Performance Measures for Locomotion Robots *

Kees van den Doel and Dinesh K. Pai
Department of Computer Science
University of British Columbia
Vancouver, Canada
{kvdoel | pai}@cs.ubc.ca

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Abstract

We design two performance measures for a planar locomotion robot, modeled closely after the Platonic Beast. The first measure is proportional to the energy consumption during locomotion of the robot, which we assume to be mainly due to viscous friction in the actuator gears The second measure determines the maximal speed of locomotion, for given maximal joint speeds. We compute optimal modes of locomotion on different slopes for various designs. The results indicate that a variable link length can greatly improve the ability of the robot to walk on steep slopes.

1 Introduction

What is the best way for a robot to walk on a given terrain? What is the best robot for this terrain? What configuration of the legs is optimal? The answers to these questions are critical to the design, selection, and programming of locomotion robots.

All these questions can be seen as redundancy resolution, the selection of the best way to do something from a set of possibilities. This set of possibilities can range from picking a design of a robot, to chosing a specific posture of a robot arm, for example.

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An essential tool in redundancy resolution is the use of performance measures. A performance measure of a task assigns a numerical value, the "cost", to a particular manner of executing this task. Finding the "best" way to execute the task can then be translated into an optimization problem. The nature of the performance measure depends of course on the nature of the task, the nature of the cost we want to minimize, and on the nature of the robot.

Performance measures are already widely used for design and posture optimization for robot arms. Several local measures have been proposed in the past, [26, 3, 9, 14, 19, 8, 11, 2, 12, 13, 1, 18], which are reviewed in [25, 23]

In previous work [24, 22, 25] we have constructed a geometrical theory of performance, which assigns a numerical value ("performance measure") to an elementary task.

The measures derived in our formalism are invariant under general coordinate transformations in configuration space, and therefore correspond to physical properties of the manipulator, and are not just mathematical constructs.

The key idea of our performance measures is to assign a distinct metric structure to the configuration space and to the work space of the robot. The metric on the configuration space measures the "cost" to move from configuration to configuration, and the metric on work space measures how much is "achieved" with a a motion. The nature of the "cost" and "achievement" is dictated by the nature of the task, and may be kinematic or dynamic. We showed how natural geometric functions derived from Riemannian metrics on configuration and work space can be interpreted as performance measures.

Useful new measures suggest themselves naturally in our framework. Among these are "redundancy" measures, which measure how easy it it for a redundant manipulator to reconfigure itself without moving its end-effector, and "non-linearity" measures, which measure the degree of non-linearity in the robot dynamics.

In [24, 22, 25] we computed various measures and optimized postures for the three link redundant planar arm, the SARCOS arm [20, 21], and the Platonic Beast [15, 16].

Our previous work has focused on local measures, and in this paper we shall construct global performance measures for locomotion robots. We shall focus on a planar version of the Platonic Beasts [16, 17], and determine energy efficiency and maximum speed of gaits, which we optimize.

The remainder of this paper is organized as follows. In section 2 we discuss the Platonic

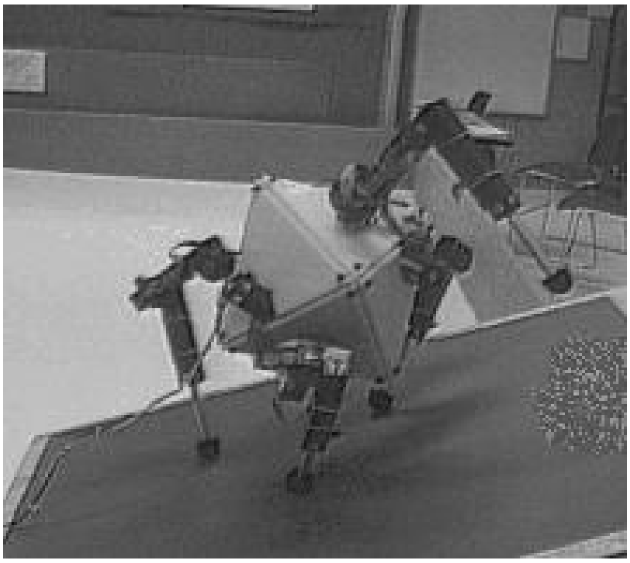

Figure 1: Platonic Beast robot walking on an incline

Beast, and a planar version of this walking robot, called the planar 3-beast. In section 3 we discuss periodic gaits of the planar 3-beast. In section 4 we construct a measure for the energy efficiency and a measure for the maximum attainable speed of the robot. In section 5 we determine optimal gaits for various designs for walking on different slopes. Conclusions are presented in section 6.

2 Walking Machines

2.1 Platonic Beasts

We have designed a family of symmetric robots called Platonic Beasts [16, 17]. The first prototype robot in this family, Mark I, has four 3DOF limbs arranged with spherical symmetry. It has been built and locomotion using a novel "rolling gait" has been demonstrated (see figure 1). The Mark II robot has the same limb construction and differs mainly in the computing and communication architecture.

The limbs of the Mark I and Mark II are built using UBC-Zebra link modules that which have a two-stage gear reduction consisting of an integral gearhead followed by a worm gear. Hence the motion of joints is dominated by friction and the robot is only capable of statically stable gaits. These robots are designed to be fault tolerant and are highly redundant for the walking task. For instance the 4-beast prototype has 12 controlled degrees of freedom.

