Composing graphical models with neural networks

(most slides stolen from Matthew James Johnson)

Outline

- Motivation and Examples
- Stochastic Variational Inference
- Structured Variational Auto-Encoders

Example: Generative Clustering

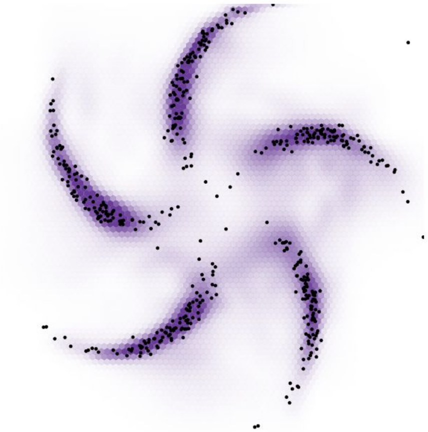
Data

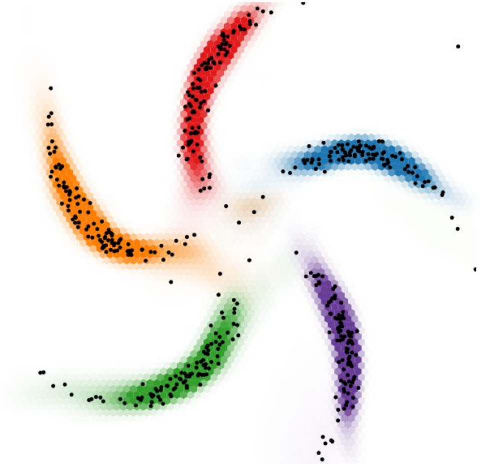
GMM

$$\pi \sim \mathrm{Dir}(\alpha), \qquad (\mu_k, \Sigma_k) \overset{\mathrm{iid}}{\sim} \mathrm{NIW}(\lambda)$$

$$z_n \mid \pi \overset{\mathrm{iid}}{\sim} \pi \qquad y_n \mid z_n, \{(\mu_k, \Sigma_k)\}_{k=1}^{K} \overset{\mathrm{iid}}{\sim} \mathcal{N}(\mu_{z_n}, \Sigma_{z_n})$$

Example: Generative Clustering





$$\gamma \sim p(\gamma) \qquad x_n \overset{\text{iid}}{\sim} \mathcal{N}(0, I)$$

$$y_n \mid x_n, \gamma \overset{\text{iid}}{\sim} \mathcal{N}(\mu(x_n;\gamma),\, \Sigma(x_n;\gamma))$$

$$\pi \sim \text{Dir}(\alpha) \qquad (\mu_k, \Sigma_k) \overset{\text{iid}}{\sim} \text{NIW}(\lambda),$$

$$z_n \mid \pi \overset{\text{iid}}{\sim} \pi \qquad x_n \overset{\text{iid}}{\sim} \mathcal{N}(\mu^{(z_n)}, \Sigma^{(z_n)})$$

$$y_n \mid x_n, \gamma \overset{\text{iid}}{\sim} \mathcal{N}(\mu(x_n;\gamma),\, \Sigma(x_n;\gamma))$$

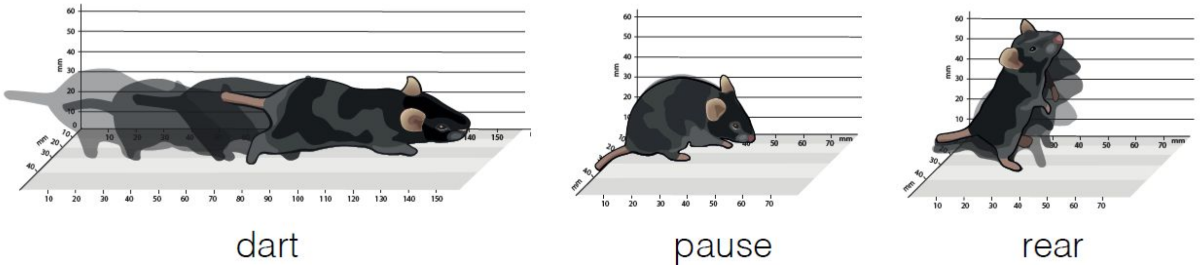
Probabilistic graphical models

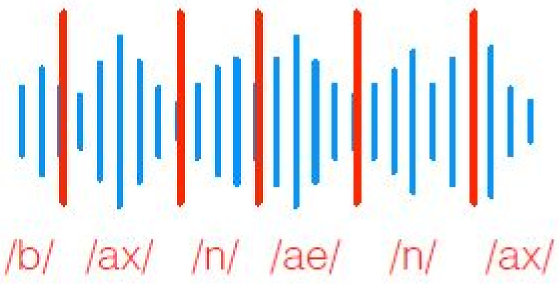
- \+ structured representations
- \+ priors and uncertainty
- – rigid assumptions may not fit
- – feature engineering
- \+ arbitrary inference queries
- \+ data and computational efficiency within rigid model classes
- – more flexible models can require slow top-down inference

