A Tool for the Design of Audience Space with Predetermined Visibility Performance.

The Design of Seating Surfaces by the Use of an Iseidomal Criteria.

by

MERT CRAMER

Computer Graphics Laboratory

Department of Computer Science

University of Waterloo

Waterloo, Ontario, Canada N2L 3G1

ABSTRACT

In the past, the designer of a theatre had only "rule of thumb" recommendations culled from the literature as a guide to the proper design of the audience seating. This essay describes a prototype of an interactive CAD tool called "Seating Planner" which provides to the auditorium designer a quantitative "figure of merit" for the average visibility available to the audience. Using anthropometric data for head and eye heights, and a probabilistic model, the Seating Planner predicts the chances of successful viewing from any seat according to the criterion that every viewer should have a known probability of seeing a specified critical point on the stage.

With this system a designer can "tune" the shape of the audience surface to meet size constraints or to favor some part of the audience. This is accomplished by establishing a contour with a uniform probability of viewing from each seat, or by arbitrarily adjusting the viewing probability of each row or a group of rows. The system can also be used to analyze the performance of an existing floor contour.

A discussion of the implementation of the system is presented. Particular emphasis is given to calculating the probability of viewing as a quantitative "figure of merit", and to the design of an effective interactive user interface. Possible future extensions to this system and problem areas with the current version are discussed.

Table of Contents

Table of Figures

Acknowledgments

The Seating Planner is the latest step in the solution of a problem that was raised in 1963 by P. S Veneklausen in his recommendations on the design of the acoustics of the Los Angeles Music Center. In 1981, W. deV. Keet reawakened my interest in the question of proper design of auditorium seating at the SAITT Conference on Computerization in the Theatre, at Pretoria, South Africa.

With the encouragement of K. S. Booth, this project started originally as an application of the Evans and Sutherland MPS calligraphic line drawing system. As the project developed, the solution of the anthropometric and probability questions became more important. The advice of R. H. Bartels, A. U. Brender, J. H. Johnson and J. G. Kalbfleisch were most helpful in clarifying the nature of the problem and in producing an algorithm for its solution.

As must be evident from the contents of this paper, a number of individuals who are concerned with seating design for theatres have made contributions to this work. Conversations with G. T. Howard, W. Russell, and A. Shaw were most useful. In addition, I. Cole, I. Hardtke, and D. H. Kohanek provided a translation of K. Wever's article. The presence of the drawings in this paper are in large part due to the advice and assistance of I. Telford, the work of F. H. Gunn, and the example provided by R. H. Cole.

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1. Introduction

> *Not only do anthropometric dimensions vary amongst individuals but the sitting postures adopted by each one are not susceptible to a discipline which would be convenient for mathematicians.*
>
> *In other words, the problem of the large lady with a hat sitting in front of a small shy gent will still remain.*
>
> Roderick Hamm

With any building design the architect has the problem of supplying the most value for the money. When planning a theatre, opera house or similar facility, a major concern is that the audience be provided with seating that gives the best feasible visibility of the stage. The classical technique for establishing the contour of the audience floor has not provided the architect with a "figure of merit" with which to judge the proposed design. Thus, in the past, the architect had to depend on the "rule of thumb" recommendations in the literature. He knew that this would produce a reasonable result; a recommendation that produced poor results would have been discarded long ago. But if the circumstances required some deviation from the formula, there has been no way to explicitly evaluate the results of the changes.

It is possible to quantify a "figure of merit" for the property of the view seen by the audience. It is the purpose of this essay to show a method of calculating this measure of the audience's success in viewing the stage. The essay discusses an interactive CAD tool called the Seating Planner, which has as its goal to make the evaluation of audience floor contours faster and easier.

The audience visibility calculation depends on a number of parameters including the separation between rows of seats, the width of the seat backs, the vertical rise between rows, whether seats are arranged "in line" or are staggered row-to-row, and the probabilities that audience members can see over the heads of others. All of these factors (and many more) influence the design of an audience space. Each has an impact on the chance for successful viewing. Changes in the design which alter the viewing affect the total enclosed volume of the building and consequently its cost. Until now the methods for developing good viewing conditions for an audience have not provided the design team with a technique that allows the design to be "tuned" to provide the best possible result within the budget. Designers have not had an easy way of assessing the effect of changes, and thus have not been fully aware of the impact of their decisions.

The problem of adequate viewing for the audience has a number of major components. This essay and the the Seating Planner system consider only the most fundamental of these. Rather than solve the full three-dimensional problem, the problem has been simplified to a consideration of only the single cross section of the seating contour that lies along the central axis from the stage to the back of the auditorium, effectively reducing the computation to two dimensions. The center line cross-section is only part of the design problem. The questions of oblique sections and the distortion of viewing caused by displacement away from the center-line are beyond the scope of this essay. Indeed, such factors can only be considered after the center-line contour is developed.

