On the Empirical Time Complexity of Random 3-SAT at the Phase Transition

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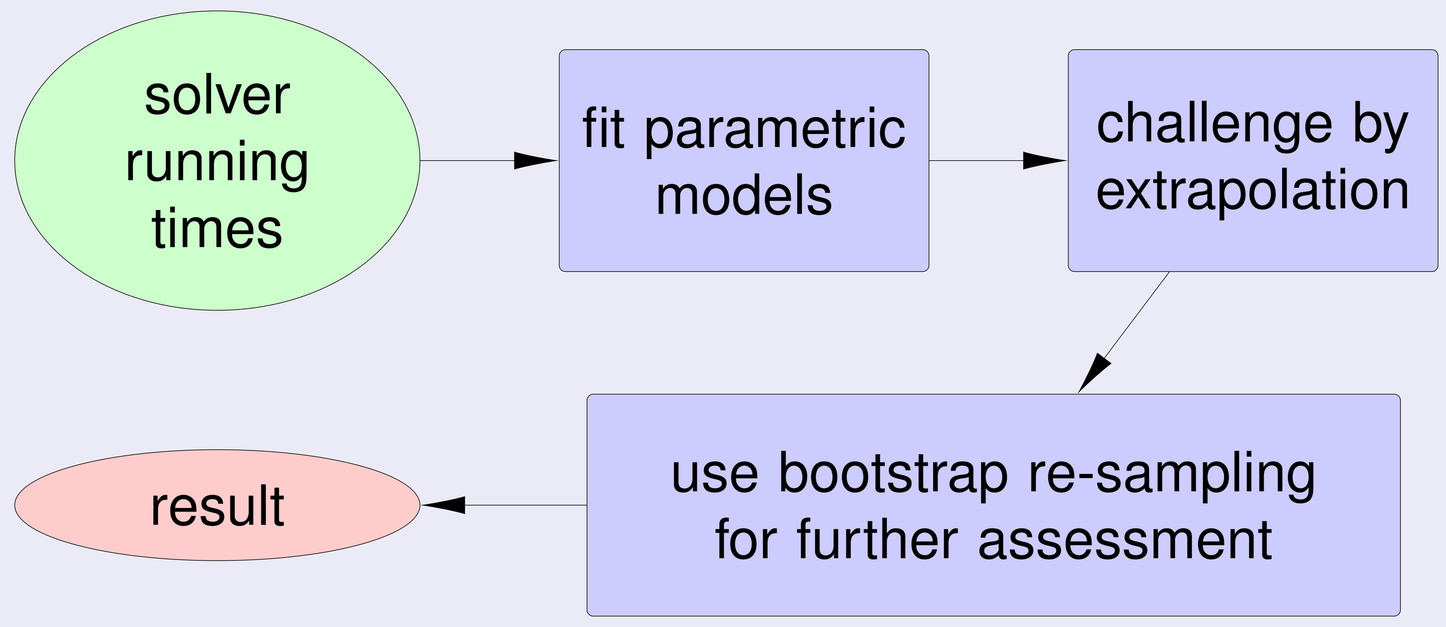
Main Results

To solve phase-transition random 3-SAT instances, the running times:

- scale polynomially for SLS-based solvers; no significant difference among scaling models
- scale exponentially for DPLL-based solvers; two march-variants scale significantly better than kcnfs
- are smaller by constant factor for DPLL-based solvers when solving satisfiable instances