Deep learning

- – neural net “goo”
- – difficult parameterization
- \+ flexible, high capacity
- \+ feature learning
- – limited inference queries
- – data- and compute-hungry
- \+ recognition networks learn how to do inference

Application: Parse mouse behaviour



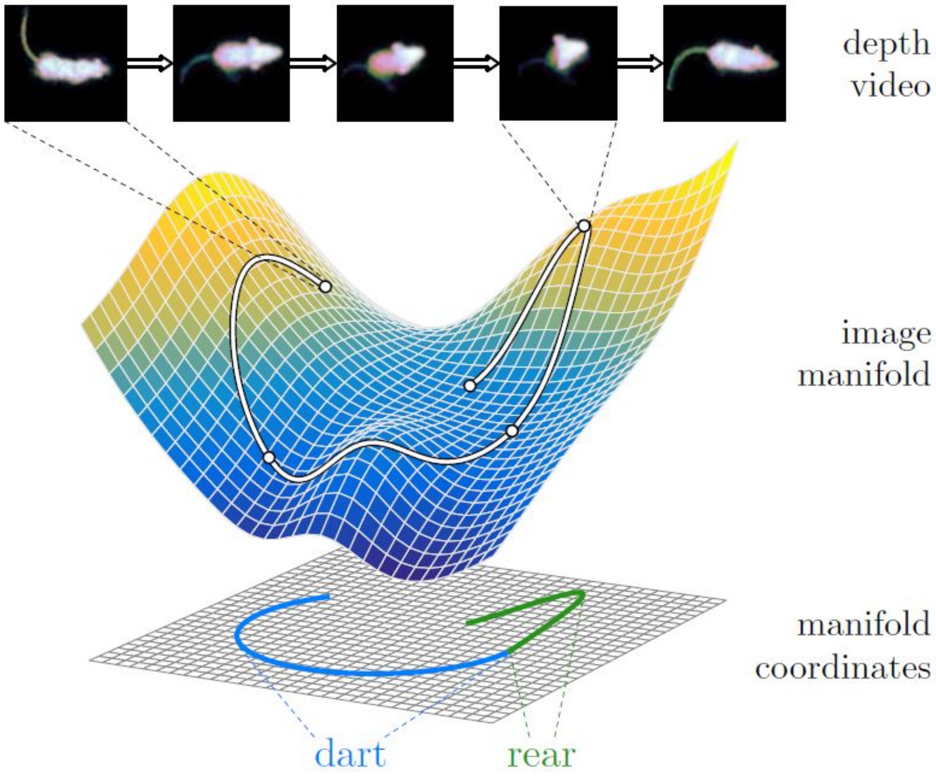


Aim: Generative model for different mouse behaviour

Application: Parse mouse behaviour

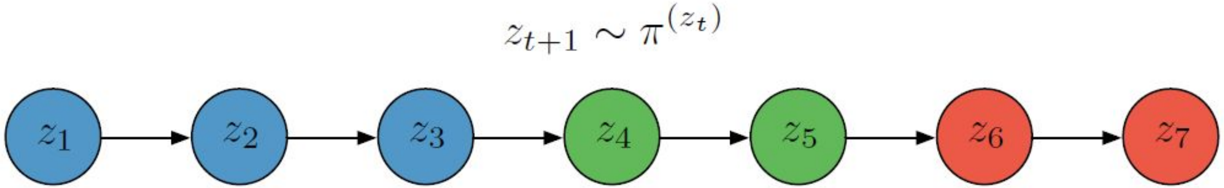
Input: Videos

