CPSC 520
Numerical methods for time-dependent differential equations

U. M. Ascher
ascher@cs.ubc.ca
604-822-4907
Fall 2012

This course explores numerical methods based on finite difference and finite volume discretizations for solving time-dependent partial differential equations (PDEs) and ordinary differential equations (ODEs). Such mathematical models arise in many diverse applications, e.g. fluid flow, image processing and computer vision, animation, mechanical systems, earth sciences and mathematical finance. We will methodically develop tools for understanding and assessing methods and software for these problems.

The course will start at a relatively basic level; elementary knowledge of PDEs and numerical methods is a sufficient prerequisite. Talk to me if you are uncertain. Towards the end of the course we will discuss advanced techniques required for problems with nonlinearities, multi-dimensions, interfaces, conservation laws, invariants and discontinuities. Many examples will be given.

Text: *Uri Ascher*, **Numerical Methods for Evolutionary Differential Equations**, SIAM, 2008. For an e-copy, search in http://epubs.siam.org/ebooks/

Course home page: *http://www.cs.ubc.ca/spider/ascher/520.html*

Tentative course outline

1. Initial value PDE problems   [1 week]
   (a) Well-posed initial value problems
   (b) A taste of finite differences
2. Methods and concepts for initial value ODEs [2 weeks +]
   (a) Linear multistep and Runge-Kutta methods
   (b) Stability and stiffness
3. Finite difference and finite volume methods   [1 week +]

   (a) Semi-discretization
   (b) Boundary conditions
   (c) Full discretization

4. Stability for constant coefficient PDEs [1 week -]
   (a) Fourier analysis
   (b) Explicit, implicit, and semi-Lagrangian methods

5. Variable coefficient and nonlinear PDEs [2 weeks]
   (a) Freezing coefficients and dissipativity
   (b) Schemes for hyperbolic systems in one dimension
   (c) Upwind and centered schemes
   (d) Nonlinear stability and "energy methods"
   (e) Lagrangian and semi-Lagrangian methods

6. Hamiltonian systems and long time integration [1 week]
   (a) Hamiltonian systems
   (b) Symplectic methods

7. Dispersion and dissipation [1 week]
   (a) Dispersion
   (b) The classical wave equation
   (c) Nonlinear examples

8. More on handling boundary conditions [1 week -]
   (a) BC for hyperbolic problems
   (b) Infinite or large domains

9. Problems in several space variables and splitting methods [2 weeks]
   (a) Extending the methods we already know
   (b) Solving large algebraic systems for implicit methods
   (c) Splitting methods