SCALE-BASED DESCRIPTIONS OF PLANAR CURVES

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Abstract

The problem posed in this paper is the description of planar curves at varying levels of detail. Five necessary conditions are imposed on any candidate solution method. Two candidate methods are rejected. A new method that uses well-known Gaussian smoothing techniques but applies them in a path-based coordinate system is described. By smoothing with respect to a path-length parameter the difficulties of other methods are overcome. An example shows how the method extracts the major features of a curve, at varying levels of detail, based on segmentation at zeroes of the curvature, κ. The method satisfies the five necessary criteria.

1. The Problem: Detail and Scale

Achieving a proper notion of detail in a domain is a prerequisite for the construction of useful descriptions of domain elements. Such descriptions allow, for example, efficient coarse-to-fine matching. In vision, the problem of image detail is often reduced to the problem of scale. One approach to that problem extracts the locations of zero-crossings in the second derivative of the Gaussian-smoothed signal, varying the width of the Gaussian kernel to obtain multiple descriptions of the signal. This method has been used to extract "edge" elements at different spatial frequencies in an image intensity function I(x,y) of two independent variables [1]. It has also been used to perform automatic peak selection in histograms [2] and generalized to extract a new description, the scale space image, of signals that are functions of one variable [3].

Here, we are concerned with the problem of detail at a higher level in the visual system for the description of edges and other contours rather than for the extraction of edge locations from sensory data. We pose the problem of scale-based descriptions of planar curves. In our cooperative interpretation project [4], we are faced with the task of, for example, matching shorelines, roads and rivers extracted from aerial and satellite imagery, at varying scales, and from sketched maps. To do this successfully for a shoreline, say, we want to extract scale-based descriptions of it as an alternating sequence of headlands and bays.

2. Necessary Conditions on Any Method

. In artificial intelligence, we often settle for sufficiency conditions, simply finding a method that will do the job. A more powerful methodology specifies criteria that *any* adequate solution method must satisfy. Here, we propose five such criteria.

Criterion 1 The method must be computational, preferably using local support techniques.

Criterion 2 The method must produce essentially the same result regardless of the coordinate system imposed on the curve. This implies that the descriptions must be well-behaved under rotation, translation, reflection and uniform expansion of the coordinate system (or the curve itself).

Criterion 3 It must not be ill-conditioned. Small changes in the curve should not cause large changes in its descriptions.

Criterion 4 The descriptions should correspond to human performance on the task.

Criterion 5 The method must not require arbitrary choices that affect the descriptions.

These criteria may not all be easy to justify or trivial to verify; however, if accepted, they impose stringent requirements on the class of all acceptable methods.

Our first candidate method was based on the detail hierarchy for curves used in the Mapsee project since its origin [5,6]. That hierarchy is a binary tree of straight line approximations to the curve. The initial approximation joins the end points. Subsequent approximations recursively refine the initial approximation by breaking the approximation at the point on the curve farthest from the straight line joining its end points.

This method violates criterion 3. If two points are roughly equidistant outliers from the current approximation then a small movement of one of them could cause a large change in the description. Criterion 5 is also violated. If the curve is closed then a purely arbitrary choice of end points is required which has a drastic effect on the description.

A second candidate method considers the curve to be a function of one variable y(x). If that function is multivalued, break it into several piecewise single-valued functions. Then apply Witkin's techniques [3] to y(x) to extract smoothed functions and mark the points of inflection. The problems with that method would include the handling of the boundary conditions at the end of each break in the curve. Even if those serious problems were solved the method would still not satisfy criterion 2. For example, after a reflection, of the coordinate system (or the curve) through the line $y = x$ the method would not produce essentially the same result. Smoothing x(y) with respect to y is quite different from smoothing y(x) with respect to x. Similar arguments apply to rotation transformations.

These considerations suggest using a description based on curvature [7,8] but one that is elaborated to analyze the curve at varying levels of detail in scale space.