Motivated by the analysis of the locomotion of the prototype, we analyze a simplified version, namely a planar Platonic 3-Beast, which consists of a triangle with three limbs, see figure 2. Such an exercise is useful because it may give us a feel for what can be achieved with a theoretical analysis of locomotion, both from a design perspective and from a control perspective. Because of the simplified nature of the two-dimensional case, results can be obtained fast, and different approaches can be tested without investing a great deal of resources in the analysis.

We shall call the planar version of the Platonic Beast the planar 3-beast. Its body is an equilateral triangle, and at each vertex a two-jointed limb is attached.

We will consider locomotion of the planar 3-beast on an inclined line and will design a performance measure to minimize energy consumption, and a performance measure to maximize speed.

2.2 Description of the Planar 3-beast

The body of the planar 3-beast consists of an equilateral triangle. The length of a side is denoted by D. The three vertices of the body are labeled $1, 2, 3$ in the counterclockwise direction.

A limb with two actuated joints is attached to each vertex of the triangle. The two joints will be called the hip and knee joint. The length of the upper and lower links are denoted by l_1 and l_2.

The robot has nine degrees of freedom: two for each of the three limbs, and three for the position and orientation of the body.

The configuration of a limb i is denoted by the joint angle vector

$$\boldsymbol{\theta}^i = (\theta_H^i, \theta_K^i),$$

where the upper index i labels the joint, and the lower index takes on the values H (hip) and K (knee). See figure 2. We take $-\frac{2}{3}\pi < \theta_H^i \leq \frac{2}{3}\pi$ (so the limb does not penetrate the body), and $-\pi < \theta_K^i \leq \pi$.

The configuration of the body is described by the position of the center $\boldsymbol{B} = (B_x, B_y)$ of the body in the ground-frame, and the angle Ψ, with $-\pi < \Psi \leq \pi$, as defined in figure 2.

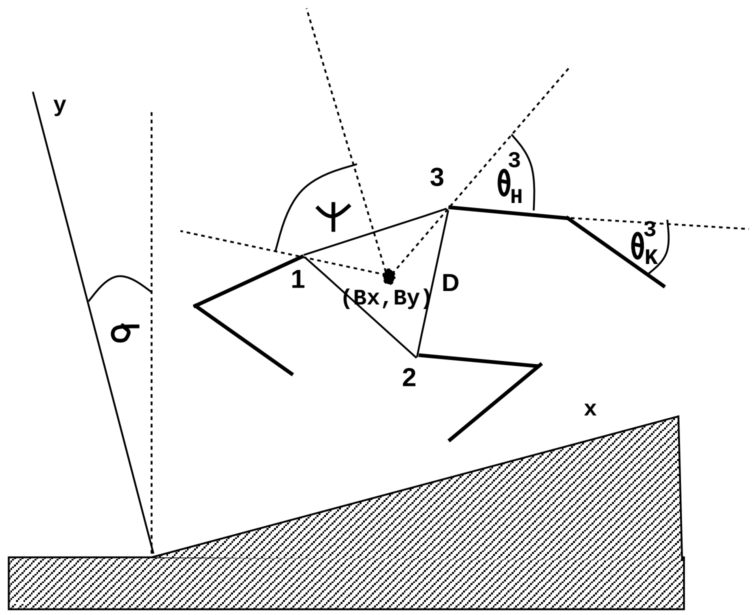

Figure 2: Conventions and notation for the planar 3-beast.

The ground is allowed to be tilted, with slope σ, so that the direction of the gravity vector in the ground-frame is given by $(-\sin\sigma, \cos\sigma)^T$.

A posture of the robot is denoted by

$$\mathcal{P} = \begin{pmatrix} \boldsymbol{B} \\ \Psi \\ \boldsymbol{\theta}^1 \\ \boldsymbol{\theta}^2 \\ \boldsymbol{\theta}^3 \end{pmatrix}.$$

3 Periodic Gait of the Planar 3-beast

The robot can walk on the ground in the manner depicted in figure 3. It is standing on two legs, with its center of mass somewhere between the two legs. We call this position $\mathcal{P}_0$. The distance between the legs is L. It will then move its body to the right until the center of mass passes the rightmost support leg. At that moment the leftmost leg will move up and the limb that was in the air will come down and become the rightmost leg. It will will then move to a position $\mathcal{P}_1$ which is identical to $\mathcal{P}_0$ except for a rotation by -120^0, a translation

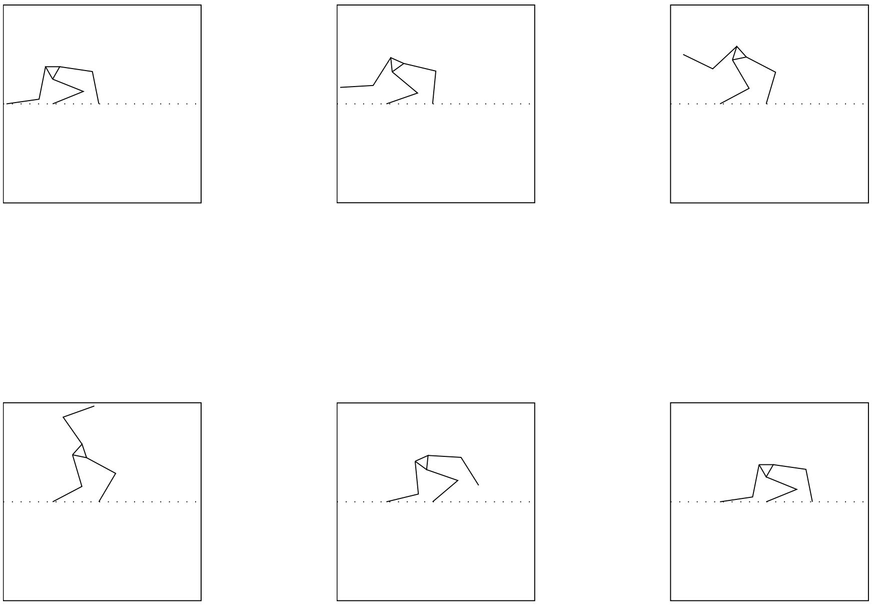

Figure 3: Walking planar 3-beast.

over L, and a cyclic interchange of the joint angles. The cycle then repeats.