Aim: Come up with a generative model for the behaviour and the video frames



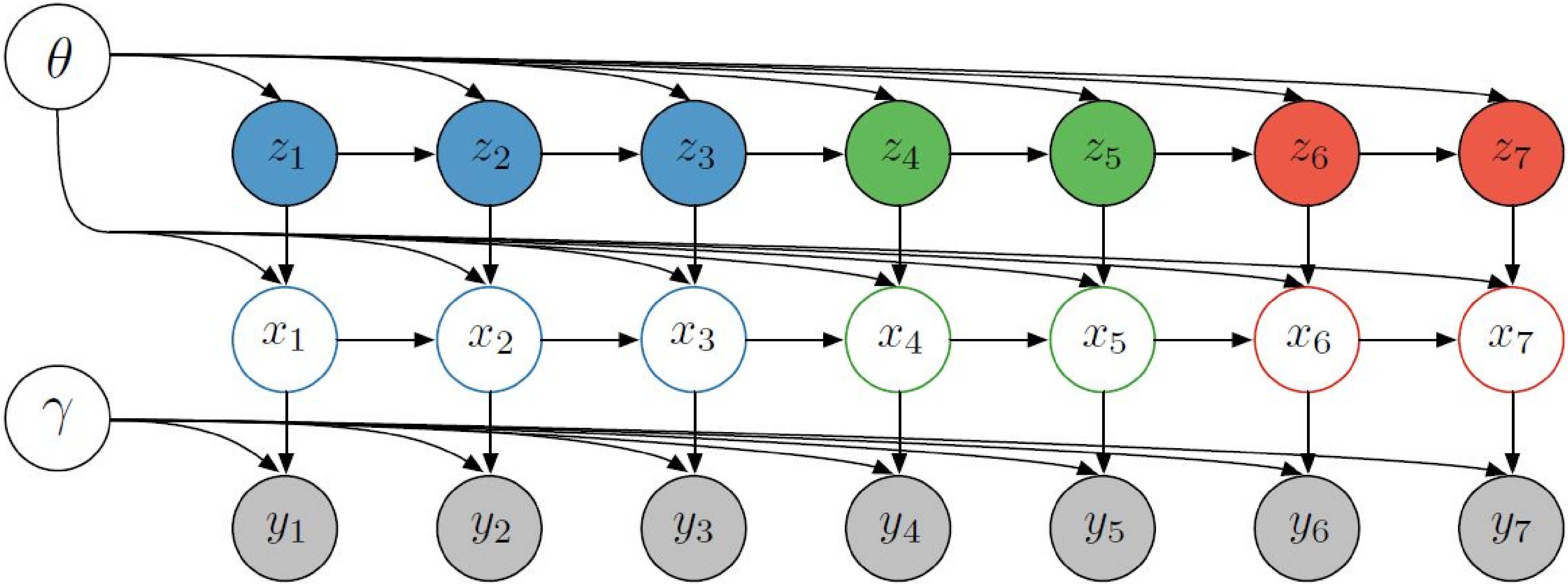
Example: Switching Linear Dynamical System

$$\pi = \begin{bmatrix} — & \pi^{(1)} & — \\ — & \pi^{(2)} & — \\ — & \pi^{(3)} & — \end{bmatrix}$$

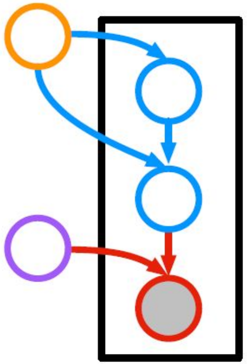


$$x_{t+1} = A^{(z_t)} x_t + B^{(z_t)} u_t \qquad u_t \overset{\text{iid}}{\sim} \mathcal{N}(0, I)$$

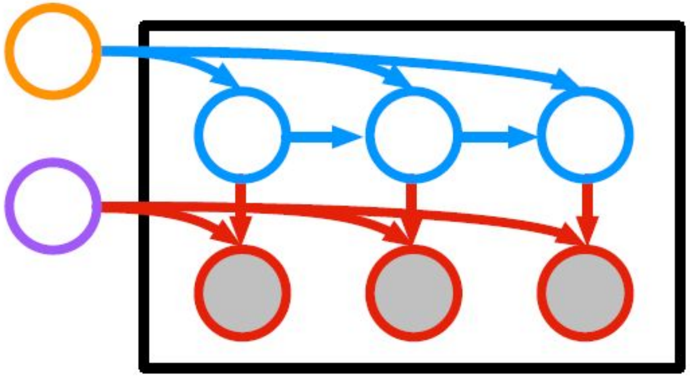
Example: Switching Linear Dynamical System