3. A Method

To satisfy criterion 2, we have to use a path-based coordinate system for a curve C. Consider the parameterization

$$C = \{(x(t), y(t)) \mid t \in [0,1]\}$$

In this section we consider only closed curves so

$$x(0) = x(1) \quad \text{and} \quad y(0) = y(1).$$

The parameter t is a linear function of s, the path length along the curve from (x(0),y(0)), scaled to range over [0,1]. x(t) and y(t) may be considered defined over (-∞,∞) with periodic behaviour:

$$x(t+1) = x(t) \quad \text{and} \quad y(t+1) = y(t)$$

The method requires smoothing the functions x(t) and y(t) by convolution with a Gaussian kernel of width σ.

$$\text{Define } X(t,\sigma) = x(t) \circledast g(t,\sigma) = \int_{-\infty}^{\infty} x(u) \frac{1}{\sigma\sqrt{2\pi}} e^{\frac{-(t-u)^2}{2\sigma^2}} du$$

and define $Y(t,\sigma)$ similarly.

The smoothed curve C_σ is simply:

$$C_\sigma = \{(X(t,\sigma), Y(t,\sigma)) \mid t \in [0,1]\}.$$

Notice that $C = C_0 = \lim_{\sigma \to 0} C_\sigma$.

The points of particular interest on C_σ are the points of inflection. The curvature κ of a planar curve at a point on the curve is the inverse of the radius of curvature of an osculating circle tangent at that point with its sign indicating the direction of curvature. Define

$$y' = \frac{dy}{dx} \qquad y'' = \frac{d^2y}{dx^2}$$

Then

$$\kappa = \frac{y''}{(1+(y')^2)^{3/2}}$$

The zeroes of κ are of interest. Since to obtain C_σ we are smoothing x(t) and y(t), it is cumbersome to return to the image domain to compute $y'(x)$ and $y''(x)$, in order to compute κ; moreover, there would be difficulties when $y' \to \infty$ or $y'' \to \infty$ and when y(x) is multivalued. Accordingly we wish to express κ purely as a function of derivatives of x(t) and y(t). Those derivatives can then be computed directly using appropriate masks. Define

$$\dot{x} = \frac{dx}{dt} \qquad \ddot{x} = \frac{d^2x}{dt^2}$$

$$\dot{y} = \frac{dy}{dt} \qquad \ddot{y} = \frac{d^2y}{dt^2}$$

Then

$$y'(x) = \frac{\dot{y}}{\dot{x}}$$

$$y''(x) = \frac{dy'}{dx} = \frac{\frac{d}{dt}\left(\frac{\dot{y}}{\dot{x}}\right)}{\dot{x}} = \frac{\dot{x}\,\ddot{y} - \ddot{x}\,\dot{y}}{\dot{x}^3}$$

and

$$\kappa = \frac{\dot{x}\,\ddot{y} - \dot{y}\,\ddot{x}}{(\dot{x}^2+\dot{y}^2)^{3/2}}$$

In the smoothed curve $\dot{X}(t,\sigma) = \frac{\partial X(t,\sigma)}{\partial t}$, $\dot{Y}(t,\sigma)$, $\ddot{X}(t,\sigma)$ and $\ddot{Y}(t,\sigma)$ are needed to compute $\kappa(t,\sigma)$. They can be obtained directly from x(t) and y(t) using,

$$\dot{X}(t,\sigma) = \frac{\partial X(t,\sigma)}{\partial t} = \frac{\partial[x(t) \circledast g(t,\sigma)]}{\partial t} = x(t) \circledast \left[\frac{\partial g(t,\sigma)}{\partial t}\right]$$

and

$$\ddot{X}(t,\sigma) = \frac{\partial^2 X}{\partial t^2} = x(t) \circledast \left[\frac{\partial^2 g(t,\sigma)}{\partial t^2}\right]$$

and similarly for $Y(t,\sigma)$. Using these equations $\kappa(t,\sigma)$ may be computed directly from convolutions performed on x(t) and y(t).

