Type Uncertainty in Ontologically-Grounded Qualitative Probabilistic Matching

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Abstract. This paper is part of a project to match real-world descriptions of instances of objects to models of objects. We use a rich ontology to describe instances and models at multiple levels of detail and multiple levels of abstraction. The models are described using qualitative probabilities. This paper is about the problem of type uncertainty; what if we have a qualitative distribution over the types. For example allowing a model to specify that a meeting is always scheduled in a building, usually in a civic building, and never a shopping mall can help an agent find a meeting even if it is unsure about the address.

1 Introduction

In a recent paper [1], we described a system for matching instances and models of realworld phenomena. These instances and models have been described by different people using controlled vocabularies (using an ontology) which allow descriptions of model and instances at varied levels of abstraction (using more general or less general terms) and detail (describing objects in terms of parts and subparts or not).

In one practical domain, geological surveys of various countries or provinces publish descriptions of mineral occurrences (e.g., twelve thousand in British Columbia) at widely varying levels of abstraction and detail. Other people spend careers developing models of archetypical mineral deposits that can help determine where certain minerals can be found³. These models are often at different levels of abstraction and detail from the mineral occurrence descriptions. The problem that we consider is to determine which mineral deposits fit which models, so that explorers can focus their exploration. This is a case where humans have to make decisions, but they are overwhelmed by the combinatorics. The aim of the computer system is to find the best fitting models to a mineral occurrence or to find the best fitting mineral occurrences to a model. As it is humans who are making the decisions, it is more important to have good explanation facilities and to return multiple potential matches than to return the "best" match according to the computer. Computers help by narrowing down the search space and explaining and justifying the potential matches.

³ See http://www.yukonmineraltargets.com/ for examples using about 2000 mineral occurrences ("anomaly clusters") in the Yukon, matched against 80 models. It shows the best matching instances for each model and the best matching models for each instance.

The main problems are the integration of (qualitative) probabilities with rich ontologies and reasoning about multiple objects with multiple subparts. For this paper we'll use the OWL [2] notation where appropriate.

In previous work [1], we make the assumption that different descriptions refer to different objects. This assumption is relaxed in this paper. In particular, we allow for uncertainty in the types and allow for qualitative distributions over hierarchically structured classes. By type we mean membership in a class, where the classes are organised hierarchically and are specified as part of an ontology.

This work is quite different to other work on combining probability and ontologies (e.g., P-Classic [3]) because we are using the ontologies to construct a rich hypothesis space rather than (or as well as) having probabilities over subclasses in the ontologies.

2 Qualitative Probabilistic Matching

The general problem is, given a model and an instance, to determine a qualitative value for P(model|instance) that can be used by a human to make decisions. This section gives an overview of our previous paper [1].

We decided to use qualitative order-of-magnitude probabilities based on the kappa calculus [4, 5]. The kappa calculus can be described in terms of surprise; the kappa values correspond to the level of surprise. When probabilities multiply, the corresponding kappa values add, and summing in probability corresponds to minimisation in the kappa calculus.

The kappa calculus can be seen as a crude approximation to probability [6]. This is useful when the probabilities are not available as it gives a rough answer and leaves only a few of the possible matches for a human to evaluate; the implausible matches can be ignored by the human. Note that we use the kappa calculus for the first-level of approximation; we use some finer distinctions to distinguish matches that may have the same values in the kappa calculus. These are described when used.

2.1 Ontologies

We assume that we have an ontology that specifies the vocabulary and partly specifies the meaning of some of the terms (in terms of how some of the terms relate to each other and some restrictions on allowed values). For this paper, we assume that we are given an ontology that specifies:

- a class hierarchy. Formally a class is a set. The top class is "Thing". We use subClassOf to denote the subclass relation. We assume that the class hierarchy is a tree; siblings in the class hierarchy form disjoint sets of individuals.
- a property hierarchy. Like OWL-DL, we assume that objects, classes and properties are distinct. Each property is either a *datatype* property or an *object* property.
- domains and ranges for properties.
- declarations that properties are functional.

