D-Charts: Quasi-Developable Mesh Segmentation

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Motivation

• Mesh segmentation into compact charts that unfold with minimal distortion

• Applications
  – Parameterization for mapping
    • Textures, Bumps, BRDFs, displacement maps, etc.
    • Geometry Images
  – Patterns for sewing, metal forging
  – CAD
    • Reverse engineering
    • Surface reconstruction
Developable surfaces

- Surfaces that unfold onto the plane with zero distortion
- Gaussian curvature is zero at every point

Find a quasi-developable segmentation
Previous work

- Texture atlas generation
  - [Levy et al. ‘02], [Zhou et al. ‘04]
  - [Garland et al. ’01], [Sander et al. ’03]
- Feature based
  - [Katz and Tal ‘03]
  - [Gelfand and Guibas ‘04]
- Patterns:
  - [McCartney et al. ‘99]
  - [Mitani and Suzuki ‘04]
- Developable surfaces
  - [Leopoldseder and Pottmann ‘98]
  - [Pottman et al. ’01]

Planar

Not developable

Not compact

Not segmentation
Lloyd based segmentation

- We use Lloyd segmentation approach
- Introduced by [Cohen-Steiner et al. ‘04] - planar charts
- Various extensions presented at EG ‘05
- Charts represented by proxies:
  - Normal to plane
  - Seed triangle
- Challenges:
  - Developable proxies
  - Bound error
Lloyd based segmentation – Framework

- Lloyd iterations:
  1. Select random triangles to act as seeds
  2. Grow charts around seeds using a greedy approach
  3. Find new proxy for each chart
  4. Repeat from step 2 until convergence

[Cohen Steiner et al. ‘04]
Devlopable surfaces of constant slope

- Developable surfaces – Hard to capture
  - Start with subset, broaden later
- Constant angle between surface normal and axis → Developable chart
- Proxy: <axis, angle> $\langle N_c, \theta_c \rangle$
Fitting error

- Measures how well triangle fits a chart
  \[ F(C, t) = (N_C \cdot n_t - \cos \theta_C)^2 \]
- Combine with compactness
  \[ C(C, t) = \pi \frac{D(S_c, t)^2}{A_c} \]

\[ \text{Cost}(C, t) = A_t F(C, t)^\alpha C(C, t)^\beta \]
Algorithm overview

- Bounded Lloyd iterations
- Hole Filling
- Merging
- Post-Processing & Parameterization
Bounded Lloyd iterations

- Initialization
  - Random / Furthest point seeds
  - Compute initial proxy
- Bounded Growing/Reseeding iterations
- Termination
Bounded Lloyd iterations – Growing

- Use greedy approach
  - Prioritize by $\text{Cost}(C, t)$
- Bound Fitting Error
  - Guarantee (nearly) developable charts
Bound Lloyd iterations – Reseeding

• Find new proxy

\[
\min_{N_C, \theta_c} \frac{1}{A_C} \sum_{t \in C} A_t F(C, t) \text{ s.t. } \|N_C\| = 1
\]

• Find new seed
  – Minimal *Fitting Error*
  – Close to center of chart
Algorithm overview

- Bounded Lloyd iterations
- Hole Filling
Hole filling

- Bound on *Fitting Error*
  \[\text{Unclassified triangles}\]
- Fill holes
  - Large holes \(\rightarrow\) New proxy
  - Small holes \(\rightarrow\) Grow neighbors
Algorithm overview

1. Bounded Lloyd iterations
2. Hole Filling
3. Merging
Merging

- Broaden set of captured developable surfaces
- Reduce number of charts
Algorithm overview

Bounded Lloyd iterations → Hole Filling → Merging 

Post-Processing & Parameterization
Post processing

- Straighten boundaries
- Darts/Gussets relax stress
  - Add seams toward high error regions
- Verify disc topology
- Parameterization

[Sander et al. ’02] [Sheffer et al. ’05]
Example results – CAD
Example results – Fandisk

Iso-Charts, [Zhou et al. ’04]
MCGIM, [Sander et al. ’03]
D-Charts

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<th>Fandisk</th>
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Example results – irregular meshes

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<tr>
<th>Gargoyle</th>
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Soft and paper-craft toys
Summary

- Segment mesh into nearly developable charts
- A simple metric of developability for surface charts – *The Fitting Error*
- Use bounded Lloyd iterations
- Use Holes / Merging to correct no. of charts
Thank you

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