Several remarks about the method are appropriate. First, notice that for closed curves, treating x(t) and y(t) as periodic eliminates all edge effects. Second, if C is closed then the choice of the point on C at which $t = 0$ is purely arbitrary but has no effect on the description in terms of zeroes of κ or the smoothed curve. Third, the use of a path-based parameterization of the curve gives the desired invariance with respect to rotation, translation, reflection and uniform scale change of the curve. The curvature κ is invariant under rotation, translation and reflection. If the curve is scaled by a factor w then $\kappa' = \frac{1}{w}\kappa$. In particular the shape of the smoothed curve and the relative locations of the zeroes of κ will be invariant. One way to see this is to realize that linear coordinate transforms commute with linear smoothing operations. Fourth, small changes in the original curve may perturb the zero-crossing description for small σ but for larger values of σ their effects will disappear.

4. An Example

This method is applied to the coastline of Africa in Figure 1 for successively doubled values of σ. Beside each C_σ the functions $X(t,\sigma)$, $Y(t,\sigma)$ and $\kappa(t,\sigma)$ are displayed. The domain of t, the interval [0,1], has been divided into 1024 equally-sized subintervals for this experiment. The values of σ are given in terms of the number of subintervals. The locations at which $\kappa = 0$ are marked on each curve. As $\sigma \rightarrow \infty$ the curve asymptotically approaches its centre of mass. Notice also that as σ becomes larger the major headlands and bays emerge as dominant. At this point we can only appeal to the reader's intuitions to justify the claim that the results correspond to human performance.

5. Extensions

We have only discussed the application to closed curves so far. However, in our application, curves do not always close. They may also have free ends and junctions, or they may extend beyond the bounds of the map or satellite image - the frame problem. The only difficulty in extending our method to these curves lies in specifying the correct boundary conditions. Extensions to space curves and surfaces in higher dimensionality spaces should be pursued.

6. Conclusion

We have posed a problem of scale-based description of planar curves, proposed five criteria to judge any solution method and described a method that satisfies those criteria.