The global manner of walking is thus determined completely by specifying a valid starting posture $\mathcal{P}_0$. We call the move from $\mathcal{P}_0$ to $\mathcal{P}_1$ a *stride*, with associated stride function $\boldsymbol{\mathcal{S}}$. See also [5]. The stride function acts on a posture by

$$\boldsymbol{\mathcal{S}} \begin{pmatrix} \boldsymbol{B} \\ \Psi \\ \boldsymbol{\theta}^1 \\ \boldsymbol{\theta}^2 \\ \boldsymbol{\theta}^3 \end{pmatrix} = \begin{pmatrix} \boldsymbol{B} + L\boldsymbol{e}_x \\ \Psi - \frac{2}{3}\pi \\ \boldsymbol{\theta}^3 \\ \boldsymbol{\theta}^1 \\ \boldsymbol{\theta}^2 \end{pmatrix}, \tag{1}$$

where L is the distance between the two feet (the stride length) and $\boldsymbol{e}_x$ is a unit vector in the positive x-direction. This describes walking to the right. Walking to the left is simply described by the inverse of $\boldsymbol{\mathcal{S}}$.

Without loss of generality we shall assume limbs 1 and 2 to be on the ground in $\mathcal{P}_0$, and we assume the center of mass to be exactly over the first foot, which we will position at the origin. For right-walking this means we are considering a posture where limb 3 is just moving into the air, and limb 2 has just touched down. The positions of feet 3 and 2 are thus given by $(-L, 0)^T$ and $(L, 0)^T$.

The initial posture $\mathcal{P}_0$ can thus be parametrized by the triple $\boldsymbol{w} = (L, h, \Psi)$, where L is the distance between the feet, $h = B_y$ (i.e. the height of the body), and Ψ is the initial orientation of the body. B_x is determined by the condition that the center of mass, which we assume to be located at the center of the body, is exactly above foot 1. It is given by $B_x = B_y \tan(\sigma)$.

For a given initial posture, there may be infinitely many realizations of the stride, corresponding to different paths from $\mathcal{P}_0$ to $\mathcal{P}_1$.

There are also different modes of walking. For example, the robot may execute a sequence of different strides. Such strides have a more complicated stride operator than the periodic gaits described by equation 1. We shall not consider these here though.

4 Gait Measures for the Planar 3-beast

We shall now construct two performance measures for a stride.

The energy consumption of the actuators of the Platonic Beast [16] is dominated by viscous friction, as was discussed in section 2. In this case, the energy consumption for a given motion is proportional to the sum of the joint movements. The first measure we will construct will measure the energy consumption of the beast. It is the sum of the total joint movements over a stride, divided by the length of the stride.

The second measure we consider determines the optimal speed the robot can attain. If all joints are driven by actuators of the same type, they will have a maximum speed, say χ. The time taken to execute a stride is determined by the joint that has to move most. If we denote the maximum joint movement by $\Delta\theta$, the maximum speed attainable is $V_{\max} = \frac{L}{\Delta\theta\chi}$. We then take $1/V$ as our measure to be minimized.

We shall follow the general approach of our previous work [24, 22, 25] by defining distance on an extended configuration space, restricting it to the "real" configuration space, and minimizing motions in the sense of this metric.

The nine dimensional extended configuration space $\hat{\mathcal{C}}$ is the set of all configurations $\mathcal{P}$. It can be visualized as all postures of the beast, irrespective of limbs interpenetrating, and irrespective of the ground. The restricted configuration space $\mathcal{C}$ is the space in which the robot moves when performing a stride. It is the set of configurations $\mathcal{P} \in \hat{\mathcal{C}}$ that satisfy the following conditions.

1. All the limbs are non-intersecting with themselves and with the body.

2. No part of the robot is below the ground.

3. Foot 1 is at position $(0,0)^T$ in the ground-frame.

4. Foot 2 is at position $(L,0)^T$ in the ground-frame, where L is a constant.

The space $\mathcal{C}$ is five dimensional. The space $\mathcal{C}$ is sometimes called a restricted configuration space, as we do not allow bodies to penetrate each other. The four constraints on the feet on the ground have eliminated four degrees of freedom. Three degrees of freedom correspond to the position and orientation of the body and the remaining two degrees of freedom correspond to limb three, which can move freely.

We now define a "distance" ds_E between two configurations with an infinitesimal separation $d\mathcal{P}$. A natural definition is

$$ds_E = \sum_{i=1}^{3} \sum_{j=H,L} |d\theta_j^i|, \tag{2}$$

which, up to a multiplicative constant, is the energy dissipation due to viscous friction for this change in the joint angles. We assume all joints to have identical friction. We have ignored Coulomb friction. For a discussion of viscous friction in robots, see for example [4].

Note that this is not a Riemannian metric, which was considered in our previous work [24, 22, 25]. A metric of the form given in equation 2 is sometimes called a Manhattan metric.