The approach taken in this essay starts from the traditional technique in which a sighting point is established on the stage and line-of-sight lines are used to determine visibility from each seat in the audience. From this basic approach the notions of probability theory and the use of anthropometric measures provide a new way of determining the contour of the audience surface that provides an acceptable view of the defined critical point. "Seating Planner", an interactively controlled CAD system implemented on a calligraphic display, makes it possible to gain a better understanding of the issues which affect the chances for an adequate view of the stage. It provides the architect with a definite, quantitative measure (the "iseidomal measure") and provides the means to explore the effects of alterations to a room design without the need for the laborious lofting exercises characteristic of more traditional approaches. Seating Planner is an extension of earlier work by the author [CRAM68], this time using both the utility of interactive computing and the notions of probability theory to provide a tool to architects and theatre designers.

Two issues were paramount in the development of the Seating Planner system. First, a quantitative figure of merit is needed to assess the possible designs for an audience space. We will work with the idea of an *iseidomal measure.* The dictionary defines "iseidomal" as "applied to a curve passing through points (in a theatre, etc.) from which a spectacle may be seen equally well" [OED71]. The second issue is the requirement for interactive control of the Seating Planner to make for convenient evaluation of alternative designs. The major contribution of the work reported here is the application of probability theory, in an effective interactive environment, to the problem of theatre design.

This essay is divided into four sections. The classical approach to the design of audience spaces and some recent developments are the topics of Section One. Section Two discusses the calculation of row elevations in a more complete way than used heretofore, and defines the visibility performance "figure of merit" based on the iseidomal criterion and a probabilistic interpretation of anthropometric data. The development of the Seating Planner as an interactive CAD tool is discussed in Section Three. This system is only a prototype. Problems with the current implementation and future possibilities are then treated in Section Four.

1.1 The Audience Space

Even though most theatres and concert halls may seem nearly alike on casual inspection, the designer of each room must make a number of choices when he plans the space. A variety of somewhat specialized terms and a common set of symbols will be used throughout this essay. These are defined here.

Aisles vs. Continental Seating

Prior to the Second World War, theatres were usually built with the audience surface divided into blocks of seats separated by aisles [BURR49]. A major consideration in the planning of the seating layout is the portion of the Fire or Building Code concerned with the safety of audiences leaving the building during of a fire or other emergency. The code usually specifies the width of aisles, spacing between rows, and limits on the maximum distance a patron may be from an aisle. A fairly common arrangement, dictated by requirements of the fire code, is that no seat be more than six seats from the aisle. This results in a seating plan having rows of fourteen seats in a line with an aisle on either side. Rows are allowed to be adjacent to walls or pillars, but with only seven seats in a line.

The fire code also specifies the minimum value of the spacing between rows, again to insure that in emergency situations the audience will be able to exit quickly and safely. With aisle seating this inter-row spacing is fairly small. Many newer buildings use so-called *Continental Seating* where the line of seats extends from one side wall of the room to the other, with no aisles breaking the rows of seats. Entry and exit are through doors at both ends of the rows. This style of seating has the property that all of the floor space is used for seating, increasing utilization. This is not entirely free. With most fire codes the removal of the interior aisles requires a larger minimum row-to-row spacing so that Continental Seating may actually require more area per audience member than the older aisle seating.

In-line vs. Staggered Seats

Often the seats in adjacent rows are arranged so that the center of each seat is on a line from the back wall of the room to the stage. An alternate scheme is to align the center of the back of each seat with the armrest dividing the two seats of the row directly in front. In this way, a person always looks between the two people seated in the row directly in front, but over the head of the person two rows in front. This staggered arrangement reduces the rise in the floor between rows, thus reducing the height of the back row and hence the total volume of the building and hopefully its cost.

Transverse Aisles (Cross-Aisles)

When aisle seating is used, there are often *transverse aisles* which allow the audience to cross from one side of the room to the other. Another name for this type of aisle is the *cross-aisle*. These must be taken into account when sighting calculations are made.

The Sighting Point

The fundamental idea behind iseidomal viewing is that there should be a point which all patrons are able to see. This *sighting point* is defined with respect to the first row of seats. It is some selected point in front of the front row of seats (presumably on the stage) at a specified distance above or below the floor level of the front row. The theatre designer must raise the height of successive rows enough to allow one patron to see the sighting point over the heads of the others. The height of the last row has a direct effect on the overall height of the building and consequently on the cost of construction, maintenance and heating. Thus, all things being equal, the designer wishes to minimize the height of the back row.

Balconies

The reduction in back row height also lowers the height of the balcony. This can be important, as the higher the balcony front row, the steeper the slope of the balcony floor. There are limits imposed by the fire code as to the maximum angle of the floor slope that will be allowed. In addition, a patron seated at the back of a steeply sloped balcony floor will see mostly the top of the actors' heads and a foreshortened view of their faces. The limiting vertical angle of view before severe visual distortion becomes a problem is between 30 and 35 degrees [BURR64][HAMM72]. The height of the first row of the balcony depends on the height of the floor below it. The lower the first row of the balcony, the greater the number of rows of seats before this critical vertical angle is reached. Behind all of this lurks the fact that the more rows of acceptable seating, the greater the potential income from a performance and hence the greater the justification for the building's construction and the architect's fee.

