Deforming Surface Features Lines in Intrinsic Coordinates

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Abstract

Significant local structure of terrain surfaces can be described by structural lines which are connected sets of points where the surface is approximately cylindrical, i.e., the ratio of the principal curvatures is large. At each point curvature is maximal in the curvature direction associated with the curvature with larger absolute value. These lines form the skeleton of the surface for constructing triangulated approximations.

Significant structures are best identified at coarse scales but need to be deformed to fine scale before use. Standard snake algorithms using proximity in the image plane do not suffice. Earlier work used the maximal curvature field as an intrinsic (curvature) coordinate system. This leads to shrinkage of the open curves under internal forces. A better solution is to restrict the movement of the lines to the principal curvature directions forming an intrinsic coordinate system over the surface.

We evaluate the results of deforming lines by utilizing the deformed lines as structural lines for triangulation. Points at coarse scale move along curvature lines to the proper location at fine scale. The resulting fine scale lines are better positioned for terrain representation than those derived from proximity alone and produce more compact triangulations.

1 Introduction

Many terrain surfaces have prominent structural lines, such as ridges, cliffs, channels, and slope breaks, that form the surface skeleton and are important in constructing Triangulated Irregular Network (TIN) models from grid models of elevation, often called digital elevation models (DEM)[5, 4, 8]. TINs are used in surface visualization and interaction, where the number of points in the triangulation determines its rendering time. Structural lines are critical for applications ([9]) in hydrographical and terrain analysis. Many techniques for approximating a DEM with a TIN incrementally improve an initial triangulation[5, 4] by adding points.

[3] first identify surface and slope discontinuities, based on the local differential structure of the surface, which is independent of the choice of coordinate system, and include them in the resulting approximation.

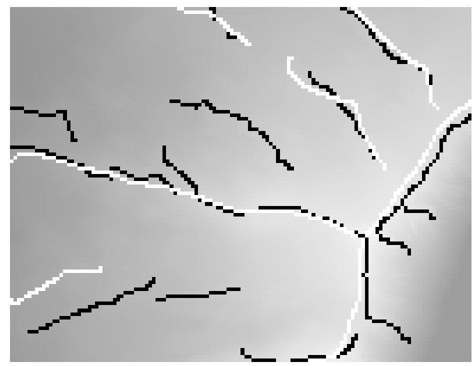


Figure 1: (Top) Terrain with p-lines (black:$\sigma = 1.0$, and white:$\sigma = 4.0$).

[7] also inserts "crest" lines into adaptive meshes to improve stereo-driven surface approximation. [4] found ridge and channel lines and inserted them in the triangulation. Little and Shi[8] showed how to extract structural lines based on local curvature descriptions and use them as the basis for a constrained Delaunay triangulation of the surface. The structural lines for a small section of the Crater Lake DEM are shown in white in Fig. 1.

We control the scale of features by varying smoothing. At small scales, i.e., when σ of the Gaussian smoothing is small, there are many local "creases" in the surface (black lines in Fig. 1), whose inclusion needlessly increases the size of the triangulation; white lines show structural lines when $\sigma = 4.0$. Smoothing the surface isolates more important "structural" lines[8], at the cost of displacement of the lines from their position at fine scale. Any location error introduced in the initial points/lines forces the triangulation method to introduce further points. To achieve maximum compaction, the coarse lines must be better localized.

In [2], edges at a series of closely spaced scales are deformed successively from coarse to fine scale. The method assigns each edge point at coarse scale to the nearest point at the next finer scale. But image plane proximity does not lead to the best localization; it replaces continuous surface variation with distance to the binary structural line. We provide an analysis of the local differential structure of terrain curvature. We then explain how to deform coarse scale structural lines along lines of curvature to fine scale position, resulting in improved localization.

2 Curvature Descriptions

The following section describes the differential geometry of structural lines. The tangent plane at a point is orthogonal to the surface normal $\vec{n}$. Cutting the surface by a plane containing $\vec{n}$ defines a direction $\vec{v}$ in the tangent plane; in that direction the *normal* curvature is the curvature of the curve formed by the intersection of the surface and the plane. The *principal directions* are the two orthogonal directions $\vec{t_1}$ and $\vec{t_2}$ where the value of the normal curvature reaches its maximum and minimum values, k_1 and k_2. k_1 is the curvature of maximum absolute value and $\vec{t_1}$ and $\vec{t_2}$ are vectors in the local tangent plane pointing the direction of maximum and minimum curvature. A point is *elliptic* when both curvatures are positive, *hyperbolic* when their signs are mixed, and *umbilic* when both are equal. At a *cylindrical* point one curvature is zero.