The length of a path is obtained by integrating ds over the path. The distance between finitely separated configurations is defined as the length of the shortest path connecting them. A peculiar property of a metric of the form 2 is that the length of a path depends only on the endpoints of the path, as long as the path is monotone, i.e. every coordinate θ_j^i either increases or decreases monotonely on the path. Let us denote the distance between configurations as defined above by $d_E(\mathcal{P}_1, \mathcal{P}_2)$.

The first walking measure for our planar 3-beast is now defined by

$$\mu_E(L, h, \Psi, \sigma) = d_E(\mathcal{P}_0, \boldsymbol{\mathcal{S}}\mathcal{P}_0)/L, \tag{3}$$

where the initial posture $\mathcal{P}_0$ is parametrized by L, h, and Ψ, as explained in section 3, and σ is the slope of the ground. The measure is not defined if there is no path in $\mathcal{C}$ between $\mathcal{P}_0$

and $\mathcal{SP}_0$, or if the parameters L, h, Ψ, and σ are not realizable in $\mathcal{C}$, i.e. without violating the constraints. The measure given in equation 3 gives the energy consumption per unit length for walking.

If there is a monotone stride-path, the measure 3 is given by

$$\mu_E(L, h, \Psi, \sigma) = \sum_{i=1}^{3} \sum_{j=H,L} |\theta_j^i - \theta_j^{(i+1) \bmod (3)}|,$$

as follows from the properties of the stride function $\mathcal{S}$ as given in equation 1.

The second measure for the maximum attainable speed with given bounds on the joint velocities is constructed along similar lines. Instead of the Manhattan metric given in equation 2 we define the distance as

$$ds_V = \max_{i,j} |d\theta_j^i|. \tag{4}$$

Unlike for the Manhattan metric, the length of a path between two points is not the sum of the lengths of segments of the path. The length of a path between finitely separated points $\mathcal{P}_1$ and $\mathcal{P}_2$ is defined as

$$d_V(\mathcal{P}_1, \mathcal{P}_2) = \max_{i,j} \int_{\text{path}} |d\theta_j^i|.$$

We define the second measure as

$$\mu_V(L, h, \Psi, \sigma) = d_V(\mathcal{P}_0, \mathcal{SP}_0)/L, \tag{5}$$

which is proportional to the inverse maximum speed.

For monotone paths, this measure is also independent of the actual path. [1]

If there is a monotone stride-path, the measure 5 is given by

$$\mu_V(L, h, \Psi, \sigma) = \max_{ij} |\theta_j^i - \theta_j^{(i+1) \bmod (3)}|.$$

The fact that these measures are independent of the path (as long as a monotone path exists) allows us to find the optimal stride by searching in the space of initial stride postures. If the measure also depended on the path of the stride, we would have to find an optimal path to determine the value of the measure for a given initial stride posture, which would greatly increase the processing time required to find optimal postures.

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[1] In fact, in this case monotonicity is sufficient, but not necessary.

5 Optimal Gaits for the Planar 3-beast

We have written a set of MATLAB routines to compute the optimal initial stride posture of a given planar 3-beast on a given slope, and its measure.

The beast is defined by specifying the dimension D of its body and the limb lengths l_1 and l_2. We call these the design parameters. These parameters, and the slope are considered given. We assume the center of mass is located at the center of the body, which is a valid approximation if the mass of the limbs is much smaller than the mass of the body.

We implemented a routine to compute a measure $\hat{\mu}$ for given (L, h, Ψ), which is an extension of the measures 3 or 5. If (L, h, Ψ) lies in $\mathcal{C}$, $\hat{\mu} = \mu_E$ (or μ_V), otherwise it returns a penalty function which is a measure of the constraint violation, i.e. a measure of how much interbody penetration the configuration has. The penalty function is constructed to have large values compared to the measure. If no configuration corresponding to (L, h, Ψ) exists, even allowing constraint violation, $\hat{\mu} = \infty$.

Note that there may be up to four solutions for the posture of the beast for a given (L, h, Ψ). This is because the inverse kinematics of a two-link arm can yield up to two solutions. We have restricted our selves here to configurations with positive knee-joint angles. Configurations with negative knee-joint angles are equivalent (we just have to reverse left and right), and configurations with mixed solutions for the front and back leg are not efficient, as the front leg will have to change from one type of solution to the other, requiring additional joint motion. So we can safely exclude these.

We start by finding a rough estimate of the posture by doing a linear search in a discretized (L, h, Ψ) space, i.e. we compute the measure for all points on a grid and find the posture with minimal $\hat{\mu}$. We then perform a local minimization of $\hat{\mu}$ starting from this configuration. If the minimum thus found is larger than 10π (corresponding to maximum joint motions) it is considered to be violating the constraints, and we conclude there is no walking posture. This happens if the slope is too steep, or if the design parameters are poorly chosen.

The value thus found is a lower bound on the true measure, which may be larger if no monotone stride path exists from the given configuration.