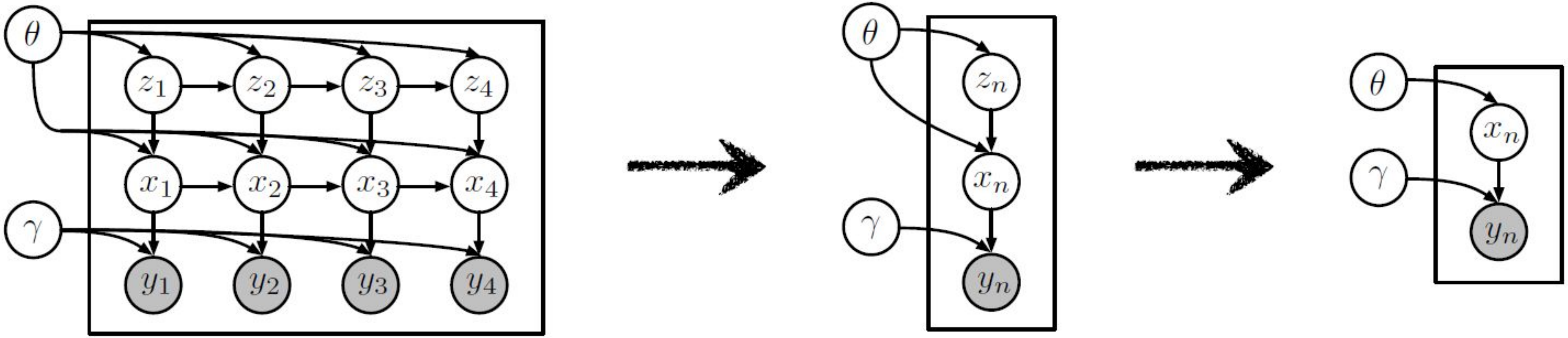
Special Cases

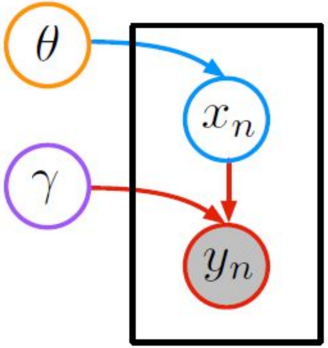


GMM



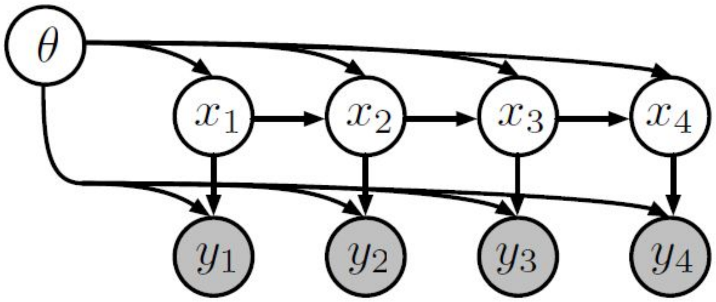
LDS





$p(\theta)$	conjugate prior on global variables
$p(x \mid \theta)$	exponential family on local variables
$p(\gamma)$	any prior on observation parameters
$p(y \mid x, \gamma)$	neural network observation model

Stochastic Variational Inference



$p(x \mid \theta)$ is a linear dynamical system
$p(y \mid x, \theta)$ is a linear-Gaussian observation
$p(\theta)$ is a conjugate prior

Mean Field Approximation

$$q(\theta)q(x) \approx p(\theta, x \mid y)$$

Objective function

$$\mathcal{L}(\eta_\theta, \eta_x) \triangleq \mathbb{E}_{q(\theta)q(x)}\left[\log \frac{p(\theta,x,y)}{q(\theta)q(x)}\right]$$

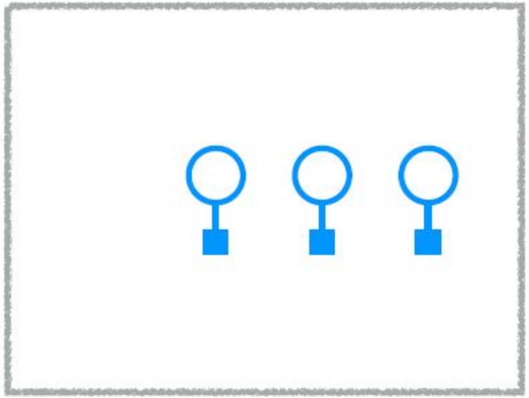
$$\eta_x^*(\eta_\theta) \triangleq \arg\max_{\eta_x} \mathcal{L}(\eta_\theta, \eta_x)$$

