# Defocus Deblurring and Superresolution for Time-of-Flight Depth Cameras (Supplementary)

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## 1. Algorithm Details

This section provides implementation details for Algo. 2 (Amplitude update) and Algo. 3 (Depth update) in the main paper. The symbol $\nabla$ defines the derivative operator, $\mathsf{T}$ defines the matrix transpose, and $\mathbf{I}$ defines the identity matrix.

**Algo. 2, Line 2**:

$$\mathbf{a} = \underset{\mathbf{a}}{\operatorname{argmin}}\, \rho||\mathbf{c} - \mathbf{A}\mathbf{a}||_2^2 + \lambda_1 \rho_a ||\nabla\mathbf{a} - \mathbf{y} - \mathbf{p}_1 + \mathbf{u}_1||_2^2 \tag{1}$$

equals to the solution of the linear equation system:

$$(\rho\mathbf{A}^{\mathsf{T}}\mathbf{A} + \lambda_1\rho_a\nabla^{\mathsf{T}}\nabla)\mathbf{a} = \rho\mathbf{A}^{\mathsf{T}}\mathbf{c} + \lambda_1\rho_a\nabla^{\mathsf{T}}(\mathbf{y} + \mathbf{p}_1 - \mathbf{u}_1) \tag{2}$$

and we solve it by the left division function in Matlab.

**Algo. 2, Line 3**:

$$\mathbf{y} = \underset{\mathbf{y}}{\operatorname{argmin}}\, \lambda_1||\nabla\mathbf{a} - \mathbf{y} - \mathbf{p}_1 + \mathbf{u}_1||_2^2 + \lambda_2||\nabla\mathbf{y} - \mathbf{p}_2 + \mathbf{u}_2||_2^2 \tag{3}$$

equals to the solution of the linear equation system:

$$(\lambda_1\mathbf{I} + \lambda_2\nabla^{\mathsf{T}}\nabla)\mathbf{y} = \lambda_1(\nabla\mathbf{a} - \mathbf{p}_1 + \mathbf{u}_1) + \lambda_2\nabla^{\mathsf{T}}(\mathbf{p}_2 - \mathbf{u}_2) \tag{4}$$

and solved by the left division function in Matlab.

**Algo. 2, Line 4**:

$$\mathbf{p}_1 = \underset{\mathbf{p}_1}{\operatorname{argmin}}\, ||\mathbf{p}_1||_1 + \rho_a||\nabla\mathbf{a} - \mathbf{y} - \mathbf{p}_1 + \mathbf{u}_1||_2^2 \tag{5}$$

is a soft shrinkage problem and has closed form solution:

$$\mathbf{p}_1 = \text{soft-shrinkage}(\nabla\mathbf{a} - \mathbf{y} + \mathbf{u}_1, \frac{0.5}{\rho_a}) \tag{6}$$

where the soft-shrinkage operator is defined as:

$$\text{soft-shrinkage}(\mathbf{x}, \epsilon) = \begin{cases} \mathbf{x} + \epsilon; \mathbf{x} < -\epsilon \\ \mathbf{0}; -\epsilon \leq \mathbf{x} \leq \epsilon \\ \mathbf{x} - \epsilon; \mathbf{x} > \epsilon \end{cases} \tag{7}$$

**Algo. 2, Line 5**:

$$\mathbf{p}_2 = \underset{\mathbf{p}_2}{\text{argmin}}\, ||\mathbf{p}_2||_1 + \rho_a||\nabla\mathbf{y} - \mathbf{p}_2 + \mathbf{u}_2||_2^2 \tag{8}$$

is a soft shrinkage problem and has closed form solution:

$$\mathbf{p}_2 = \text{soft-shrinkage}(\nabla\mathbf{y} + \mathbf{u}_2, \frac{0.5}{\rho_a}) \tag{9}$$

