Department of Computer Science
CPSC 303 Midterm Examination (U. Ascher)

Oct. 25, 2010. NAME:_____________________________________

Number of pages: 4 Signature:_____________________________________

Time: 50 minutes STD. NUM: _____________________________________

You are permitted one double-sided sheet of notes to assist you in answering the questions.

For the short answer questions, be as concise as possible. The weight of each question is given in parentheses. The total number of marks is 100 (approximately 2 marks/min). Show all your work.

Good Luck!

Q1	Q2	Q3	Q4	TOTAL
15	20	35	30	100

---

1. [**15 marks**] Please circle **TRUE** or **FALSE** as appropriate.

   (a) All monic polynomials $p(x)$ of degree n satisfy $\max_{x\in[-1,1]} |p(x)| \geq 2^{1-n}$.

   TRUE / FALSE

   (b) Least squares data fitting is more common than minimax data fitting because the latter concentrates on the wrong data points.

   TRUE / FALSE

   (c) Cancellation error may magnify roundoff error propagation and can occasionally be avoided by using a different but mathematically equivalent formula.

   TRUE / FALSE

(d) Piecewise cubic Hermite interpolation is usually both smoother and more accurate than piecewise linear interpolation.

TRUE / FALSE

(e) Among monomial, Lagrange and Newton forms the Lagrange base is the most suitable for Chebyshev interpolation.

TRUE / FALSE

2. A secret function was interpolated by a piecewise polynomial at uniformly spaced points with spacing h. Maximum errors were recorded and the following table was obtained.

h	Error
.1	9.81e-6
.01	9.75e-10
.001	9.78e-14
.0001	2.54e-14

(a) **[10 marks]** It is known that the interpolant was either a “broken line” or a cubic spline. Based on the results in the table, which was it? Justify briefly.

(b) **[10 marks]** Which of the recorded error values is dominated by discretization error and which by roundoff error? Justify briefly.

3. Given $n+1$ data pairs $(x_0, y_0), (x_1, y_1), \ldots, (x_n, y_n)$, define for $j = 0, 1, \ldots, n$

$$\rho_j = \prod_{i \neq j} (x_j - x_i).$$

Also, let

$$\psi(x) = \prod_{i=0}^{n} (x - x_i).$$

The interpolating polynomial in Lagrange form is then given by

$$p_n(x) = \psi(x) \sum_{j=0}^{n} \frac{y_j}{(x - x_j)\rho_j}.$$

(a) **[10 marks]** Show that

$$\psi(x) = 1 \ / \ \sum_{j=0}^{n} \frac{1}{(x - x_j)\rho_j}.$$

[Hint: what polynomial has the value 1 at all abscissae x_j?]

(b) **[10 marks]** Assuming (a), conclude that

$$p_n(x) = \sum_{j=0}^{n} \frac{y_j}{(x - x_j)\rho_j} \ / \ \sum_{j=0}^{n} \frac{1}{(x - x_j)\rho_j}.$$

[This formula uses the so-called barycentric coordinates of $p_n(x)$.]

(c) **[15 marks]** Use the barycentric formula of Part (b) to evaluate the straight line interpolating the points $(0, 1)$ and $(1, 0)$ at $x = 0.25$. Show your derivation.

4. The Taylor expansion $f(x_0 + h) = f(x_0) + hf'(x_0) + \mathcal{O}(h^2)$ leads, as we saw in class, to the 1st order (i.e., $\mathcal{O}(h)$ discretization error) formula $\frac{f(x_0+h)-f(x_0)}{h}$ for approximating $f'(x_0)$, which experiences significant roundoff error problems for $0 < h \ll 1$.

   (a) **[bonus 10 marks]** Suppose that the function $f(z)$ is infinitely smooth on the complex plane and $f(z)$ is real when z is real. Assume that x_0 is real and denote $\imath = \sqrt{-1}$. Show by a Taylor expansion of $f(x_0 + \imath h)$ about x_0 that

$$f'(x_0) = \Im[f(x_0 + \imath h)]/h + \mathcal{O}(h^2).$$

   (b) **[15 marks]** Give two reasons why the approximation $\Im[f(x_0 + \imath h)]/h$ is better than $\frac{f(x_0+h)-f(x_0)}{h}$. Here, $\Im(a)$ is the imaginary part of the complex number a.

   (c) **[15 marks]** Give two reasons why the approximation $\Im[f(x_0 + \imath h)]/h$ cannot always be used where $\frac{f(x_0+h)-f(x_0)}{h}$ can.