Example 1. As part of the ontology, we can specify a class hierarchy, part of which may be written as tuples:

 $\begin{array}{l} \langle boardroom, subClassOf, meeting_room \rangle \\ \langle meeting_room, subClassOf, room \rangle \\ \langle room, subClassOf, enclosed_space \rangle \end{array}$

We can also specify properties and property attributes, such as:

 $\langle held_in, subPropertyOf, has_location_in \rangle$ $\langle held_in, domain, meeting \rangle$ $\langle held_in, range, room \rangle$

2.2 Describing Instances

Instances are things in the world that are described using the vocabulary of the ontology. Instances are given (internal) names.

We make the open world assumption: we do not assume that we are told everything about an individual, and so do not conclude anything from the lack of information. If we want to say that something is not true, we need to say explicitly that its value is absent, or say that there are no other values. Instances have property values that are marked as "present" or "absent".

We use the term *object* to denote both an instance object and model object. The room at the North-East Corner of 123 Pretty St, Vancouver, may be an instance of a kitchen. A room in the model of houses that Joe likes may be a model of a kitchen.

Instances are described using tuples of the form:

 $\langle Object, Property, Value, TruthStatus, Reference, Comment, EnteredBy \rangle$

where *Frequency* is either *present* or *absent*. *Reference*, *Comment* and *EnteredBy* specify the source of the information and human-readable comments. The rest of this paper ignores these fields, but it is important for us to attribute the source of all knowledge.

If *Property* is a datatype property, *Value* is a primitive data type (such as a string or a number or a pair of numbers representing a range). If *Property* is an object property, *Value* is (a reference to) an object.

Example 2. Assume we are using the ontology partly specified in Example 1. Consider the following instance of a meeting which we will call m_1 :

 $\langle m_1, held_in, rmCS123, present \rangle$ $\langle m_1, organised_by, david, present \rangle$ $\langle m_1, start_time, 5: 00pm, present \rangle$ $\langle m_1, attended_by, bill, absent \rangle$

This meeting refers to a room rmCS123, which can similarly be described:

 $\begin{array}{l} \langle rmCS123, type, boardroom, present \rangle \\ \langle rmCS123, located_in, CSbuilding, present \rangle \\ \langle rmCS123, capacity, 50, present \rangle \\ \langle rmCS123, contains, lectern, absent \rangle \end{array}$

2.3 Describing Models

Models are specified using tuples of the form:

 $\langle Object, Property, Value, Frequency, Reference, Comment, Entered By \rangle$

where the attributes are as before, and *Frequency* specifies the qualitative probability that the *Object* will have *Value* for *Property* in the current model. After feedback from domain experts, we use a 5-value frequency scale⁴ to describe models. Suppose p represents the proposition "*Object* has *Value* for *Property*", the frequency is one of:

- *always*: you are very surprised if p is false⁵.
- *usually*: you are somewhat surprised if p is false.
- sometimes: you aren't surprised if p is true or if it's false
- rarely: you are somewhat surprised if p is true.
- *never*: you are very surprised if p is true.

These provide a language for *inputting* uncertainty. We output a numerical value in the range [0,100] where 100 is the score for the best possible match and 0 is for the worst possible match. Internally we use a reasonably arbitrary numerical scale.

Example 3. We can specify a model of a particular type of meeting that we should attend. It is usually in a boardroom, always organised by an administrator, usually attended by a department head. This can be specified using tuples:

 $\langle ImportantMeeting, held_in, RoomOfImportantMeeting, usually \rangle \\ \langle RoomOfImportantMeeting, type, boardroom, always \rangle \\ \langle ImportantMeeting, organised_by, OrganizerOfImportantMeeting, always \rangle \\ \langle OrganizerOfImportantMeeting, type, administrator, always \rangle \\ \langle ImportantMeeting, attended_by, ADepartmentHead, usually \rangle \\ \langle ADepartmentHead, type, department_head, always \rangle$

2.4 Matching

If not for different levels of abstraction and different levels of detail, to compute the qualitative counterpart of P(instance|model), we add the *surprises* of the instance with respect to the qualitative probabilities specified in the model. The main contribution of [1] was to show how the kappa calculus could be combined with rich ontologies that let us describe models and instances at various levels of abstraction and detail.