Methodology

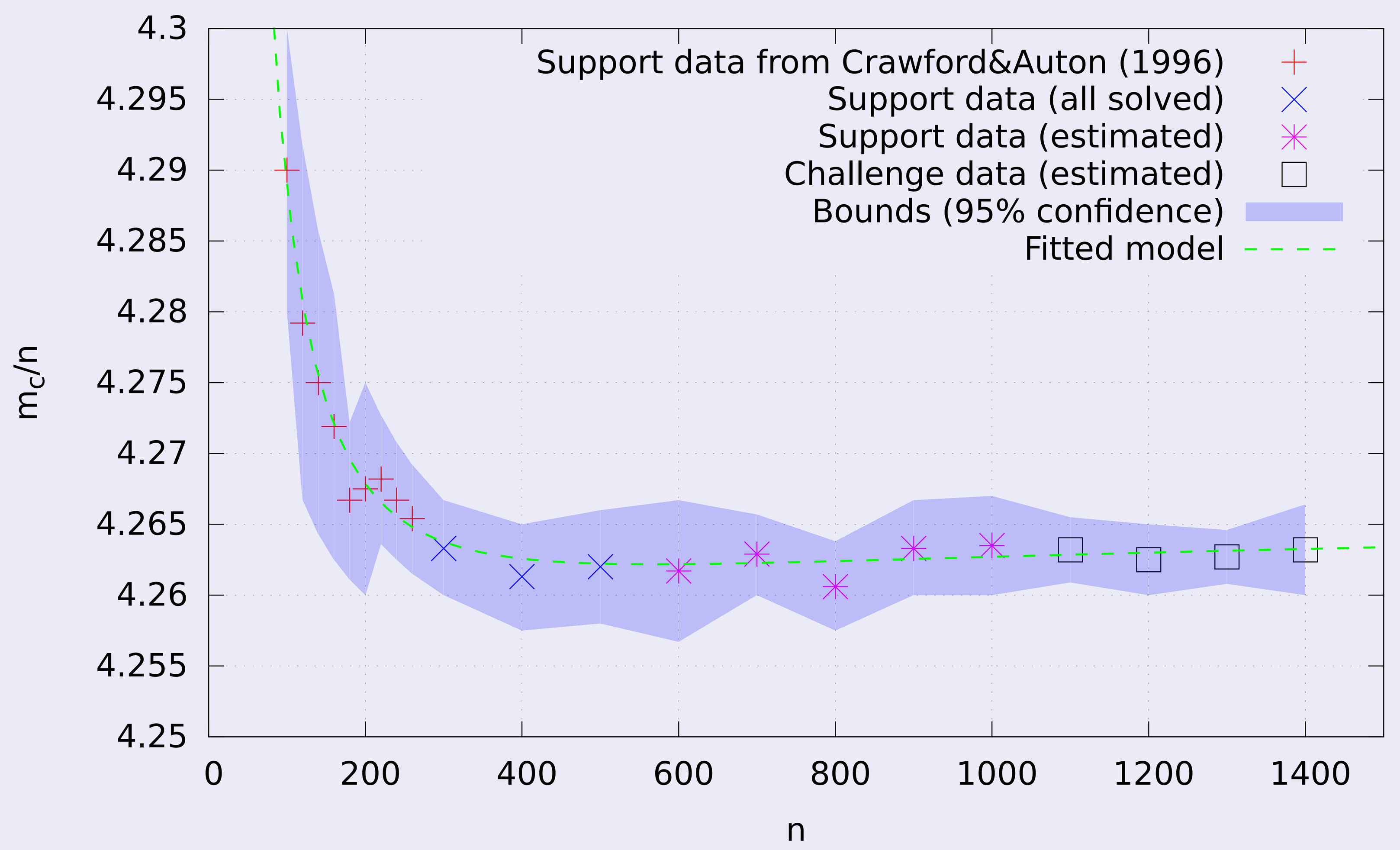


Extensions to existing methodology [2, 3]:

- Use conf. intervals of observed data to assess models
- Compare scaling models of two solvers based on conf. intervals of observed/predicted data