Basic Parameters for Discussing Audience Seating Design

There are a set of physical measurements which will be used to describe the problem. They are indigenous to the design of audience surfaces. The particular symbols and labels attached to the various parameters are not standardized in the literature. The ones used here are convenient for our purposes.

a	The location of the sighting point above (or below) the row one floor level.
b	The distance of the sighting point from the front of row one.
β	The distance of the sighting point expressed in rows ($\beta = b/h$).
c	The width of a cross-aisle.
h	The row-to-row spacing.
i	The index of a row.
l_{head}	The height of the top of the viewer's head above the seat plane.
l_{eye}	The height of the viewer's eye above the seat plane.
l_{diff}	The difference ($l_{head} - l_{eye}$).
l_{stand}	The height of the top of the head of a standing viewer.
n	The total number of rows in the seating area.
s	The height of the seat plane above the floor.
w_i	The height of the eye in the ith row above the row one floor level.
y_i	The height of the floor level at the ith row above (or below) the first row level.
z	An arbitrary distance between the ith eye position and the floor level of the ith row.

1.2 Russell's Method

John Scott Russell (1808-1882) was the first to discuss the proper shape for a large lecture hall [RUSS38]. As Professor of Natural Philosophy at the University of Edinburgh, his major concern was how to design ship hulls to reduce losses due to friction. He of course lectured to engineering students and thus became concerned with the proper acoustic design of the rooms he was using. He and his students performed a series of tests to determine the floor slope which allowed auditors adequate reception. The result was that Russell was the first to discuss a method to develop the layout of the floor slope to an iseidomal or isoacoustic criterion.

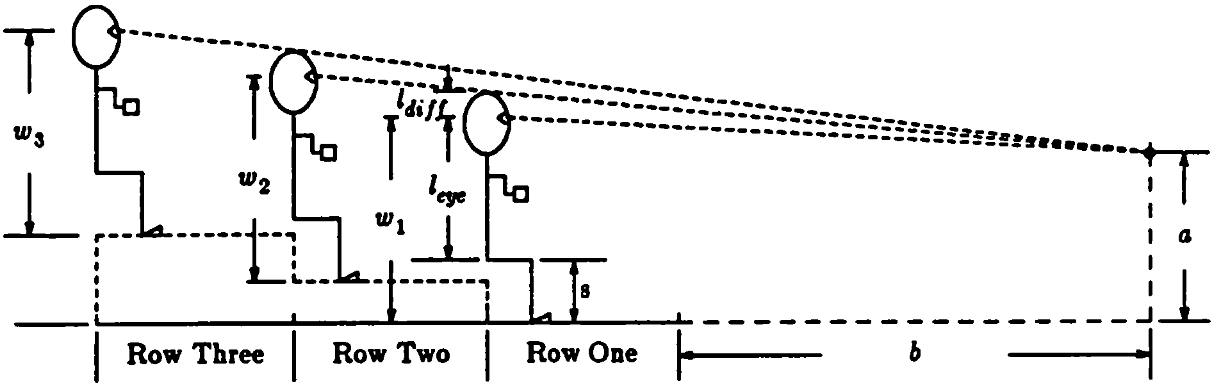


Figure 1.1

Russell's Method of Establishing an Iseidomal Floor Slope.

His work, and that of W. C. Sabine at Harvard seventy years later, form the beginning of the science of architectural acoustics. Sabine was concerned with reverberation (the long-term storage of energy within a room), Russell was concerned with tracing the line of sight from the speaker to the hearer or viewer. Russell's isoacoustic and iseidomal criteria differ only in the choice of using the ear or eye as the reception point when the calculation of the floor contour is made. In our case the visibility of the speaker by the audience is the key issue, so the iseidomal criterion will be used.

Russell's layout procedure is usually performed as a drafting exercise with the result derived by the use of a straight-edge and a scale. The relation between row elevations depends on the use of a constant given in the literature. The performance, in the sense of successful viewing, is not explicitly stated. Russell's procedure has been restated in books concerned with the architecture of theatres ever since. That the prescribed values usually given in the literature produce, in fact, an acceptable result will be shown in Section 2.3 [BURR64][HAMM72][IZEN77]. As there is no quantitative measure provided by Russell's method, what is left unanswered is how the chances for good visibility are changed by deviating from one of the recommended values.

Russell's construction is a simple one.

1. The two critical points needed to start Russell's layout procedure are the location of the eye of the person seated in the first row and the location of the sighting point which everyone should be able to see. Once these are picked, the location of the following rows proceeds in a step by step manner.

2. For each row, starting with the second, draw a line of sight from the sighting point (in Russell's case the mouth of the speaker) grazing the head of the person in front of the viewer. Locate the eye of the viewer at the point where the line off sight intersects the row. The floor level for the row is established a fixed distance below the eye point.

