CPSC 303 Midterm Solution

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Question 1

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Question 2

The condition that $g[x_0, x_1, x_2] \equiv 1$ means that $g(x)$ is a quadratic with $g''(x) = 2,\ \forall x$. Using also $g(0) = 1$, we can write $g(x) = 1 + c_1 x + x^2$. Now the remaining condition $g(1) = 0$ implies $c_1 = -2$, so

$$g(x) = 1 - 2x + x^2 = (1 - x)^2.$$

Question 3

(a) A quartic won't work, because 5 smoothness conditions at a knot mean that this is not a knot for a quartic (i.e., the two quartic pieces form one polynomial across such knot). So we need a quintic piecewise polynomial, $m = 5$. For $x_i \le x < x_{i+1}$ we then have

$$v(x) = s_i(x) = a_i + b_i(x-x_i) + c_i(x-x_i)^2 + d_i(x-x_i)^3 + e_i(x-x_i)^4 + f_i(x-x_i)^5.$$

In total there are therefore $6r$ corefficients to determine.

(b) We spend as usual $2r$ conditions just to obtain a continuous interpolant. Then there are the smoothness requirements that $s'_{i-1}(x_i) = s'_i(x_i),\ s''_{i-1}(x_i) = s''_i(x_i),\ s'''_{i-1}(x_i) = s'''_i(x_i),\ s''''_{i-1}(x_i) = s''''_i(x_i)$, for $i = 1, 2, \ldots, r-1$. These are $4(r-1)$ conditions, so in toal there are $6r - 4$ conditions to satisfy, which means that there are $6r - (6r - 4) = 4$ free parameters.

(c) Just like for the complete cubic spline, we use the values of the first derivatives at each end. Here we need 4 additional conditions, so match $v'(a) = q'(a),\ v''(a) = q''(a)$ at the left end and $v'(b) = q'(b),\ v''(b) = q''(b)$ at the right end.

Question 4

Obviouslty there is no guarantee that q_2 can be represented as a uni-valued function of q_1. Rather, a parametric curve is required. Here it is natural to use time t as the parameter τ. Thus, interpolate for $q_1(t)$ and $q_2(t)$ separately, obtaining say $v_1(t)$ and $v_2(t)$, respectively, and form the phase plane plot $q_1(t) \times q_1(t),\ a \leq t \leq b$.

Question 5

(a) It's the third option (iii).

(b) The extremum (max and min) values are ± 1, so it can't be Legendre. Furthermore, the spacing of the roots in x is uniform, so it can't be Chebyshev either.