Location of Phase Transition

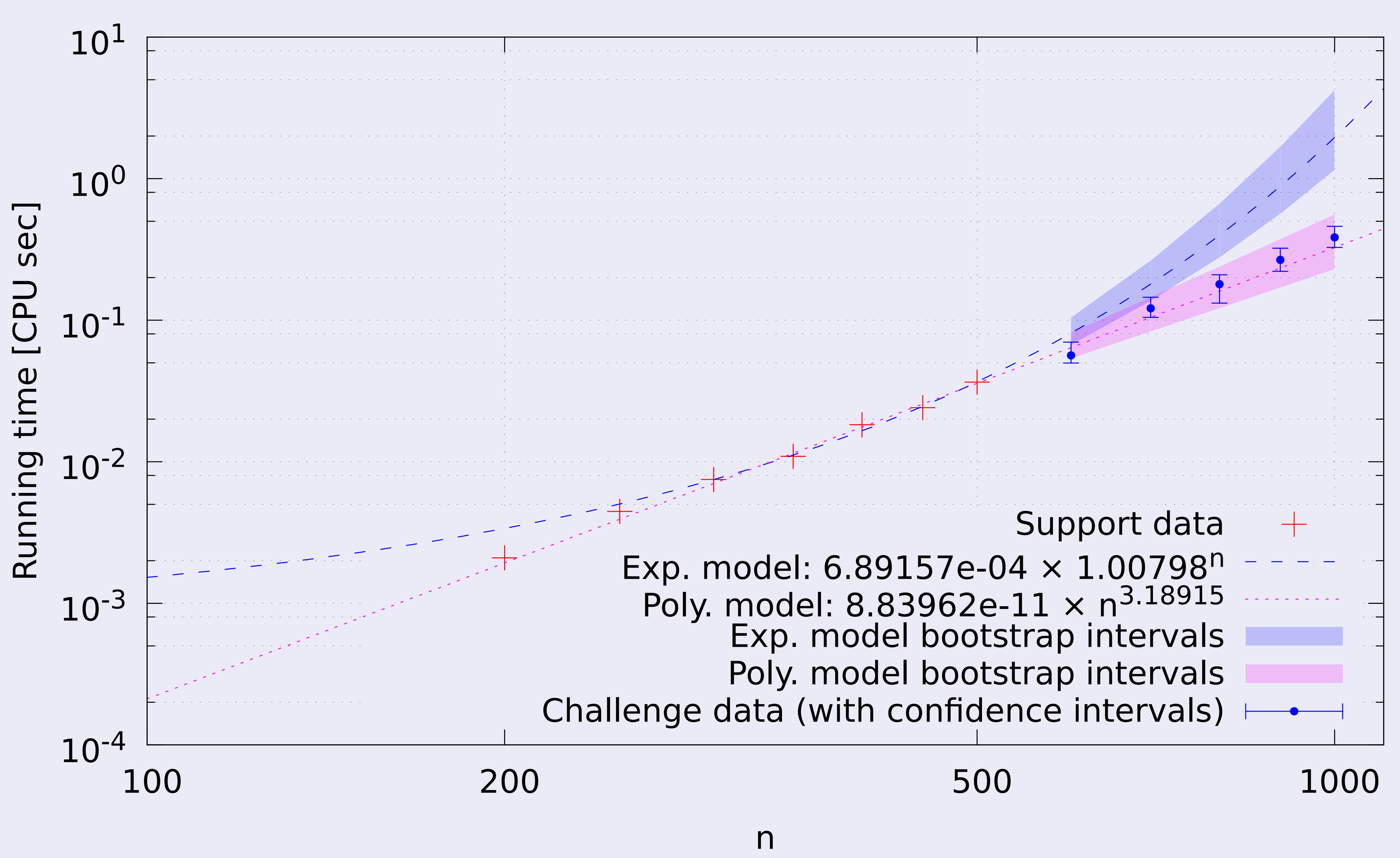
Empirical bounds on location of phase transition, m_c/n:



Refined model based on [4] & empirical data (partly obtained from [1]):