The principal directions on the surface define a mesh except in umbilic regions. At a slope break, k_1 will be large and k_2 will be small; the surface will be approximately cylindrical. Figure 2(a) shows a surface with the local geometrical structure.

To find structural lines we determine whether the maximum curvature k_1 at each point is locally maximal in the direction $\vec{t_1}$. Figure 2(b) shows the structural lines for a surface with sinusoidal cross section; (c) shows $|k_1|$ for the Crater Lake region. Non-maximum suppression identifies locally maximal points; it looks in the direction of maximum curvature, $\vec{t_1}$, and marks points where $|k_1|$ is greater than neighboring points along the line of curvature.

A *p-line* connects points of locally maximal positive curvature. An *n-line* has negative curvature. To find these lines, we track lines and connect the points, employing hysteresis with thresholding, using the magnitude of the maximum curvature. Tracking, followed by pruning short lines, produces the p-lines shown in Fig. 1. As σ increases the number of p-lines decreases.

2.1 Snakes: Deforming Large Scale Lines

Figure 1 shows how smoothing eliminates small surface perturbations (Fig. 3)(a). Lines at a small scale ($\sigma = 1.0$) are too numerous and are not necessarily globally significant. The lines at a coarse scale ($\sigma = 4.0$) may have been displaced by smoothing. The crest on a hill where slope on one side is significantly steeper will be displaced toward the shallower side (Fig. 3)(b). When displaced lines are used as the skeleton, triangulation includes undesirable "corrective" points near the p-line to model the actual location of the crest.

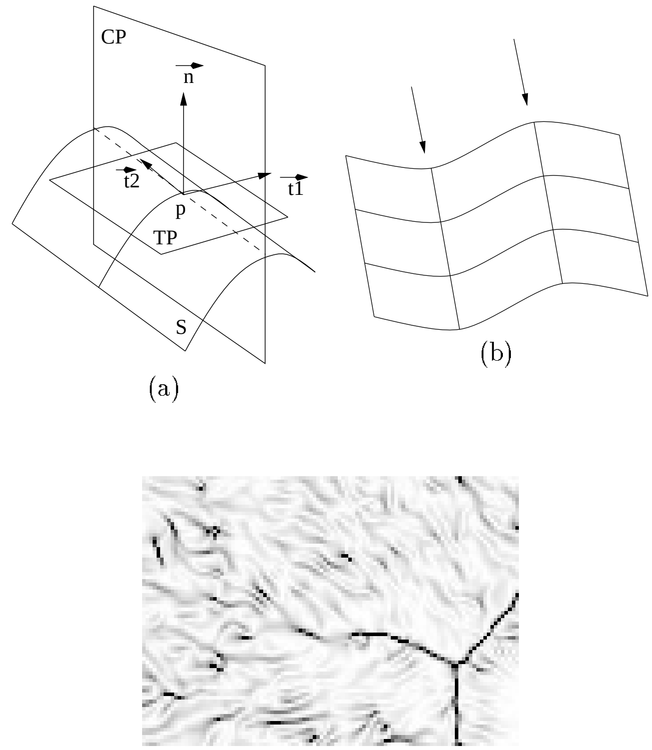


Figure 2: (a)Local surface geometry: the principal directions are orthogonal; $\vec{t_1}$ is the direction of maximum curvature. (b) Surface structural lines where curvature is locally maximal in direction $\vec{t_1}$. (c) $|k_1|$, maximal curvature, for the terrain section; darker is higher.

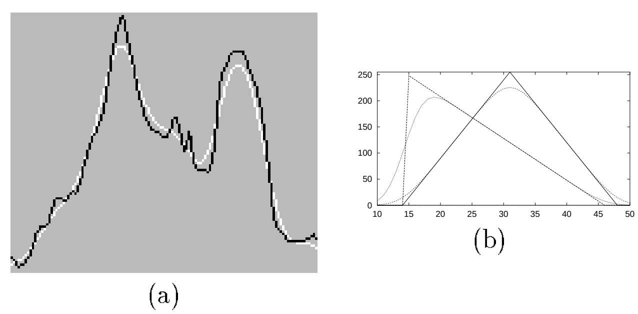
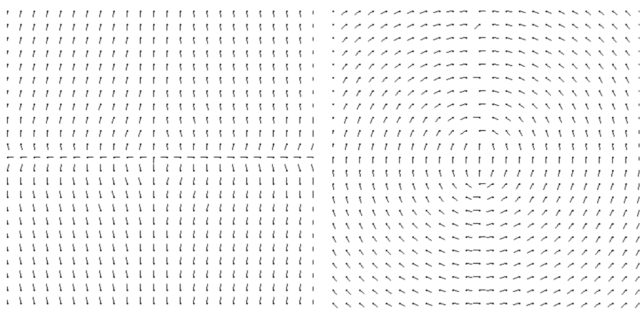