We do an explicit search for a monotone stride path as follows. The third joint-angles can move unrestricted to their target values, as this limb is not in contact with the ground. Thus the problem is reduced to a closed loop linkage consisting of limb 1, the body, and limb 2. The system has three degrees of freedom, which we chose to be the position and orientation of the body. The corresponding configuration space is denoted by $\mathcal{C}_3$. The problem is to find a path in $\mathcal{C}_3$ such that the four joint angles are monotone. An explicit search of a discretized $\mathcal{C}_3$, as was done for instance in [10] for planar motion planning, will run into problems in this case, because of the requirement of monotonicity. A given cell in $\mathcal{C}_3$ may have no monotone path to any of its neighbors. For example, if one of the four joint-angles moves very little (which happens often in practice for optimal postures), an effective constraint is imposed on $\mathcal{C}_3$, reducing its dimension. We have overcome this problem by allowing transitions from a cell in $\mathcal{C}_3$ to any cell in some bounded region around the initial cell, so that a monotone stride can be found. By increasing this region, more directions can be explored. The search is performed with an A^* search algorithm [7, 6].

We have checked the optimal postures generated, and have not found any posture that does not allow a monotone path. We conjecture that such a path always exist, but have not been able to prove this. In the presence of more restrictive joint limits than we have considered here, monotone paths may not exist.

We have computed the optimal postures and their measures for various designs and for various walking slopes. To compare various designs, we normalize the overall dimension of the beast. We have taken our "standard" beast to have a body dimension of $D = 0.25$, and the limb lengths are both 1.0. The maximum distance of a foot from the center of the body is $2 + \frac{1}{4\sqrt{3}}$, which we will keep fixed when changing design parameters.

We have computed optimal posture for slopes ranging for -40^0 to 65^0 for body dimensions $D = 0.25$ and $D = 0.5$, for five combinations of $l_1/l_2 = 1/3, 3/5, 1, 5/3, 3$. For angles outside this range, no stride postures exist for the planar 3-beast.

The results for the energy measure are presented in tables 1 and 2 where we show the values for the measure of the optimal postures found. A dash means that for this slope no stride posture exists. Some of the optimal postures are drawn in figs 4 and 6.

The results for the velocity measure are presented in tables 3 and 4, and some of the optimal postures are drawn in figs 5 and 7.

We give two examples of a monotone stride path in figures 8 and 9, where we also plot the six joint-angles over the stride.

This raw data can be used in many ways to aid the design and control of the walking robot. We draw the following conclusions.

- In many cases the optimal postures are the same for both measures, but not always.

- The optimal design for the link lengths for walking on a given slope, appears to be the same for both measures that we considered.

- Comparing the slope and design ranges where postures were found of table 1 and table 2, or table 3 and table 4 indicates that the smaller body gives more mobility. Note that this vconclusion is independent of the measure used, as it is determined solely by the kinematics of the design.

- The symmetric design with equal limb lengths is the most flexible, in that its slope range is centered around 0^0.

- For steep slopes, one should choose unequal link lengths, a long upper limb being favoured for walking downhill (or, equivalently, uphill with negative knee joint-angles), and a long lower limb is better for walking uphill (or downhill with negative joint-angles). However, the precise ratio should be carefully chosen.

- There may be gaps in the slope range of a given design. This happens when the geometry of the design is such that the limbs get in each others way.

- For walking on flat terrain, the best design is a small body with equal limbs. the energy consumption increases only a little for moderate slopes (up to 20^0 for $D = 0.25$, up to 15^0 for $D = 0.5$.

- Surprisingly, the optimal locomotion we found is on a 45^0 slope with $D = 0.5$ and $l_1/l_2 = 1/3$. This particular configuration uses the body in the manner of a "wheel", which can be visualized by inspecting figure 9, where we show the intermediate postures of the stride.

	$(0.5, 1.5)$	$(0.75, 1.25)$	$(1, 1)$	$(1.25, 0.75)$	$(1.5, 0.5)$
-40	-	-	-	-	21.27
-35	-	-	-	-	9.65
-30	-	-	9.24	-	-
-25	-	-	7.62	10.21	15.33
-20	-	-	5.94	8.06	13.10
-15	-	8.58	4.80	4.58	11.44
-10	-	-	4.44	4.44	9.33
-5	8.20	5.71	4.23	4.89	7.72
0	7.19	5.15	4.07	5.45	9.13
5	6.60	4.89	3.98	5.95	-
10	6.11	4.55	4.02	5.93	12.00
15	5.75	4.22	4.31	6.37	-
20	5.54	4.00	4.43	7.48	-
25	5.53	3.90	4.69	9.94	-
30	5.74	3.82	-	-	-
35	6.58	3.75	-	-	-
40	9.06	3.83	-	-	-
45	-	3.93	-	-	-
50	-	4.02	-	-	-
55	-	3.95	-	-	-
60	-	4.33	-	-	-
65	-	3.89	-	-	-

Table 1: Values of energy measure for optimal postures for $D = 0.25$ for different slopes (top to bottom) and different link lengths (l_1, l_2) (left to right).

	$(0.46, 1.39)$	$(0.70, 1.16)$	$(0.93, 0.93)$	$(1.16, 0.70)$
-25	-	-	-	4.48
-20	-	-	6.46	4.52
-15	-	-	5.94	4.66
-10	-	8.45	5.35	4.90
-5	-	7.34	5.07	6.18
0	-	6.74	4.50	8.31
5	12.97	6.30	4.57	12.43
10	10.03	5.68	4.73	-
15	8.33	5.05	4.91	-
20	6.69	4.56	9.20	-
25	5.51	4.43	-	-
30	4.79	4.50	-	-
35	4.27	4.76	-	-
40	3.95	5.17	-	-
45	3.68	5.75	-	-
50	3.80	-	-	-
55	4.09	6.24	-	-

Table 2: Values of energy measure for optimal postures for $D = 0.5$ for different slopes (top to bottom) and different link lengths (l_1, l_2) (left to right). No postures exist for $l_1/l_2 = 3$.