To establish the line of sight, the seated head height and the seated eye height dimensions are required. The problem is that the architect cannot, in general, predict the eye height of the person seated nor the head height of the person in front. An approximation is employed. As shown in Figure 1.1, l_{diff} is the necessary dimension. The value used in the literature, typically 5 inches (12.7 cm), is nearly, but not exactly, the difference between the mean value of the head height (l_{head}) and the mean value of the eye height (l_{eye}). This is discussed in Section 2.3.

The calculation gives the position of the eye of the patron seated in the *ith* row. But where is the floor level? The height of the floor underneath the seat is set by subtracting a fixed distance, z, from the eye height. The z that appears in the literature is an arbitrary value of 44.0 inches (112 cm) [BURR64][HAMM72]. After subtracting the 16.5 inches for the usual seat-to-floor distance, the remaining 27.5 inches is less than the mean eye height for men or women. In fact, almost all men and most women have a seated eye height which is greater than this value. Thus, Russell's method as found in the literature contains two arbitrary constants (z and l_{diff}). They appear to be derived from anthropometric data but there is no indication as to how the particular values were chosen. The difficulties presented by the uncertainties in the selection of l_{diff} and z will be left for later.

The results of the drafting exercise can be be expressed mathematically as a recurrence relation. The grazing incidence line from the eye over the head in front is one side of a pair of similar right triangles with a common vertex at the sighting point. Figure 1.1 shows the geometric relationship between the rows that is stated in Equation 1.1.

$$\frac{w_i - a}{i\,h + b} = \frac{w_{i-1} + l_{diff} - a}{(i-1)h + b} \qquad \text{for } i \geq 2 \tag{1.1}$$

Solving for w_i (the height of the eye in the *ith* row) gives Equation 1.2.

$$w_i = \left(\frac{i+\beta}{i-1+\beta}\right) w_{i-1} + \left(\frac{i+\beta}{i-1+\beta}\right)(l_{diff}) - \left(\frac{a}{i-1+\beta}\right) \tag{1.2}$$

This recurrence can be used to calculate the height of any row relative to row one. Using the floor level at row one as a point of reference for all measurments sets the eye height of the first row at $w_1 = z$. We can then reference the succeeding row heights to the first row floor elevation. This allows the location of any row to be determined, assuming uniform spacing between rows. This method has been used in the majority of discussions of the design of audience seating surfaces.

There are several objections to Russell's method:

1. The critical factor for the establishment of the height of the eye position at each row is the difference between the head height and the eye height of seated audience members. The value recommended in the literature appears to be an approximation to the difference in the mean values of the head and eye heights. But this difference has one value for men and another for women. Worse, the difference of the mean heights depends on the composition of the audience. An all male audience has a mean difference of 4.9 inches (12.4 cm.), while an all female audience would have a mean value 4.6 inches (11.7 cm.). It would seem that a composite audience of men and women would have a mean value somewhere in between the two extremes, but when a man sits behind a woman the mean difference is only 1.9 inches (4.8 cm.), while in the case of a woman sitting behind a man the mean difference is 7.6 inches (19.3 cm.). In fact, the situation is more complex because, in general, there is no way to predict where the men and women of the audience will be seated. What is recommended in the literature is a value which appears to produce an "acceptable" result.

2. Russell's method does not provide the designer an understanding of the effect of using a particular value of the constant l_{diff}. Indeed, for the recommended l_{diff} the visual performance is not specified by any quantitive measure. More important, the question of what happens when a different value must be used is left unanswered. In either case, the designer is left to take his chances blindly and without a firm idea of the impact of his decisions.

3. Russell's construction plots a set of points which represent the eye positions of the occupants of the seats. The floor line for each row is established by dropping down a fixed distance. What is the recommended value? If the seat height (16.5 inches) from the floor is subtracted, Table 2.1 shows that the remaining quantity (27.5 inches) is not the mean value of the eye height for men nor for women. In fact, 99.9 percent of male audience members and 95 percent of the female audience members will have an l_{eye} exceeding this value. The impact of altering z is not clear. The literature gives no clue to how its value was derived and the assumptions made in the value's choice seem hidden.

1.3 Recent Developments

While the great majority of the writers on the subject of audience viewing have followed Russell's work (or reinvented it themselves), there have been several recent alternative approaches to the design of audience viewing surfaces.

A simplified method for the floor slope calculation.

Bodycombe [BODY81] developed an extension of Russell's design technique in an attempt to avoid the use of computers and to simplify the problem of compliance with the sections of the fire code concerned with public safety. Most fire codes prefer linear ramps or stairs with constant riser heights, to minimize the chance of patrons stumbling in the case of an emergency. To calulate these linear slopes Bodycombe uses a complex nomogram which can determine the elevation or a row by use of a ruler. The designer lays down a set of lines which cross the scales of the nomogram. From the intersection points of the lines a final value can found. The same objection applies to Bodycombe's approach as with Russell's original work. No measure of successful viewing is produced. Beyond that objection, Bodycombe only approximates the correct contour because he takes straight line segments which are centered on the correct slope. The result is that some rows are above the desired slope, and some are below it. It seems a pity that Bodycombe rejects the computer as tool in the establishment of the proper audience surface as it is just what he needs to replace the complex nomograms used for his calculations.