As we are adding surprises, and returning the topmost (least surprising) match, the zero point is arbitrary. We can define zero to be the level of the empty $match^{6}$ (i.e., a

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⁴ None of the theory or results in this paper depends on using this scale, but we will use it in all of our examples. In practice, we have found that experts are happy using this scale, and find it very natural.

⁵ This is *not* the always of modal logic. Even though our experts described things as "always" true, we allow for exceptions due to errors in descriptions.

⁶ This is for the case of the open-world assumption, where we don't assume that a complete description is given. We do allow someone to specify that "silence implies absence"; that a part or property that is not described is false. In this case an empty description does not have value zero. It is positive as we expect nothing else and found nothing else.

match with an empty description), then we have positive rewards when there is a better match than this and negative rewards (penalties) for those matches that are worse than this.

For each model qualitative probability and for each instance value "present" or "absent", we will have a numerical reward or penalty. Thus, for example, we will talk about the always-present reward (which gives the reward received when a model property that has qualitative probability "always" matches an corresponding instance property that is present) or a usually-absent penalty (when the model property is "usually" present, but it is absent in the instance). For example, if a model specifies a room that is always a bedroom and usually pink and we have an instance that is a bedroom that is not pink (i.e., bedroom is present and pink is absent), we get both the always-present reward and the usually-absent penalty.

Given an abstraction hierarchy of classes, it is important to distinguish the description of an instance from the instance itself. For example, if something is described as a building, it must be some sort of building (generic buildings don't exist). One of the differences between an instance and a model is, when given a general concept, such as "building", in an instance we *don't know* what sort of building it is, but if the same term is used in a model, we *don't care* what sort of building it is [1]. When we want the probability of an instance, we don't want the probability of the description. The probability of a more abstract description is more likely than the that of a more specific instance. For or example, a house is a kind of building, so for any evidence e, P(building|e) > P(house|e). However if the model specifies a house is always present, and instance 1 is described as a building and instance 2 is described as a house, then instance 2 definitely fits the model, but it is less likely that instance 1 fits the model.

Our previous paper [1] made two assumptions that we relax in this paper:

- The type of objects was known.
- In a single description, different descriptions of parts (or other property values) pertain to different parts (or property values).

3 Type Uncertainty

In many real examples, we may have uncertainty about the type of an object. We would like to have a qualitative distribution over which class an object is a member of. For example, the model may specify that a place that can take the role of a home office is always a room, usually a bedroom and rarely a master bedroom.

We also need to distinguish between one object satisfying multiple type restrictions and multiple objects satisfying them. Consider the following two contrasting examples:

Example 4. Suppose we had a model of "a room that Sam likes" that says the room is "usually red and never pink". This could mean either

- it has a single colour that is usually a non-pink shade of red, or
- there could be multiple colours; as long as one is red and one is non-pink they would be happy. So a blue and pink would be good; they just don't want all-pink or no shade of red.

In the first case, the description is referring to a single colour and in the second to multiple colours.

Example 5. Suppose we have a model for a house that always contains a bathroom and always contains a room that is not a bathroom. In this case, we don't want this to refer to the same room (there is no room that is both a bathroom and is not a bathroom), but rather to two rooms.

We need a way to specify we are referring to a single colour or room or to multiple colours or rooms.

In summary, we need a representation to specify what objects are assumed to exist and what type distributions are over the objects. The same issues arise when specifying functional properties (as in colour above). We motivate the problems in terms of types, but then treat type as a (functional) property in the algorithm.

In order to understand the issues, we will give a detailed example of a specific example of matching the type description of an instance and a type description of model type. This example will be used a prototype for the general case.

Example 6. Consider the class hierarchy of Figure 1. Some of the relations that are true

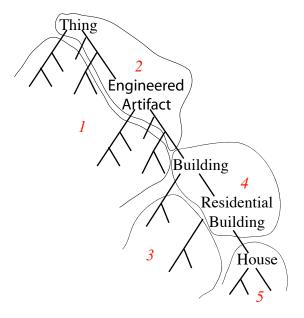


Fig. 1. A class hierarchy. Different exceptional regions for model M_1 are numbered (see Example 6).