7. Acknowledgements

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8. References

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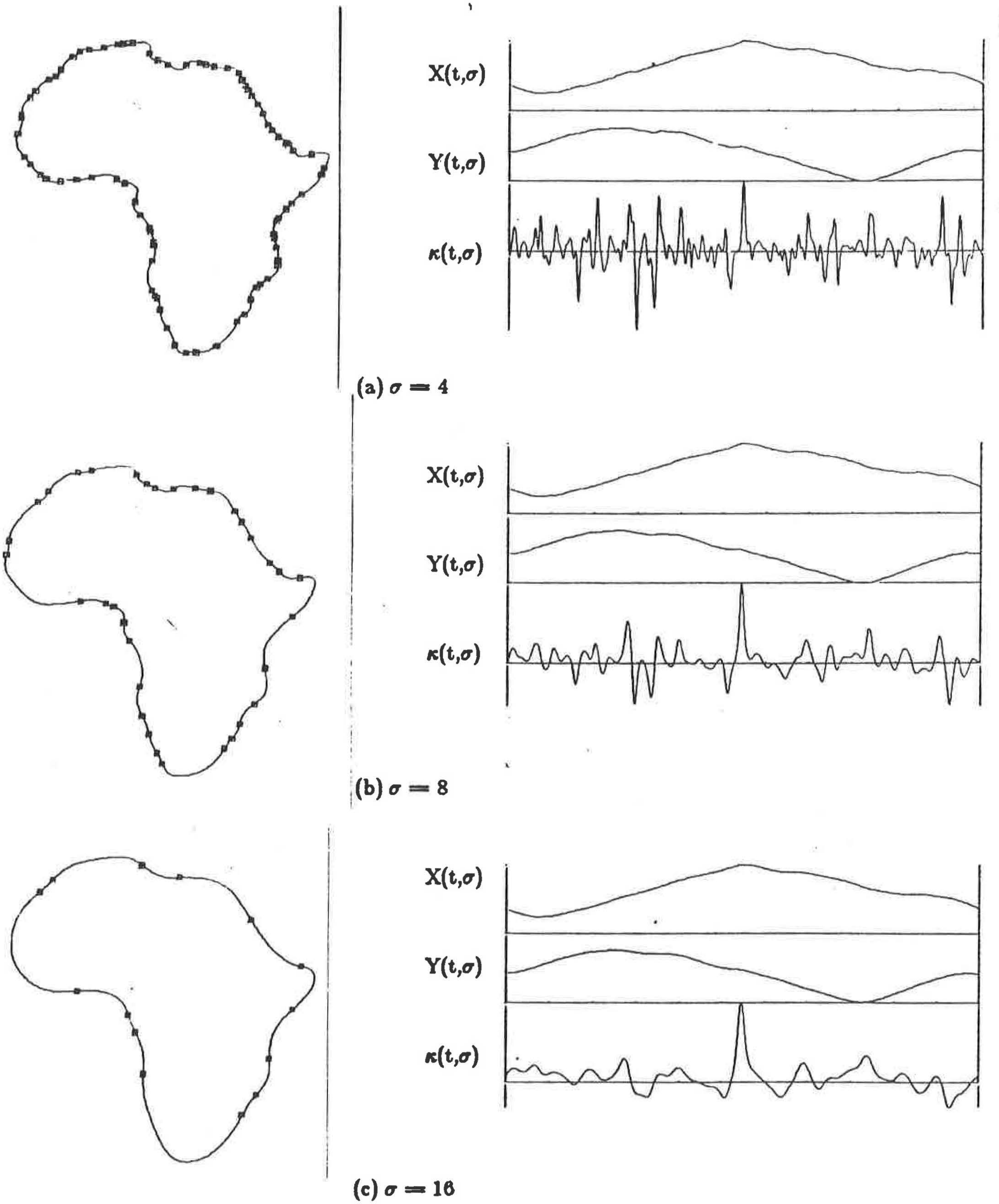
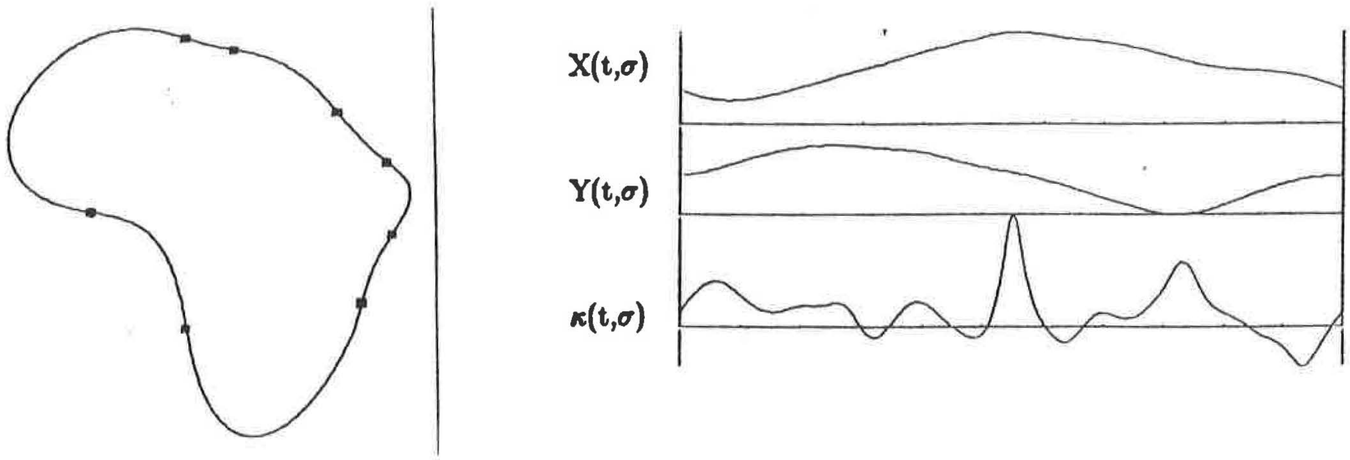
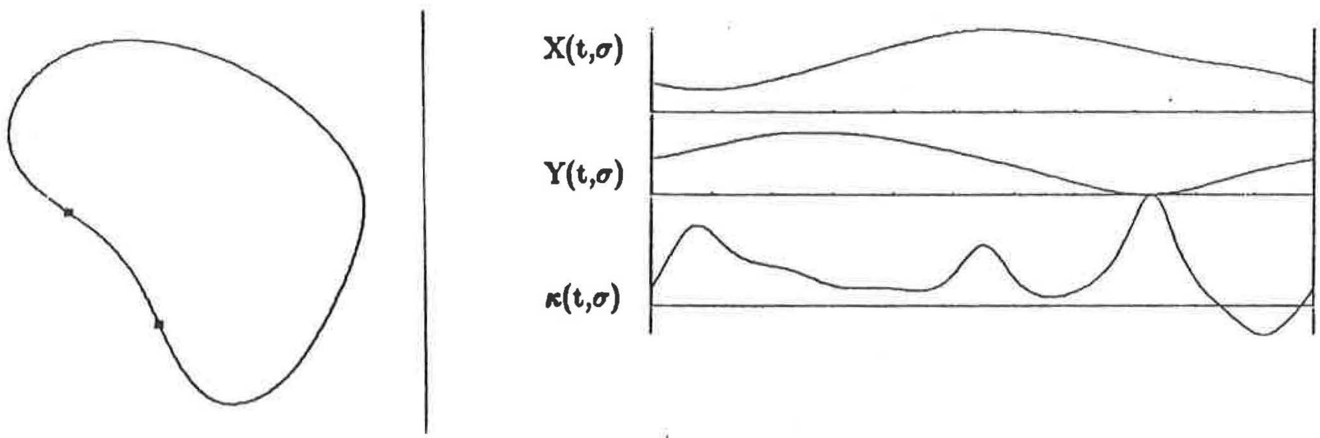