Klaus Wever's Isovirtual criteria.

The work of Klaus Wever [WEVE68] is a radical departure from the previous ideas. He takes a global view of the visual accommodation for an audience and attempts to develop a "simple mathematical technique" that allows the designer of a theatre space to lay out the seating based on the average eye height of a standing actor and recognized attributes of visual perception. His system uses only the average values of certain human measurements and the question of iseidomal viewing as an attribute of the room is submerged in his notion of an "Isovirtual Principle" which defines a boundary of acceptable viewing. The design procedure is based on a set of radii that leads to the construction of a particular spherical shell section which is the audience surface. The constants he uses are based on his observations, and measurements of thirty unspecified theatres. After building a set of calculations based on human perception limits and the geometry of the room, he "adjusted" his formulae to agree with some theatres that he thought were good examples.

Wever employs three basic criteria that are specified (by him) with tolerances that vary between ± 5% and ± 15%. The use of a particular value within the tolerance band would presumably depend on the experience and skill of the designer. The effect of adjustments or changes to the Isovirtual design are not discussed.

Throughout Wever's discussion there is the implication that there is a right way to design an audience space. The presence of the scaling factors he ultimately employs and the allowed tolerance bands leaves much of the design to the planners judgement. The designer using Wever's procedure has the same problem as with Russell's; he has no way of assessing the effects of changes to the design.

2. The Probability for a Good View of the Stage

To predict the probability that an audience member will be able to see over the head of a person one or two rows in front of him, three issues must addressed:

1. An expression of the floor height at the *ith* row (y_i) using the iseidomal criterion is needed. That is, replace the "lofting out" of the solution by a mathematical expression which explicitly shows the influence of l_{head} and l_{eye}. The choice of the values used for these quantities is the key to the calculation of a "figure of merit" for the probability of viewing.
2. An algorithm is required that approximates the Gaussian distribution integral so that the calculation of values can be done in a quick and convenient way. In addition to being able to find the probability of a normalized Gaussian variable x, the inverse function will be required. These allow the calculation of the figure of merit for "successful viewing" which depends on the variables l_{head} and l_{eye}. Both of these variables have Gaussian probability distributions.
3. With the geometry described and with convenient approximations to the Gaussian probability distribution and its inverse, the height of the rows can be found. From any row the probability of viewing is a function of the floor height y_i. A fixed point iterative process can be used to find the y_i associated with a probability of viewing set by the architect. For a given sequence of elevations (y_i, for $i = < i = < n$) the probability of successful viewing can also be found in the same manner.

The resolution of these three issues is the purpose of this section.

2.1 Floor Slope Equation

The basic idea behind the iseidomal concept is that everyone should have an equally good chance of being able to see some predetermined point. Each person should be able to see over the head of the person seated in the row in front, or in the case of staggered seating, over the head of the person two rows in front.

Because of the variability in the heights of human beings, it is not possible to achieve the same viewing performance for all members of the audience. In Section 2.3 a technique for ensuring "average" probability of iseidomal viewing will be discussed.

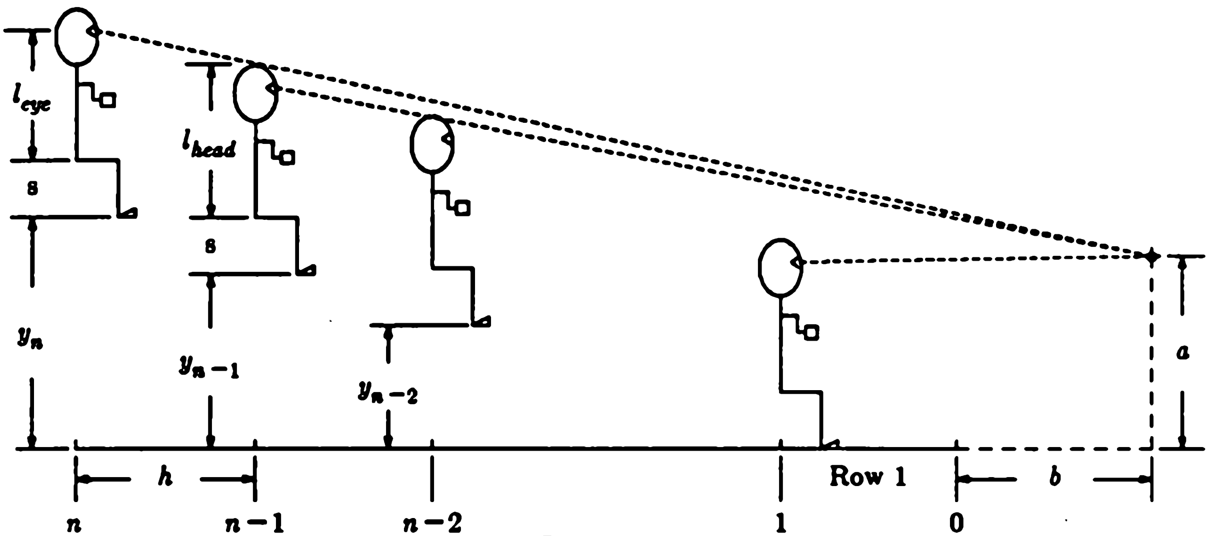


Figure 2.1

The Geometry of Row to Row Visibility.