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include:

```
\langle House, subClassOf, ResidentialBuilding \rangle
\langle ResidentialBuilding, subClassOf, Building \rangle
\langle Building, subClassOf, EngineeredArtifact \rangle
```

For this discussion, we do not intend that these are immediate subclasses. There may be many intermediate classes (e.g., that are classes that are subclasses of "Engineered Artifact" and super classes of "Building"). At the root of the tree is *Thing* which is the topmost class.

Suppose a model that specifies that the location of some activity is:

always a Building and rarely a House.

Call this model M_1 .

Given a simple description of an instance where we only give a single class that is present, let's determine how surprised we are that the location is at that instance. The description could be any position in the taxonomy, and the instance could be any leaf that is a descendant of the class that is said to be present. The figure shows the five qualitatively different regions of the taxonomy that the description of an instance could be in:

- Region 1. If the description is in region 1, the cousins of *Building* (the values in the same tree that are neither descendants nor ancestors of *Building*), the instance is not a *Building*.
- Region 2. The description is an ancestor of *Building* so the instance is perhaps a *Building*, but could be a non-building.
- Region 3. The instance is a *Building* and not a *House*.
- Region 4. The instance is a *Building* and perhaps a *House*.
- Region 5. The instance is a House.

Consider how surprised we would be that an object in each of these locations would be an instance of the model "*always* a *Building* and *rarely* a *House*".

If the instance description is in region 1, the instance would receive the alwaysabsent penalty. The model says the instance is always a building and the instance is not a building.

Suppose description d_2 of the instance is in region 2, for example it is the description *Engineered Artifact*. We don't know if the instance is a *Building* or not. To understand the qualitative probability, consider the probability of the model M_1 given the description d_2 :

$$P(M_1|d_2) = P(M_1|Building \land d_2) * P(Building|d_2)$$
$$+ P(M_1|\neg Building \land d_2) * P(\neg Building|d_2)$$

 $P(M_1|Building \wedge d_2) = P(M_1|Building)$ as $Building \wedge d_2$ is logically equivalent to *Building*. $P(M_1|\neg Building \wedge d_2) = P(M_1|\neg Building)$ as the model doesn't specify anything more general that *Building*. Thus

$$P(M_1|d_2) = P(M_1|Building) * P(Building|d_2)$$

+ P(M_1|\gamma Building) * P(\gamma Building|d_2)

In terms of the kappa-calculus (taking logs, ignoring adding by zero, and minimising):

$$\kappa(M_1|d_2) = \min(\kappa(Building|d_2), \kappa(M_1|\neg Building))$$

assuming that

- $\kappa(\neg Building|d_2) = 0$; we are not surprised that the *Engineered Artifact* is not a building. We would be surprised if it is a building as there are many more sorts of engineered artifacts than buildings.
- $\kappa(M_1|Building) = 0$; we are not surprised that a building matches the model M_1 as the model M_1 specifies the object is always a building.

Thus the "surprise" that the engineered artifact fits the model comes from either the surprise that the *Engineered Artifact* is a building or the surprise that a non-building matches the model. Our level of surprise is the minimum of these two.

We could have surprise information as part of our ontology. For each element in the taxonomy, we would have a value of how surprising each child is. For example, we could infer the surprise of *Building* given the description *Engineered Artifact*.

If we didn't have the information in the taxonomy, we can make some simplifying assumptions to estimate this value. Suppose the description d_2 is m levels in the hierarchy above *Building*, and suppose that the average branching factor of b, and that the children of any node have approximately equal probability. Then $P(Building|d_2) \approx (1/b)^m$. Taking logarithms, we see that the surprise that the instance is a *Building* should be linear in m.

We do not use the kappa calculus directly, as this would entail having a surprise that there is no description. We'd rather just ignore non-existent descriptions. This can be done by defining the surprise value of a empty description as zero. We get positive rewards for being less surprised than this and negative rewards (penalties) for being more surprised. Given that we know something exists, not specifying a value is the same as stating it has the top value (*Thing* in the above taxonomy). This then gives us a way to calibrate the surprise. The value is zero when the description is *Thing*. If the description is not *Thing*, but in region 2, then it should have a positive reward, as it is more likely a *Building* than if it were a *Thing*. Under the assumptions made above⁷, this should be linear in the depth.

