CPSC 510 – multigrid and multilevel methods

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This course explores numerical methods based on multi-level resolution for solving large, sparse systems with an appropriate local structure. Such systems arise when solving partial differential equations. Practical and theoretical aspects will be investigated and many examples given.

http://www.cs.ubc.ca/spider/ascher/510.html

Tentative course outline

1. Preliminaries I
   (a) Elliptic equations
   (b) Discretizations
2. Preliminaries II
   (a) Relaxation methods for large, sparse linear systems
3. Defect correction and multigrid
   (a) Defect correction
   (b) Coarse grid defect correction
   (c) Two-grid and multigrid methods
4. Fundamental algorithms
   (a) Complete multigrid cycle
   (b) Nonlinear multigrid methods
   (c) Nested iteration (full multigrid)
5. Multigrid analysis

   (a) A simple two-grid method for the model problem
   (b) An $O(n)$ algorithm
   (c) Local mode analysis

6. Design of a multigrid algorithm
   (a) Choice of smoothing relaxation
   (b) Choice of coarse grid correction operator
   (c) Choice of prolongation and restriction

7. Additional topics. Possible topics include
   (a) Algebraic multigrid
   (b) Singular perturbation
   (c) Multi-level preconditioning
   (d) Hierarchical bases
   (e) Multigrid in CFD
   (f) Multigrid in optimization
   (g) Time-dependent problems
   (h) Domain decomposition
   (i) Wavelets

References: Texts: [11, 2]; Simple introduction: [4]; Classics: [5, 7, 3]; Adaptive: [8, 10]; Also: [1, 9, 6].

Web resource: *http://casper.cs.yale.edu/mgnet/www/mgnet.html*

References

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