Equivalent to the Kalman Filter

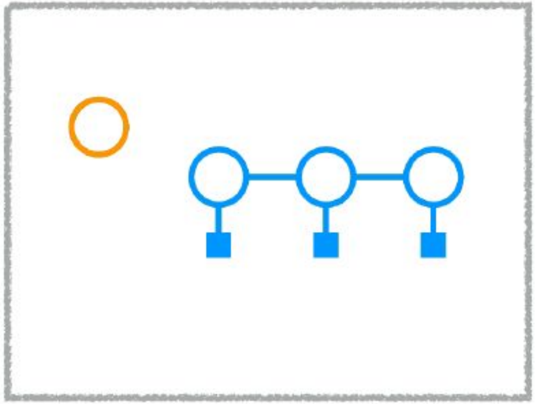
$$\mathcal{L}_{\mathrm{SVI}}(\eta_\theta) \triangleq \mathcal{L}(\eta_\theta, \eta_x^*(\eta_\theta))$$

Algorithm

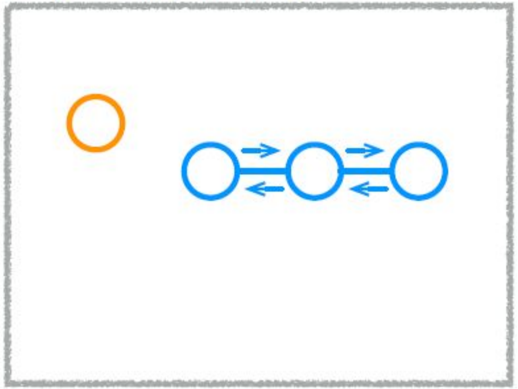
Step 1: compute evidence potentials



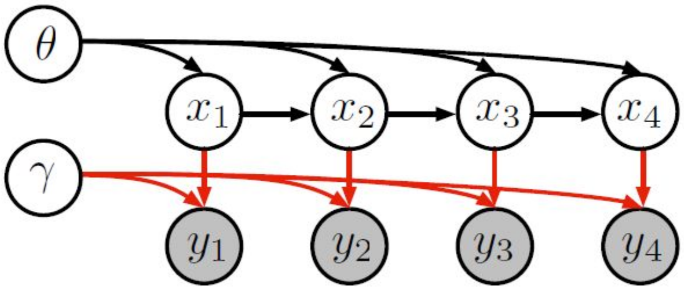
Step 2: run fast message passing



Step 3: compute natural gradient



Structured Variational Auto Encoder



$p(x \mid \theta)$ is a linear dynamical system
$p(y \mid x, \gamma)$ is a neural network decoder
$p(\theta)$ is a conjugate prior, $p(\gamma)$ is generic

Mean Field Approximation

$$q(\theta)q(\gamma)q(x) \approx p(\theta, \gamma, x \mid y)$$

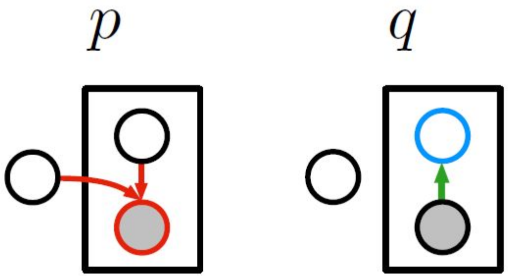
Objective function

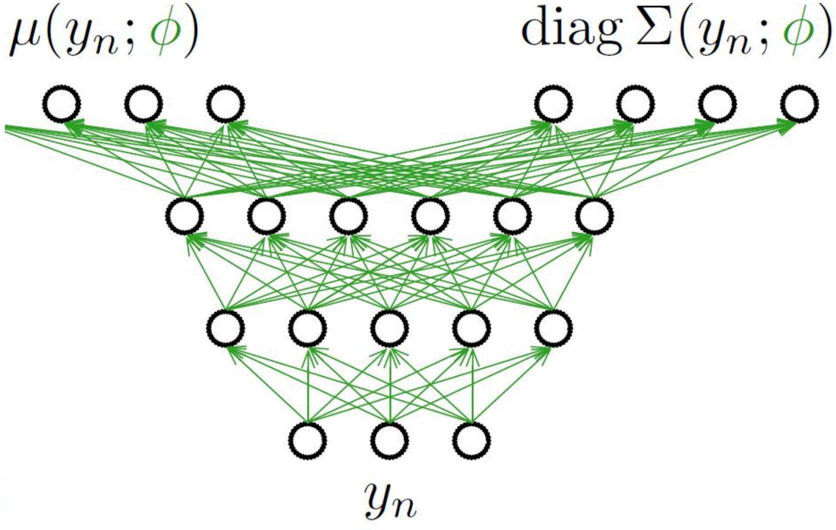
$$\mathcal{L}(\eta_\theta, \eta_\gamma, \eta_x) \triangleq \mathbb{E}_{q(\theta)q(\gamma)q(x)}\left[\log \frac{p(\theta,\gamma,x)p(y \mid x,\gamma)}{q(\theta)q(\gamma)q(x)}\right]$$