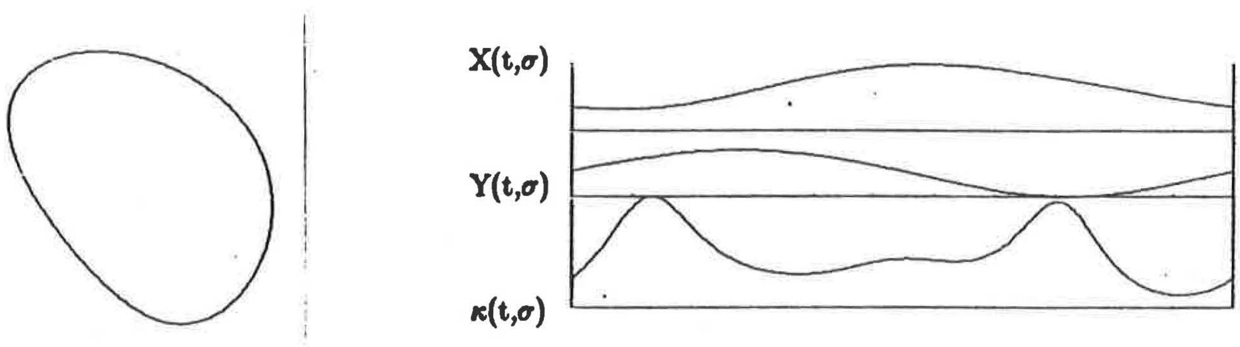
Figure 1. Smoothing a Curve: Scale-based Effects



(d) $\sigma = 32$



(e) $\sigma = 64$



(f) $\sigma = 128$

Figure 1. (Continued) Smoothing a Curve: Scale-based Effects