$$m_c = 4.26675n + 447.884n^{-0.0350967} - 430.232n^{-0.0276188}$$

Graphical Results for WalkSAT/SKC



Scaling Models for SLS-based Solvers

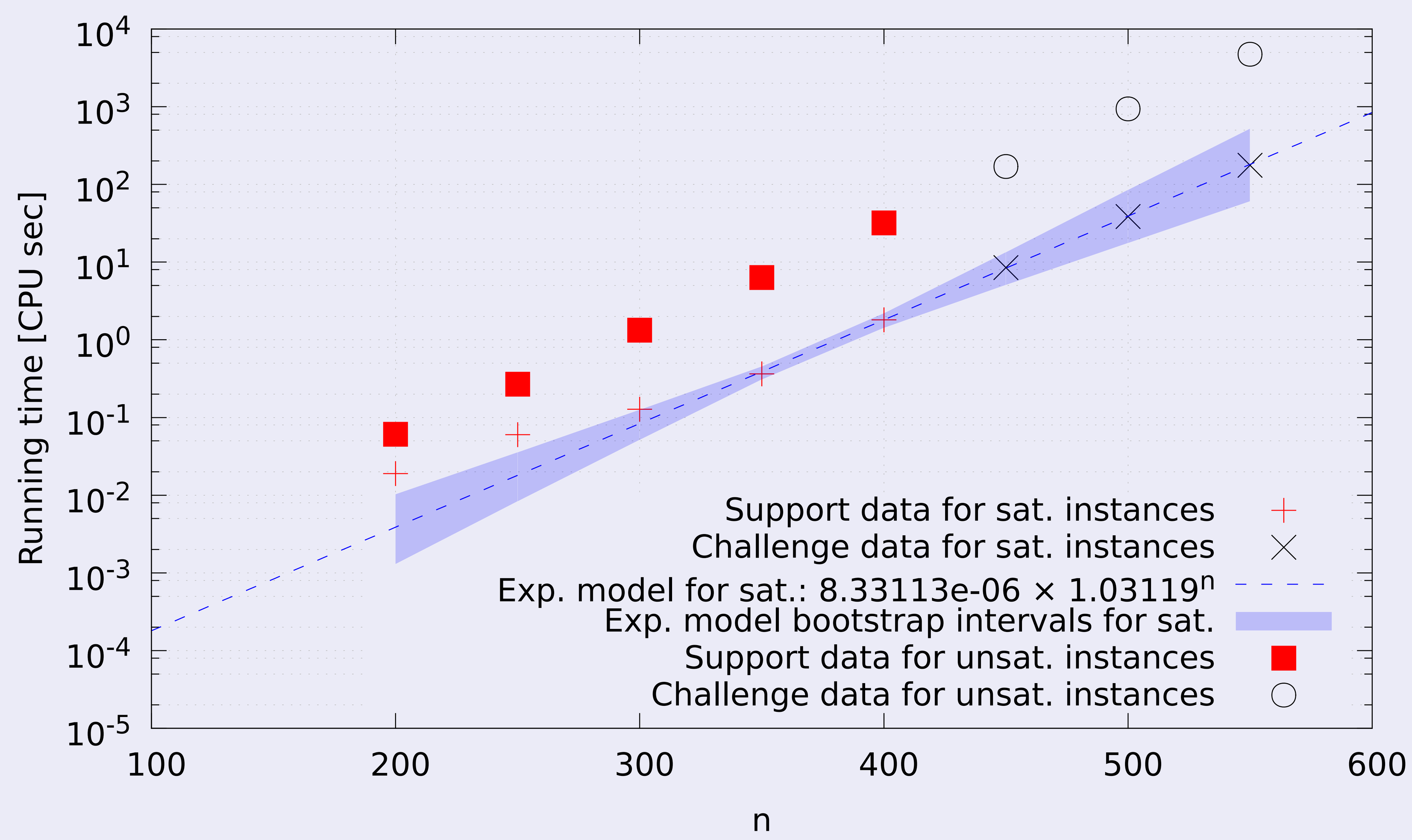
Fitted models of median running times:

		Model	RMSE (support)	RMSE (challenge)
Walksat/SKC	Exp. Model	$6.89157 \times 10^{-4} \times 1.00798^n$	0.0008564	0.7600
	Poly. Model	$\mathbf{8.83962 \times 10^{-11} \times n^{3.18915}}$	**0.0007433**	**0.03142**
BalancedZ	Exp. Model	$1.32730 \times 10^{-3} \times 1.00759^n$	0.001759	1.081
	Poly. Model	$\mathbf{5.14258 \times 10^{-10} \times n^{2.97890}}$	**0.002870**	**0.05039**
probSAT	Exp. Model	$8.35877 \times 10^{-4} \times 1.00763^n$	0.0013867	0.6487
	Poly. Model	$\mathbf{2.92275 \times 10^{-10} \times n^{2.99877}}$	**0.002285**	**0.03301**

95% confidence intervals for polynomial model parameters:

Solver	Confidence interval of a	Confidence interval of b
WalkSAT/SKC	$[2.58600 \times 10^{-12}, 8.63869 \times 10^{-10}]$	$[2.80816, 3.76751]$
BalancedZ	$[3.65984 \times 10^{-11}, 4.53094 \times 10^{-9}]$	$[2.60985, 3.41689]$
probSAT	$[5.00843 \times 10^{-12}, 1.02411 \times 10^{-8}]$	$[2.40567, 3.66266]$

Graphical Results for march_hi



Scaling Models for DPLL-based Solvers

Fitted models of median running times:

			Model	RMSE (support)	RMSE (challenge)
kcnfs	All	**Exp. Model**	$\mathbf{4.30400 \times 10^{-5} \times 1.03411^n}$	**0.05408**	**143.3**
		Poly. Model	$9.40745 \times 10^{-31} \times n^{12.1005}$	0.06822	1516
	Sat.	**Exp. Model**	$\mathbf{2.41708 \times 10^{-5} \times 1.03205^n}$	**0.02496**	**83.86**
		Poly. Model	$2.41048 \times 10^{-30} \times n^{11.7142}$	0.05600	225.8
	Unsat.	**Exp. Model**	$\mathbf{6.38367 \times 10^{-5} \times 1.03386^n}$	**0.001466**	**52.19**
		Poly. Model	$9.70804 \times 10^{-31} \times n^{12.1448}$	0.1813	2291
march_hi	All	**Exp. Model**	$\mathbf{4.93309 \times 10^{-5} \times 1.03295^n}$	**0.05449**	**460.0**
		Poly. Model	$1.05593 \times 10^{-30} \times n^{12.0296}$	0.05971	1266
	Sat.	**Exp. Model**	$\mathbf{8.33113 \times 10^{-6} \times 1.03119^n}$	**0.03035**	**3.087**
		Poly. Model	$2.44435 \times 10^{-30} \times n^{11.4789}$	0.03879	61.77
	Unsat.	**Exp. Model**	$\mathbf{7.86081 \times 10^{-5} \times 1.03281^n}$	**0.03336**	**399.7**
		Poly. Model	$2.10794 \times 10^{-30} \times n^{11.9828}$	0.1670	1912
march_br	All	**Exp. Model**	$\mathbf{6.17600 \times 10^{-5} \times 1.03220^n}$	**0.05401**	**402.4**
		Poly. Model	$5.56146 \times 10^{-30} \times n^{11.7408}$	0.05199	1253
	Sat.	**Exp. Model**	$\mathbf{1.02788 \times 10^{-5} \times 1.03048^n}$	**0.02497**	**13.72**
		Poly. Model	$1.25502 \times 10^{-29} \times n^{11.1944}$	0.03341	67.85
	Unsat.	**Exp. Model**	$\mathbf{6.10959 \times 10^{-5} \times 1.03344^n}$	**0.03230**	**262.8**
		Poly. Model	$5.18600 \times 10^{-31} \times n^{12.2154}$	0.1586	1920

95% confidence intervals for exponential model parameters:

Solver	Instances	Confidence interval of a	Confidence interval of b
kcnfs	All	$[3.33378 \times 10^{-5}, 1.07425 \times 10^{-4}]$	$[1.03136, 1.03476]$
	Sat.	$[2.02817 \times 10^{-6}, 5.85540 \times 10^{-4}]$	$[1.02283, 1.03835]$
	Unsat.	$[4.22382 \times 10^{-5}, 1.03613 \times 10^{-4}]$	$[1.03252, 1.03508]$
march_hi	All	$[2.90480 \times 10^{-5}, 1.72479 \times 10^{-4}]$	$[1.02928, 1.03433]$
	Sat.	$[7.97341 \times 10^{-7}, 7.07414 \times 10^{-5}]$	$[1.02521, 1.03765]$
	Unsat.	$[5.51043 \times 10^{-5}, 1.06014 \times 10^{-4}]$	$[1.03195, 1.03386]$
march_br	All	$[2.61030 \times 10^{-5}, 1.08165 \times 10^{-4}]$	$[1.03064, 1.03466]$
	Sat.	$[1.81911 \times 10^{-6}, 6.40234 \times 10^{-5}]$	$[1.02515, 1.03519]$
	Unsat.	$[4.27173 \times 10^{-5}, 9.18950 \times 10^{-5}]$	$[1.03230, 1.03443]$

References

[1] J. M. Crawford and L. D. Auton. Experimental results on the crossover point in random 3-SAT. *Artificial Intelligence*, 81(1):31–57, 1996.

[2] H. H. Hoos. A bootstrap approach to analysing the scaling of empirical run-time data with problem size. Technical report, TR-2009-16, Dept. of Computer Science, Univ. of British Columbia, 2009.

[3] H. H. Hoos and T. Stützle. On the empirical scaling of run-time for finding optimal solutions to the travelling salesman problem. *European Journal of Operational Research*, 238(1):87–94, 2014.

[4] S. Mertens, M. Mézard, and R. Zecchina. Threshold values of random k-SAT from the cavity method. *Random Structures and Algorithms*, 28(3):340–373, 2006.