$$\eta_x^\star(\eta_\theta, \eta_\gamma) \triangleq \operatorname*{arg\,max}_{\eta_x} \mathcal{L}(\eta_\theta, \eta_\gamma, \eta_x)$$

$$\mathcal{L}_{\mathrm{SVI}}(\eta_\theta, \eta_\gamma) \triangleq \mathcal{L}(\eta_\theta, \eta_\gamma, \eta_x^\star(\eta_\theta, \eta_\gamma))$$

Variational Auto-Encoder





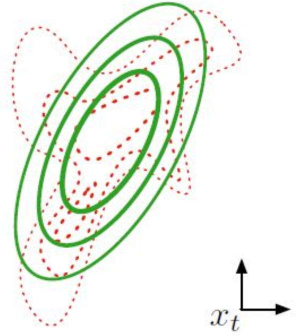
$$q^*(x) \triangleq \mathcal{N}(x \mid \mu(y;\phi), \Sigma(y;\phi))$$

$$\widehat{\mathcal{L}}(\eta_\theta, \eta_x, \phi) \triangleq \mathbb{E}_{q(\theta)q(\gamma)q(x)}\left[\log \frac{p(\theta,\gamma,x)\exp\{\psi(x;y,\phi)\}}{q(\theta)q(\gamma)q(x)}\right]$$

$$\eta_x^*(\eta_\theta, \phi) \triangleq \arg\max_{\eta_x} \widehat{\mathcal{L}}(\eta_\theta, \eta_x, \phi)$$

$$\mathcal{L}_{\mathrm{SVAE}}(\eta_\theta, \eta_\gamma, \phi) \triangleq \mathcal{L}(\eta_\theta, \eta_\gamma, \eta_x^*(\eta_\theta, \phi))$$