Figure 3: (a)A slice along column 70. Black: original DEM; white: smoothed by Gaussian ($\sigma = 4.0$). (b) Smoothing an asymmetrical hill displaces the maximum, where the p-line lies, toward the shallower slope.

The "snake" method[6, 1] moves the coarse-level line to the location of the fine-level line. The snake method allows the line to deform to minimize the sum of *internal energy*, the energy of stretching the line, and *external energy*, the attractive force applied by some external source. [8] uses the maximal curvature field computed at the fine scale (Fig. 2(c)).

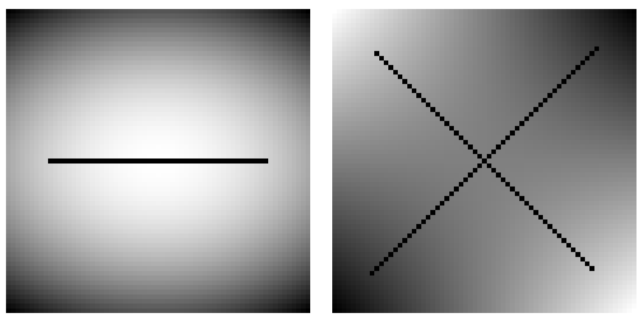
2.2 Local curvature structure

Using the maximal curvature as the external force field for snaking improves significantly upon simple proximity, but it has unforeseen effects, including



(a) (b)

Figure 4: (a) The gradient of the maximal curvature on the ellipsoid. (b) computed curvature lines for the ellipsoid, in the direction of maximal curvature.



(a) (b)

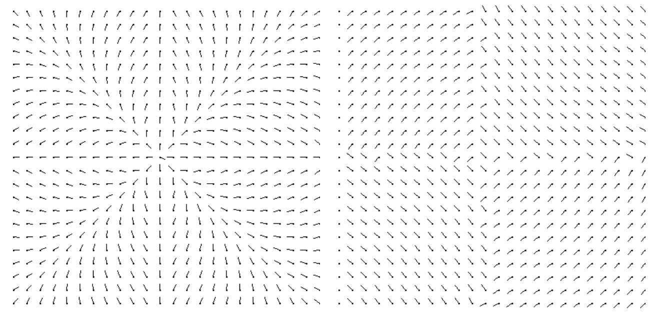
Figure 5: (a) Ellipsoid with p-line. (b) Hyperboloid with p- and n-lines.

shrinking lines. We will examine local differential geometry of simple surfaces to understand these effects.

In the ellipsoid $x^2+2y^2=1$ the p-line lies on the horizontal axis; there is no n-line (Fig. 5(a)). The gradient of the maximal curvature field appears in Fig. 4(a); the curvature lines in the direction of maximal curvature appear in (b). There are two curvature directions at all but umbilic points; we show the $\vec{t_1}$ direction of maximal absolute curvature.

There is one line of maximal curvature through a local extremum and two through a saddle. At an extremum, curvature is positive in both principal directions; only one curvature direction can be maximal. For a saddle one line has negative normal curvature and the other positive. Both are locally maximal in the absolute value of curvature; one is a positive extremum and the other negative.

Figure 6(a) shows the gradient of the maximum curvature field for a hyperbolic surface ($xy=1$); peaks are located in the upper left and lower right; a p-line runs between them and an n-line runs from lower left to upper right (Fig. 5(b)). Figure 6(b) shows its curvature lines. The normal curvature of the surface is positive in direction $x=y$ and negative in the orthogonal direction. There are two sets of curvature lines; the transition between one set and the other occurs at the



(a) (b)

Figure 6: (a) Gradient of maximal curvature. (b) maximal curvature lines on the hyperboloid.