	$(0.5, 1.5)$	$(0.75, 1.25)$	$(1, 1)$	$(1.25, 0.75)$	$(1.5, 0.5)$
-40	-	-	-	-	6.95
-35	-	-	-	-	6.42
-30	-	-	3.21	-	-
-25	-	-	2.41	3.20	4.97
-20	-	-	1.75	2.43	4.14
-15	-	2.94	1.36	1.16	3.62
-10	-	-	1.24	1.11	2.89
-5	2.58	1.78	1.12	1.21	2.30
0	2.25	1.51	1.02	1.31	3.00
5	2.06	1.42	1.03	1.63	-
10	1.86	1.23	1.10	1.93	5.20
15	1.61	1.11	1.13	2.54	-
20	1.42	1.01	1.35	3.84	-
25	1.39	1.00	1.44	3.74	-
30	1.56	0.99	-	-	-
35	1.96	1.00	-	-	-
40	2.51	1.15	-	-	-
45	-	1.31	-	-	-
50	-	1.42	-	-	-
55	-	1.54	-	-	-
60	-	1.56	-	-	-
65	-	1.72	-	-	-

Table 3: Values of velocity measure for optimal postures for $D = 0.25$ for different slopes (top to bottom) and different link lengths (l_1, l_2) (left to right).

	$(0.46, 1.39)$	$(0.70, 1.16)$	$(0.93, 0.93)$	$(1.16, 0.70)$
-25	-	-	-	1.16
-20	-	-	1.86	1.14
-15	-	-	1.69	1.21
-10	-	2.63	1.48	1.32
-5	-	2.30	1.26	1.51
0	-	2.00	1.12	2.22
5	3.96	1.82	1.17	3.95
10	3.00	1.54	1.23	-
15	2.43	1.32	1.30	-
20	1.90	1.14	2.85	-
25	1.51	1.13	-	-
30	1.23	1.17	-	-
35	1.10	1.27	-	-
40	1.02	1.44	-	-
45	0.96	1.88	-	-
50	1.28	-	-	-
55	1.62	2.17	-	-

Table 4: Values of velocity measure for optimal postures for $D = 0.5$ for different slopes (top to bottom) and different link lengths (l_1, l_2) (left to right). No postures exist for $l_1/l_2 = 3$.

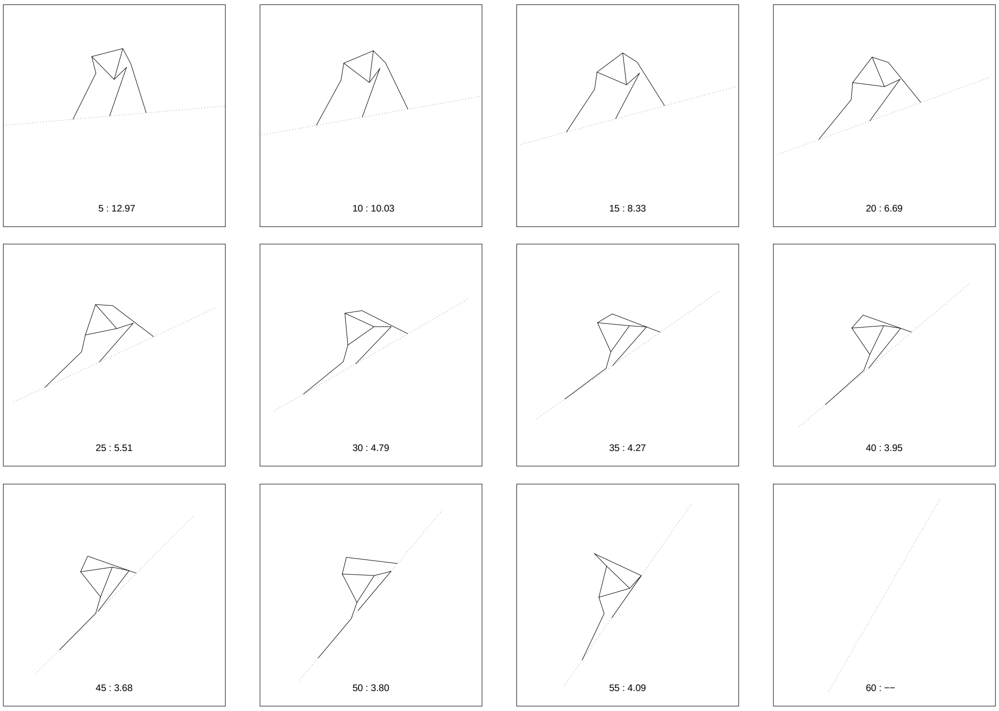

Figure 4: Optimal postures for energy measure. $D = 0.5$, $l_1 = 0.46$, $l_2 = 1.4$, slopes 5 to 60.