$\mathbb{E}_{q(\gamma)} \log p(y_t \mid x_t, \gamma)$



$\psi(x_t; y_t, \phi)$

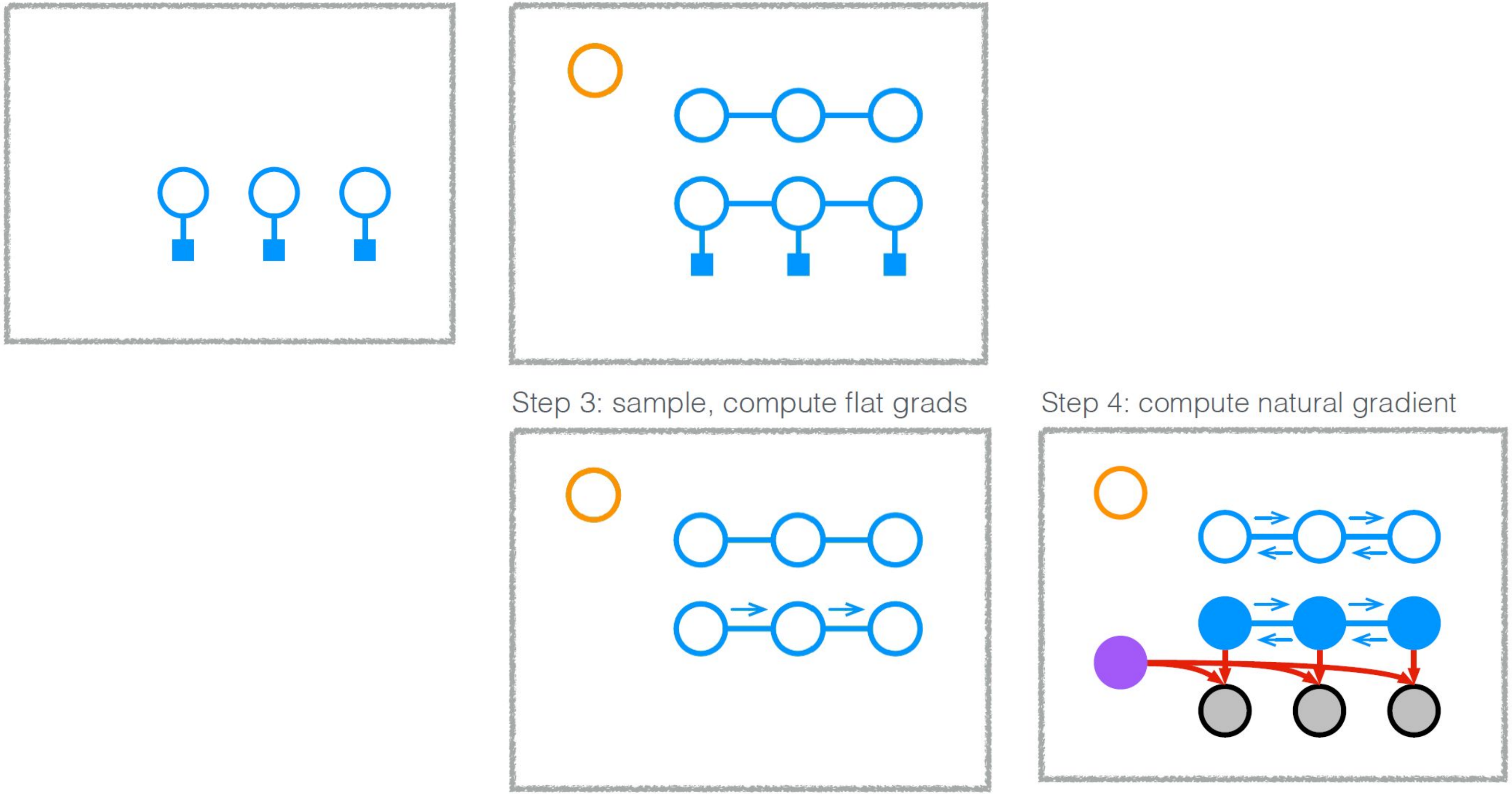
Algorithm

Step 1: apply recognition network

Step 2: run fast PGM algorithms

Step 3: sample, compute flat grads

Step 4: compute natural gradient



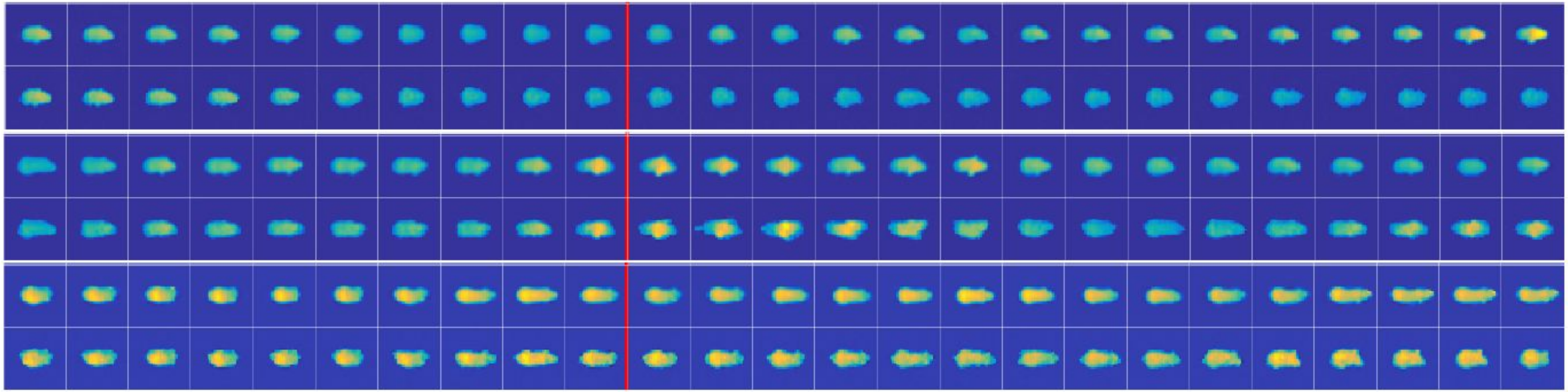
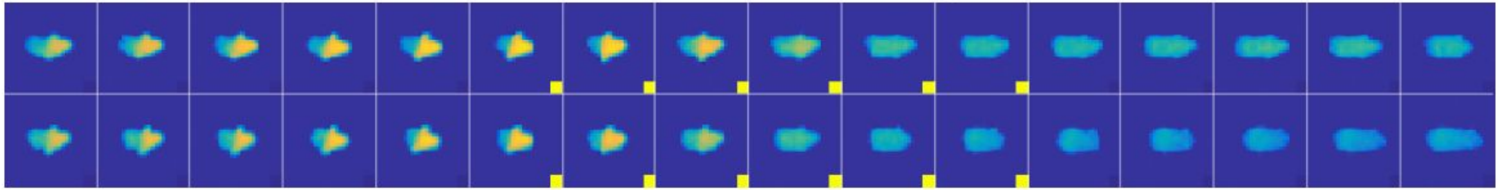
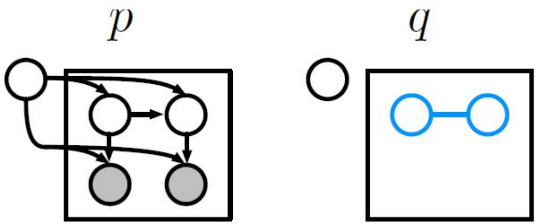