If the assumption that children are approximately equal is not a reasonable assumption, it is possible to specify the surprise values for each child in the taxonomy as part of the ontology. For example, specifying how surprised you are that residential building is a house. Note that in this case it is possible to specify a model that is surprised by a normal condition; in this case, the model should also be surprised by a empty description. For example, if things are usually engineered artifacts, but a model specifies that fits to the model are rarely engineered artifacts, then the model should be surprised by a description of an object just as *Thing*.

⁷ The main assumption is that surprises are not specified as part of the taxonomy and that all children are approximately equal. This means all children along the path from *Thing* are approximately equal, not that a child is equal to all of its sibling.

If the instance is in region 5, it gets the rarely-present penalty. We know the instance is a type of house; the model specifies that we should be surprised the location is a house.

In order to understand the reward of an instance in region 3, it is instructive to consider some more models. Suppose model M_0 is "always a Building", model M_1 is "always a Building and rarely a house" and model M_2 is "always a Shopping Centre". Then we have M_2 subsumes M_1 (given that Shopping Centre is a subclass of Building that is disjoint with house) and M_1 subsumes M_0 . If the instance is a Strip Mall (a subclass of Shopping Centre), it matches all three models. As M_0 and M_2 give the same match; an always-present reward, it seems reasonable to also give the match with M_1 the same reward. The match with M_1 should thus not also get a rarely-absent reward.

Instances in region 4, are known to be buildings and they could be houses. If we do a similar probabilistic analysis to region 2, with d_4 a description on region 4, we get:

$$P(M_1|d_4) = P(M_1|House \land Building) * P(House|d_4) + P(M_1|\neg House \land Building) * P(\neg House|d_4)$$

(as $P(Building|d_4) = 1$). If you just consider the second part of the sum, you are not surprised that the model is true for a building that is not a house (it is "always" true), you are also not surprised that a building in region 4 is not a house. Thus in terms of the kappa values, this has kappa value 0. That is, $\kappa(M_1|d_4) = 0$.

However, this is considered to be a worse match than for an instance in region 3, and so gets a small penalty that is proportional to the depth of the description. This value is designed to be dominated by the kappa values so that it only distinguished instances that have the same kappa values.

4 Matching Algorithm

Under these assumptions, there is a canonical form for the value(s) of the type of an instance. Because the declarations are implicitly conjoined, you can assume there is exactly one "present" class for any functional property (including type) and a number of absent classes that are subclasses of the present class. This is because you can always assume that the top element is present, and if a class and a subclass are both present, you can remove the superclass as present and preserve the meaning. There can't be two classes that are both "present" if one is not a subclass of the other if the hierarchies are trees (as there are no elements in common between the classes).

Similarly we can assume that for the value of a type of model object, there is at most one *always*, at most one *usually*, that frequencies go down in the hierarchy, there are no children of *never* or cousins of *always*. The only cousins of a usually are *nevers*. [Note that *sometimes* is used when we have a complete knowledge assumption; it will be ignored in this section.]

Figure 2 gives an algorithm to determine the score for matching the type descriptions of an instance object and a model object. Suppose I is an instance object that is to be matched with a model object M. We use the notation I.present to be the position in the hierarchy of the value of the instance that is declared to be present.

Similarly M.always is the position in the hierarchy of the value declared to be always true in model M. Conditions involving M.always are assumed to be false if nothing is declared to be always true in model M. Below and above refer to positions in the hierarchy (above is more general), and a node is below itself and is above itself.

One non-obvious aspect of this algorithm is when the model has an "always" above a "usually". In this case, if an instance has *present* below the the *always*, but a cousin of the *usually*, it gets just the usually-absent penalty, and no reward for the always-present. This is reasonable as, if the always was not there, this would be equivalent to having always at the top. If the instance has *present* above the *usually*, it has a reward that is linear in the depth of the *present* instance, independent of the position of the *always*.

```
procedure scoreMatch
Inputs:
     Model Description M
     Instance Description I
Returns:
     Score
begin
     if (I.present is below a M.never)
          return never-present reward
     else if (I.present is cousin of M.always)
          return never-present reward
     else if (I.present is below a M.rarely)
          return rarely-present reward
     else if (I.present is cousin of M.usually)
          return rarely-present reward
     else if (I.absent is above M.always)
          return always-absent reward
     else if (I.absent is above M.usually)
          return usually-absent reward
     else if (I.present is below M.usually)
          return usually-present reward
     else if (I.present is above M.usually)
          return \alpha \times usually-present reward
                where \alpha is depth of I.present / depth of M.usually
     else if (I.present is above M.always)
          return \alpha \times always-present reward
                where \alpha is depth of I.present / depth of M.always
```