By similar triangles, Equations 2.1 and 2.2 show the relationship between the row elevations.

$$\frac{y_i + s + l_{eye} - a}{ih + b} = \frac{y_{i-1} + s + l_{head} - a}{(i-1)h + b} \tag{2.1}$$

The restatement of 2.1 as a recurrence relation solved for $y_i (i \geq 2)$ is Equation 2.2.

$$y_i = \left(\frac{i+\beta}{i-1+\beta}\right) y_{i-1} + \left(\frac{i+\beta}{i-1\beta}\right) l_{head} - l_{eye} + \left(\frac{s-a}{i-1+\beta}\right) \tag{2.2}$$

As shown in Figure 2.1, $y_1 = 0$ is the starting value for the recurrence. The general term can be expressed as a summation.

$$y_i = (l_{head} - l_{eye}) \sum_{k=1}^{i-1} \left(\frac{i+\beta}{i-k+\beta}\right) + \left(\frac{i-1}{i+\beta}\right) (l_{eye} + s - a) \tag{2.3}$$

Non Uniform Row to Row Spacing.

For uniform row spacing Equation 2.3 is satisfactory. But in some situations, the row-to-row spacing is not a constant because of the presence of cross-aisles or because of provisions within the fire code. For the use of Continental Seating some fire codes require that, as more seats are added to the row (increasing the distance to the exit door), the row-to-row spacing must also be increased [UBC82].

In any case, the calculation of the elevation for row (i) will depend only on factors related to its location with respect to row $(i-1)$. Thus, by using Equation 2.2 with the values appropriate for that row, the elevations of successive rows can be established. The presence of a cross-aisle adds a small complication. If the fire code allows patrons to stand in the aisle, it must be expected that the worst case will occur. The standing patrons will position themselves just behind the seats in front of the cross-aisle so as to be as close to the stage as possible (Figure 2.2).

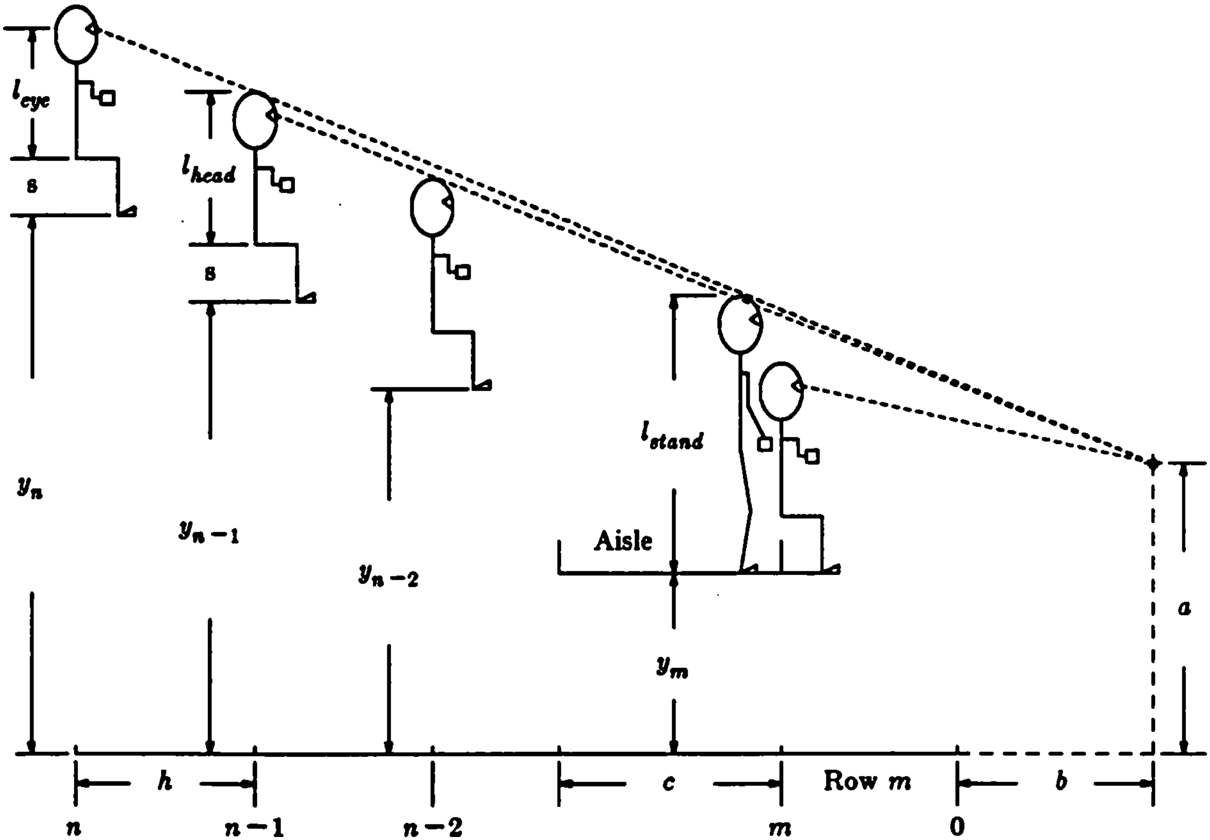


Figure 2.2

The Cross-Aisle Case.