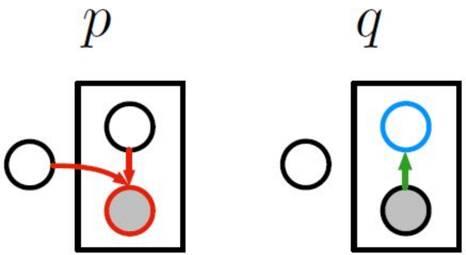
Figure 6: Predictions from an LDS SVAE fit to depth video. In each panel, the top is a sampled prediction and the bottom is real data. The model is conditioned on observations to the left of the line.

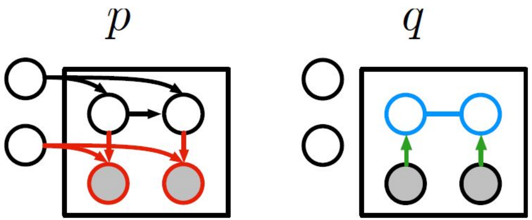


(b) Fall from rear

Figure 7: Examples of behavior states inferred from depth video. Each frame sequence is padded on both sides, with a square in the lower-right of a frame depicting when the state is the most probable.







$$q^*(x) \triangleq \underset{q(x)}{\arg\max}\, \mathcal{L}[\, q(\theta)q(x)\,]$$

– expensive for general obs.

\+ optimal local factor

\+ exploits conj. graph structure

\+ arbitrary inference queries

$$q^*(x) \triangleq \mathcal{N}(x \mid \mu(y;\phi), \Sigma(y;\phi))$$

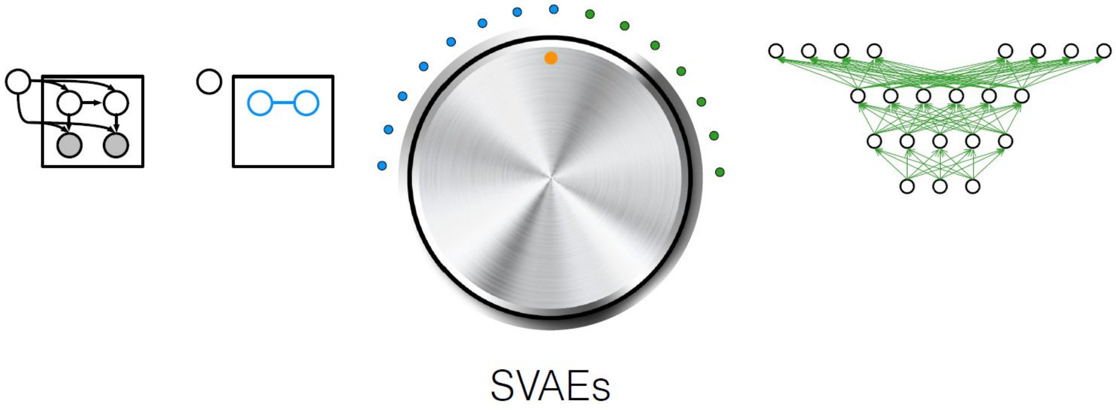
\+ fast for general obs.

– suboptimal local inference

– ϕ does all local inference

– limited inference queries

Conclusion



Possible applications

Reinforcement Learning and Bandits
Semi-supervised Learning