end

Fig. 2. Determining Reward from Type description of Instance I and Model M

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5 Matching Parts

In order to be able to match complex description, we need to consider the case where the value may be an object that has proerties with values. In the case when the model and the instance both have have complex descriptions, we need to determine the correspondencies between these complex descriptions. If the instance has multiple instances of the property (and the property is not functional), we need to determine which model values correspond to which instance values.

Object	Property	Value	Frequency
M_1	has_location	L_1	usually
M_1	starts_at	T_1	always
M_1	organized_by	P_1	always
			• • •
P_1	type	Admin_staff	usually
P_1	type	Dept_head	rarely
P_1	type	Financial_officer	never

Example 7. Consider the value of the *organized_by* property of M_1 in Figure 3. If there

Fig. 3. An example model of a research meeting

exists a person who fits the description of P_1 , there should be a reward, and if there doesn't there should be a penalty. If there are multiple people, we need to choose the most appropriate one to fill the role.

To understand how this works, it is constructive to consider a full probabilistic analysis of the probability of model M_1 given instance I_1 :

$$P(M_{1}|I_{1}) = P(M_{1}|(\exists P_{1}) \land I_{1})P((\exists P_{1})|I_{1}) +P(M_{1}|(\neg \exists P_{1}) \land I_{1})P((\neg \exists P_{1})|I_{1}) = P(M_{1}|(\exists P_{1}))P((\exists P_{1})|I_{1}) +P(M_{1}|(\neg \exists P_{1}))P((\neg \exists P_{1})|I_{1})$$

where $\exists P_1$ is shorthand for there exists a P_1 that matches the description of P_1 in Figure 3. The model gives the qualitative value $P(M_1|(\exists P_1))$. To compute the qualitative values of $P((\exists P_1)|I_1)$ we find the best match of P_1 to the values of *organized_by* in I_1 .

Taking the qualitative version of this formula, we replace multiplication by addition and the addition by minimum in the kappa calculus or maximum in our system. The works if we expect $(\exists P_1)$ to be true (the frequency of *organized_by* in M_1 is always or usually) and we find that we have positive support for $(\exists P_1)$.

Unfortunately, the other cases are not as straightforward. We cannot readily compute $\kappa((\neg \exists P_1)|I_1)$ as if $(\exists P_1)$ has no surprise, then its negation has some surprise, but we don't know how much, and if $(\exists P_1)$ has some surprise, then its negation has no surprise. We have chosen a simple scheme that gives intuitive results, as follows.

If the model specifies we are surprised that $(\exists P_1)$ (the frequency is "rarely") and we are not surprised that it true in the instance (i.e., $(\exists P_1)|I_1$ is positive), we get the surprise of the rarely (the *rarely-present* reward).

If the qualitative probability of $(\exists P_1)$ in I_1 is negative, we give the appropriate *always-absent*, *usually-absent*, ... *never-absent* reward.

For example, if the instance had multiple organisers, we need to determine which one best fulfils the role specified in the model. In terms of the kappa calculus, the distribution over which instance fulfils the role becomes a minimisation of surprised (maximisation of scores). We do this by choosing the one with the highest score. Then we consider the model frequency and how surprised we are than an organiser of the appropriate type exists.

6 Conclusion

This paper has grown out of a project to build knowledge-based decision tools in various domains such as minerals exploration, geological hazards (landslides, earthquakes), land-use planning. We needed qualitative reasoning and rich ontologies. We don't have the probabilistic knowledge or the utilities to do full decision theory, but have developed a system that uses a small but natural set of qualitative probabilities that can integrate with the ontologies being developed and with the sort of knowledge about instances and models that can be obtained. This paper outlined how we are handling cases where a functional relation has qualitative probabilistic constraints on what values it can take (some are more surprising than others). We have made some pragmatic choices that seem to work in practice, but there is much more theoretical and empirical work that needs to be carried out.

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