A patron seated in the row behind the cross-aisle will have to see over the head of a standing patron. The height of the first row beyond the cross-aisle must be high enough to accomodate l_{stand} rather than l_{head}. If there are patrons standing in the aisle, then Equation 2.1 must be modified to account for all of the aisle parameters, as in Equation 2.4.

$$\frac{y_i + s + l_{eye} - a}{ih + b + c} = \frac{y_{i-1} + s + l_{stand} - a}{(i-1)h + b} \tag{2.4}$$

If the local fire code prohibits "standing room" patrons, the only modification to Equation 2.1 is to account for the larger space between the rows bordering the cross-aisle. The calculation of the floor shape with cross-aisles is the same iterative process using the modified equation at each cross-aisle.

Staggered Rows of Seats.

One technique to reduce the total building volume is to stagger the rows of seats so that the back of the seat in the ith row is centered along the line of the armrest between the two seats directly in front of it. Since the eye of the viewer is located between the heads of the patrons in the row in front, the elevation of the row can be less; the critical sight path from the eye is over the head height of the patron *two* rows in front (or over the shoulder height of the those in the row immediatly in front).

When the staggered case is discussed in the literature, the technique for calculating the row heights is to use the standard Equation 1.1 with l_{diff} divided by 2 [FRNK68][HAMM72]. Plausible as this approach may seem, it has some problems.

While it is usually assumed that the stagger between rows is established by shifting the seats by half a seat width, this is not the only possible case. This stagger does provide the widest angle of view and therefore is the only one we will consider in detail. For the one-half seat offset, the layout of the seating elevations is quite sraightforward. There are two rows of seats which require special attention. The first row, as before, is the reference point for all vertical measurements, but the second row is also an initial row because there is no seat directly in front of it. Once these two initial heights are set, the modified recurrence relation of Equation 2.5 can be used to determine all other elevations. The value of y_2 will not, in general, be zero.

It is the head height of the patron two rows in front, or the shoulder height of the patron in the row in front, which will specify each row elevation. The y_i is the maximum of the two values. The shoulder height will be critical only in the case of floor slopes which are so steep as to be either unusable or illegal and so it will not discussed in detail. Equation 2.5 assumes that the maximum y_i occurs because of the head height of the patron two rows in front.

The geometry involving the head of the viewer two rows in front leads to Equation 2.5. The definition of β and the row numbering are unchanged from the previous discussion.

$$y_i = \left(\frac{i+\beta}{i-2+\beta}\right) y_{i-2} + \left(\frac{i+\beta}{i-2+\beta}\right) l_{head} - l_{eye} + \left(\frac{2(s-a)}{i+\beta-2}\right). \tag{2.5}$$

In order to find the height of subsequent rows, the height of row two must be set. The geometry shows that there is a range of values which might be assigned to the height for row two. The lowest eye location in row two is on the line from the sighting point just grazing the shoulder of the person in row one. The highest point is when the shoulder of the row two patron just grazes that line from the sighting point to the eye position in row three. This is quite a wide range of heights. In an attempt to narrow the range of values and to follow the slope established in the other rows, the line of sight of the eye position in row three is used as a reference for the height of row two. If a vertical plane is located along the second row, it will intersect a sighting line for row three. A projection of this intersection point is then located on a vertical line located at the center of the seat in row two. It is this intersection that locates the critical point for row two. The elevation of y_2 is then established so that the critical point is located within the span between the eye and the top of the head of the patron in row two. The floor height of row two y_2 is then constrained to

$$\frac{l_{head} + s - a}{\beta+1} \leq y_2 \leq \frac{l_{head}(\beta+2) - l_{eye}(\beta+1) + s - a}{(\beta+1)}. \tag{2.6}$$

If the planned use of the room does not require the viewers to see the stage below the horizontal plane touching the head of a patron in the first row, the lower limit in Equation 2.6 will be negative. Examples of this type of design could be classrooms, churches, or motion picture theatres.

So far the discussion has focused on the geometric part of the problem. The geometry was constructed with the assumption that values for the anthropometric quantities were known. Unfortunately, we cannot predict in advance how the members of an audience will be distributed in terms of height. The order in which seats are sold, the size of the blocks of seats sold, and the physical attributes of various audience members are all factors in determining the particular l_{head} and l_{eye} to be used in the row calculations. Because it is not practical to control these factors, the exact values of the head and eye height which should be used are not known. But these parameters can be described statistically. With the fixed geometry described, the remaining business is to assess how the random factors can be accommodated within the design procedure. This is the subject of the next sections.