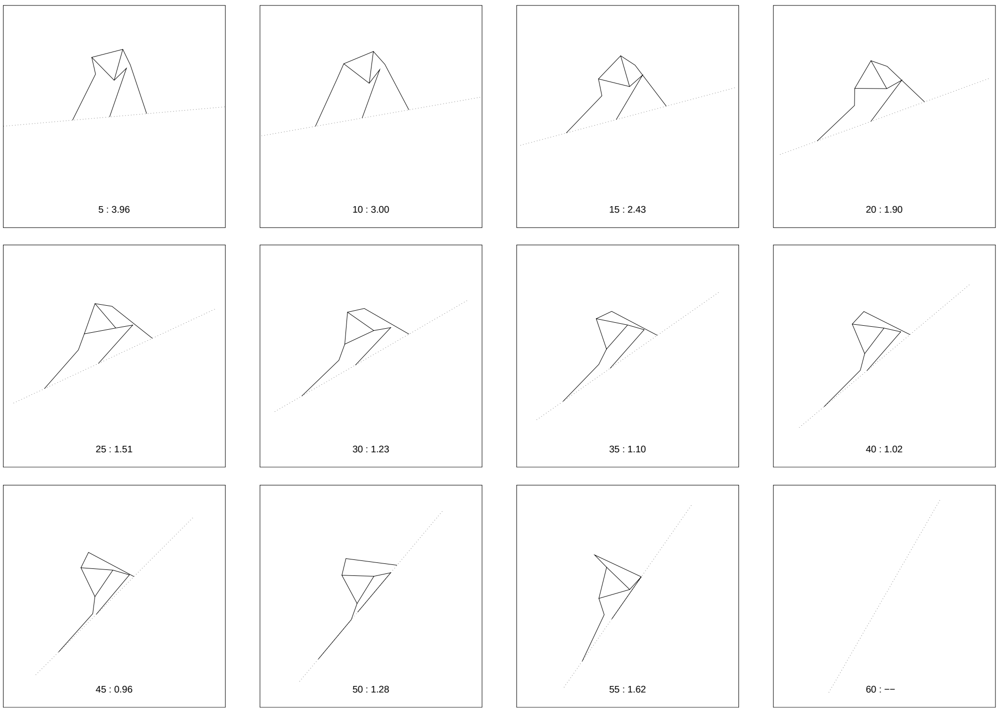

Figure 5: Optimal postures for velocity measure. $D = 0.5$, $l_1 = 0.46$, $l_2 = 1.4$, slopes 5 to 60.

Figure 6: Optimal postures for energy measure. $D = 0.25$, $l_1 = l_2 = 1.0$, slopes -30 to 25.

Figure 7: Optimal postures for velocity measure. $D = 0.25$, $l_1 = l_2 = 1.0$, slopes -30 to 25.

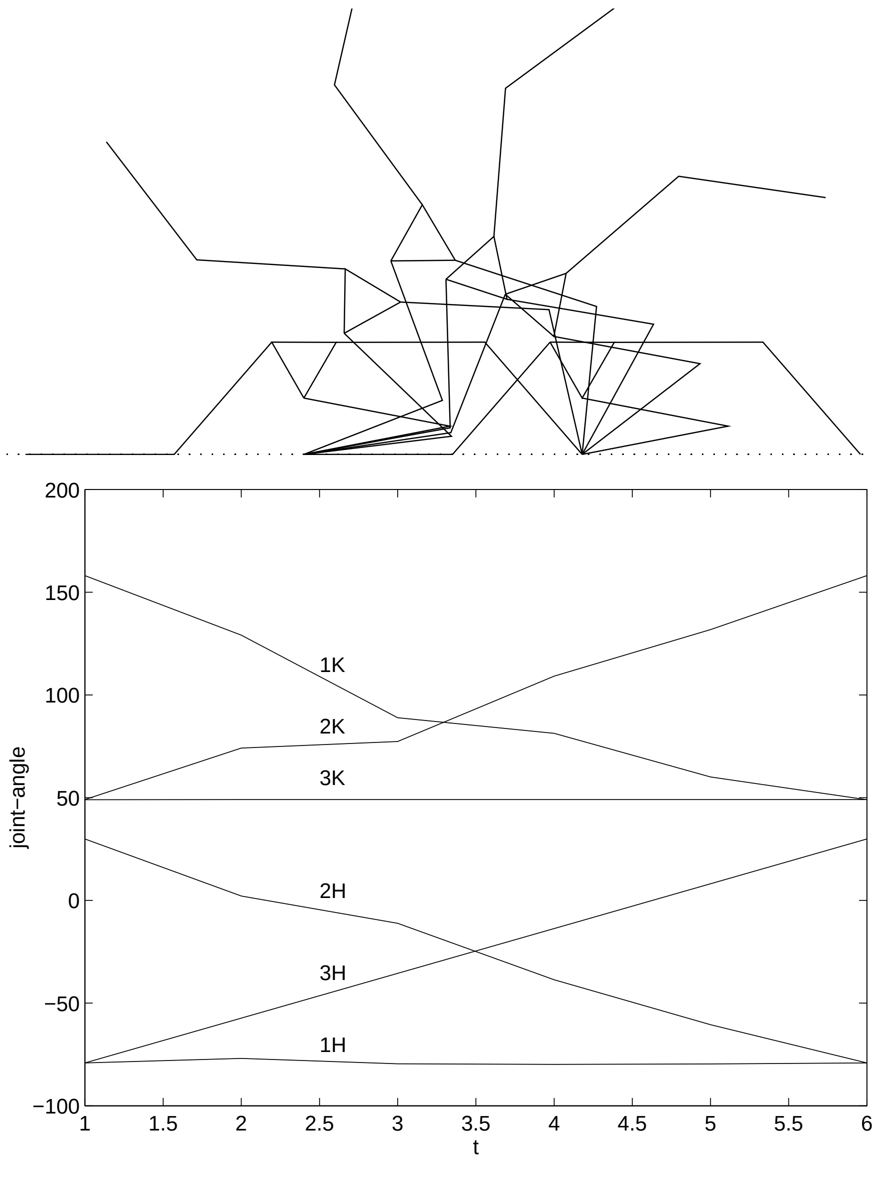

Figure 8: Optimal monotone stride path for $D = 0.25$, $l_1 = l_2 = 1$, on flat ground. We show the six joint-angles, labeled according to leg number and knee or hip. They are plotted against time t over the stride. The time-scale is chosen for convenience. This posture is the same for both measures.