**Algo. 3, Line 2**:

$$\mathbf{z} = \underset{\mathbf{z}}{\text{argmin}}\, \overbrace{\rho||\mathbf{c} - \mathbf{a}\circ\mathbf{g}(\mathbf{z})||_2^2}^{\text{data fitting constraint}} + \overbrace{\tau_1\rho_x||\nabla\mathbf{z} - \mathbf{x} - \mathbf{q}_1 + \mathbf{v}_1||_2^2}^{\text{prior constraint}} \tag{10}$$

is a nonlinear least squares problem due to the nonlinearity of the modulation function $\mathbf{g}(\mathbf{z})$. We solve this problem by the Levenberg-Marquardt method implemented in the lsqnonlin(.) function in Matlab. We provide the analytical Jacobian for acceleration:

$$J(\mathbf{z}) = \begin{bmatrix} J_{data}(\mathbf{z}) \\ J_{prior} \end{bmatrix} \tag{11}$$

where the matrix $J_{data}(\mathbf{z})$ and $J_{prior}$ define the Jacobian of the $1^{st}$ (data fitting constraint) and $2^{nd}$ (prior constraint) least squares in Eq. (10) respectively.

Since the $1^{st}$ least squares are pixel-wise separable (benefit from our splitting method explained in Sec. 3.1 in the main paper), $J_{data}(\mathbf{z})$ is simply a diagonal matrix composed of:

$$-\mathbf{a}_k \cdot \frac{\partial\mathbf{g}(\mathbf{z}_k)}{\partial\mathbf{z}_k} \cdot \sqrt{\rho} \tag{12}$$

where $k$ is the pixel index. For the ToF cameras based on cosine model modulation (see Eq. (1) in the main paper), the diagonal element in Eq. (12) becomes:

$$-\mathbf{a}_k \cdot i\frac{4\pi f}{c} \cdot e^{i(\frac{4\pi f}{c}\cdot\mathbf{z}_k)} \cdot \sqrt{\rho} \tag{13}$$

For arbitrary modulation waveforms in the future, the diagonal element in Eq. (12) can be estimated from calibration data. $J_{prior}$ is simply the matrix version of the derivative operator $\nabla$ multiplied by $\sqrt{\tau_1\rho_x}$, which is independent of $\mathbf{z}$.

**Algo. 3, Line 3**:

$$\mathbf{x} = \underset{\mathbf{x}}{\text{argmin}}\, \tau_1||\nabla\mathbf{z} - \mathbf{x} - \mathbf{q}_1 + \mathbf{v}_1||_2^2 + \tau_2||\nabla\mathbf{x} - \mathbf{q}_2 + \mathbf{v}_2||_2^2 \tag{14}$$

equals to the solution of the linear equation system:

$$(\tau_1\mathbf{I} + \tau_2\nabla^{\mathsf{T}}\nabla)\mathbf{x} = \tau_1(\nabla\mathbf{z} - \mathbf{q}_1 + \mathbf{v}_1) + \tau_2\nabla^{\mathsf{T}}(\mathbf{q}_2 - \mathbf{v}_2) \tag{15}$$

and solved by the left division function in Matlab.

**Algo. 3, Line 4**:

$$\mathbf{q}_1 = \underset{\mathbf{q}_1}{\text{argmin}}\, ||\mathbf{q}_1||_1 + \rho_x||\nabla\mathbf{z} - \mathbf{x} - \mathbf{q}_1 + \mathbf{v}_1||_2^2 \tag{16}$$

is a soft shrinkage problem and has closed form solution:

$$\mathbf{q}_1 = \text{soft-shrinkage}(\nabla\mathbf{z} - \mathbf{x} + \mathbf{v}_1, \frac{0.5}{\rho_x}) \tag{17}$$

**Algo. 3, Line 5**:

$$\mathbf{q}_2 = \underset{\mathbf{q}_2}{\operatorname{argmin}} ||\mathbf{q}_2||_1 + \rho_x ||\nabla \mathbf{x} - \mathbf{q}_2 + \mathbf{v}_2||_2^2 \tag{18}$$

is a soft shrinkage problem and has closed form solution:

$$\mathbf{q}_2 = \text{soft-shrinkage}(\nabla \mathbf{x} + \mathbf{v}_2, \frac{0.5}{\rho_x}) \tag{19}$$