was found.
  - No solution was found, and no path was pruned
  - No solution was found, and a path was pruned.

Direction of Search

- The definition of searching is symmetric: find path from start nodes to goal node or from goal node to start nodes.
- Forward branching factor: number of arcs out of a node.
- Backward branching factor: number of arcs into a node.
- Search complexity is b^n. Should use forward search if forward branching factor is less than backward branching factor, and vice versa.
- Note: sometimes when graph is dynamically constructed, you may not be able to construct the backwards graph.

Bidirectional Search

- You can search backward from the goal and forward from the start simultaneously.
- This wins as $2b^{k/2} \ll b^k$. This can result in an exponential saving in time and space.
- The main problem is making sure the frontiers meet.
- This is often used with one breadth-first method that builds a set of locations that can lead to the goal. In the other direction another method can be used to find a path to these interesting locations.

Island Driven Search

- Idea: find a set of islands between s and g.

$$s \longrightarrow i_1 \longrightarrow i_2 \longrightarrow \ldots \longrightarrow i_{m-1} \longrightarrow g$$

  There are m smaller problems rather than 1 big problem.
- This can win as $mb^{k/m} \ll b^k$.
- The problem is to identify the islands that the path must pass through. It is difficult to guarantee optimality.
- You can solve the subproblems using islands $\Longrightarrow$ hierarchy of abstractions.

Dynamic Programming

Idea: for statically stored graphs, build a table of $dist(n)$ the actual distance of the shortest path from node n to a goal.
This can be built backwards from the goal:

$$dist(n) = \begin{cases} 0 & \text{if } is\_goal(n), \\ \min_{\langle n,m\rangle \in A}(|\langle n, m\rangle| + dist(m)) & \text{otherwise.} \end{cases}$$

This can be used locally to determine what to do.
There are two main problems:

- You need enough space to store the graph.
- The *dist* function needs to be recomputed for each goal.