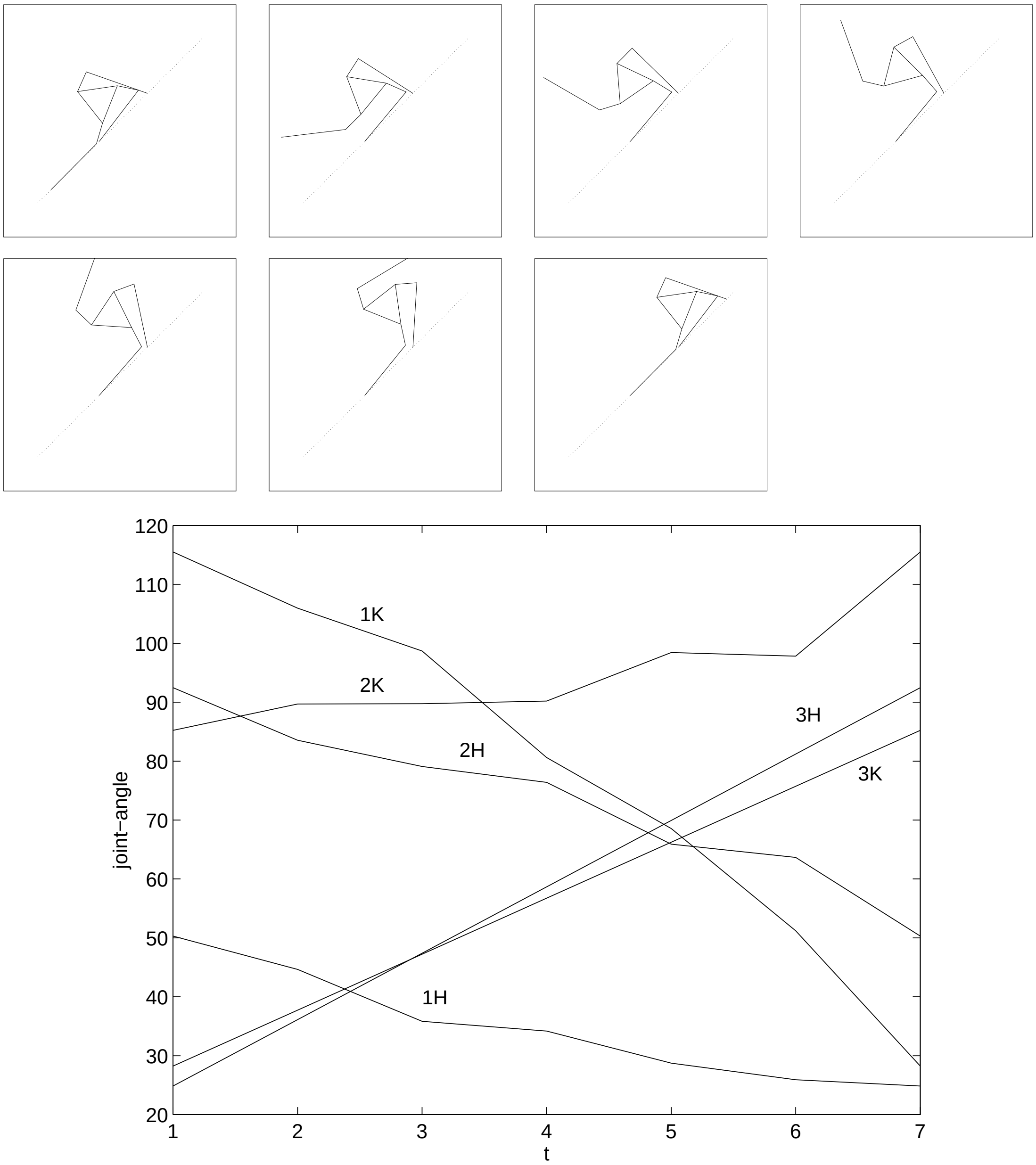

Figure 9: Optimal monotone stride path for $D = 0.5$, $l_1 = 0.46$, $l_2 = 1.40$, on a slope of 45^0 We show the six joint-angles, labeled according to leg number and knee or hip. They are plotted against time t over the stride. The time-scale is chosen for convenience. This posture is the absolute best of all postures found, for both measures.

6 Conclusions

After reviewing the general framework for constructing performance measures, we have analyzed the performance of a two-dimensional walking machine, a lower dimensional version of the Platonic Beast currently under development. We have devised a measure of the energy efficiency of the locomotion of this "planar 3-beast", assuming the main energy consumption is due to internal gear friction, independent of the load. We have also devised a measure for the maximum speed attainable by the robot.

We have computed optimal modes of locomotion for various designs on various slopes and have shown that many things can be learned from an analysis of this type.

Our measure depends only on the global shape of the stride, and not on the detailed path taken during the stride, with some restrictions (monotonicity). This means we can still optimize a secondary criterion to chose among the many paths corresponding to the same type of stride.

The analysis suggested that it may be advantageous to have adjustable link lengths, as different slopes have different requirements.

We believe that this result generalizes to the three-dimensional case, and thus provides a useful design criterion.

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