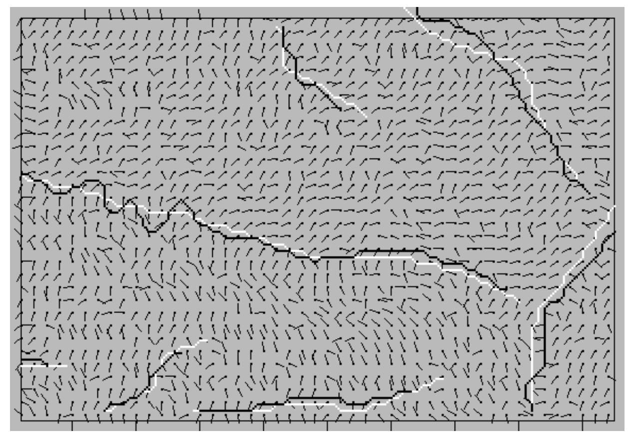


Figure 7: Terrain curvature directions ($\sigma=1.0$) with white structural lines ($\sigma=4.0$) and lines deformed using curvature lines and then snaked to $\sigma=1.0$.

vertical and horizontal axes.

Because the value of the maximum curvature is not constant along structural lines, the gradient of the maximum curvature field is not aligned with the lines of curvature. Far from the maximal curvature lines the gradient points directly to the lines, but as it approaches them it turns gradually to approach them. In contrast, curvature lines become orthogonal close to the structural line. Without internal forces (springs between points along the line), points moving to reduce external forces will move along the gradient of the maximal curvature, hence retracting or shrinking the lines.

2.3 Retraction

In a snake external forces attracts the curves and stretching links between points generates energy as the curve deforms to fit external forces. In an open curve, the curve will retract toward local maxima where the energy reduction by retraction balances the energy increase by moving away from the maximum. The curve can reduce energy by moving its end point. An endpoint at position 0 will move to position 1 only if the reduction in energy from external forces $dE=(E_1-E_0)-K$ is negative; $dE<0$ means the point will move. E_i is

Figure 8: The results of sliding along curvature lines from coarse scale to fine scale (black); standard snaking applied to the results of sliding (white).

the external energy at position i. K balances external and internal forces; $-K$ represents the reduction in stretching energy from moving one unit. If the end of the line lies near a relative maximum of attractive force, the endpoint will move toward the maximum, along the line, thus shrinking the line. Reducing K to 0 is not a solution, since the snaked line then adjusts to small local variations in the curvature field, losing its fidelity to coarse scale structure.

2.4 Moving on the curvature line

To avoid retraction, we implemented *sliding*: each point moves independently along the maximal curvature line. There are no internal forces in our simple implementation. A point on a coarse scale line moves along the finer scale curvature line until it reaches a local maximum of the maximal curvature or lies upon the structural line found at the finer scale (Fig. 8: black). The points are not restricted to moving to lattice points but can use curvature directions interpolated between the lattice points. After the points have been moved to be near or upon the fine scale lines, they are snaked by dynamic programming using the maximal curvature field(Fig. 8: white).

3 Experiments

To determine whether including structural lines can improve the resulting triangulation, we compare the size of triangulations based on structural lines deformed either by the maximal curvature field or the curvature lines. The fine scale lines are better localized, but may not be significant. The goal of deforming the lines from coarse scale to fine scale is to balance localization with fidelity to the larger structure of the surface. A good result approximates the surface well leading to a compact approximation of the DEM.

We can vary the quality of the TIN approximation to the DEM by varying the target RMS error. The metric is the relative reduction of the size of the TIN relative to the TIN without using structural lines; the lines are most effective in reducing the TIN size for rough approximations. We report the average relative size for a broad range of target RMS error and for rough RMS error. The curvature sliding method improves upon the best result reported in [8], reducing the size by 22% for broad range and 42% for rough TINs. When the final snaking stage is omitted, the reductions are 18% and 36%, indicating that final curve repositioning matters and that retraction along the lines aids the fit since it reduces the size of the lines.

Instead of deforming lines at coarse scale to fine scale we can dilate the coarse scale lines and mask the fine scale lines. The resulting subset of the fine scale lines leads to reasonable reduction on average (18%), but relatively poor rough fits (23% vs. 42%), where the benefits of structural lines are most significant.

4 Discussion

Choosing the proper intrinsic coordinate system to determine the external field for deformation improves the mapping from coarse to fine scales. Proximity in the image plane does not lead to proper tracking across scale, nor does using the gradient of maximal curvature. Selecting the curvature directions based on maximal curvature leads to a better balance between coarse scale detection and fine scale localization. This avoids retraction caused by movement along the structural lines. The resulting localized lines form an improved skeleton for triangulation and reduce the number of points needed to achieve a given error.

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