2.2 An Approximation of the Standard Error Function

The calculation of a row height depends on two measurements, the head and eye heights of a seated patron, which have the properties of random variables. Any random variable can be characterized by a set of attributes that are peculiar to the set of values that can be realized by the variable. The two most basic of these attributes are the mean value (μ), and the variance of the set of values (σ^2). The mean value is the "expected" value of the random variable. The variance is a measure of how the values of the random variable distribute about the mean value.

The physical attributes of the human population are examples of variables which exhibit the properties of the Gaussian or Normal distribution function. This is fortuitous, as this function is well understood, and its properties make the mathematical manipulations simple [CROW60][PARZ60]. In particular, for Gaussian distributed random variables any weighted sum of the different random variables is itself a Gaussian distributed random variable. The mathematical statement of this fact is the following.

Theorem:

If each x_i is a Gaussian random variable for $0 \leq i \leq n$, and for each i there is an associated scalar weighting factor α_i, then

$$x_{weighted} = \sum_{i=0}^{n}(\alpha_i x_i)$$

is a Gaussian random variable with

$$\mu_{weighted} = \sum_{i=0}^{n}(\alpha_i \mu_i) \text{and } \sigma^2_{weighted} = \sum_{i=0}^{n}(\alpha_i^2 \sigma_i^2).$$

For the calculations which follow in Section 2.3, answers to the following questions will be needed.

1. Given x, what is the probability $P(x)$ that the normalized Gaussian variable has at most the value x?
2. Given $P(x)$, what is the value of the normalized Gaussian random variable x such that the probability is $P(x)$.

The algorithms which will provide the answers, in addition to being close approximations to the mathematically correct results, must also be efficient to compute. The techniques used to get these results for a Gaussian random variable and its inverse are found in Appendix 1.

2.3 Calculation of the Heights of the Rows

The topic of this section is the problem alluded to by Hamm, "the large lady with a hat sitting in front of a small shy gent". The recognition of the probabilistic nature of the problem is the key to its solution. Ideally, there should be a line which originates at the sighting point and which just grazes the top of the head of the person seated one or two rows in front of any viewer (Figure 2.1). Because of the variability in the height of human beings, there is a chance that a patron will be too short to see the sighting point. There is no guarantee that the eye position of every audience member will be above the grazing line. Thus, there is no practical way to ensure that every patron will be able to see the critical sighting point. But it is possible to establish a probability of iseidomal viewing over the whole audience.

To find the correct value for the floor level at each row y_i there are four topics which must be explored:

1. The probability that the eye of a viewer is located at the grazing line or above
(the probability associated with iseidomal viewing);
2. The influence of the seat occupant's gender;
3. The weighted average of the gender dependent probabilities;
4. The calculation of the height of the current row.

Audience Composition

A review of anthropometric data shows that for both head and eye heights, men and women have distinguishably different distributions. Thus, in any pair of seats, one in front of the other, the gender of the occupants is significant.

There are four possibilities:

1. A man seated behind a man.
2. A man seated behind a woman.
3. A woman seated behind a man.
4. A woman seated behind a woman.

For each of these cases there is an associated pair of expected head and eye heights. This would imply that for each case there is a different value for y_i. Figure 2.5 shows the effect of the gender pair on the slope of the floor. Note that Case 2 is logically impossible as it requires a man to be sitting behind a women for every pair of adjacent rows. To do this, the occupant of each seat has to be male when the row elevation is being calculated but female when the succeeding row is under consideration. This anomaly also exists for Case 3 with the genders reversed.

		Men	Women
l_{head}	μ_{head}	36.9	33.9
	σ_{head}	1.277	1.216
l_{eye}	μ_{eye}	32.0	29.3
	σ_{eye}	1.216	1.186

Table 2.1

The Height Parameters of Seated Patrons (in inches) [NASA78]*

The Probability of Seeing Over the Head in Front

The solution of this problem starts from a probability of successful viewing $P_{desired}$ that is set by the designer. Because of the random nature of the head and eye heights of the patrons, for any y_i there is a probability that y_i will be at a height which will place the eye location at or above the grazing incidence line (Figure 2.1). The problem is to find the value of y_i which satisfies Equation 2.9.

$$P_{desired} = P\left(\frac{y_i + L_{eye} + s - a}{i\,h + b} \geq \frac{y_{i-1} + L_{head} + s - a}{(i-1)h + b}\right). \tag{2.9}$$

In this equation l_{head} and l_{eye} have been replaced by the values L_{head} and L_{eye}, the random variables which describe the head and eye heights of the two persons occupying the pair of seats. For any particular pair there is only a probability of success available. It is dependent upon the gender of the viewer and the gender of the person sitting in front of the viewer. In general, the distribution of males and females throughout the audience is not predictable. But the fraction of the audience which is male can be estimated and the likelihood of each of the four gender cases can then be established. With this

---

* Table 2.1 shows the set of data for seated head and eye heights. Earlier data listed in Damon, Stoudt and McFarland had a suspect sample for the eye height of women (443 female U.S. Navy pilots whose age was in the range of 18 to 35) [DAMO66]. Henry Dreyfuss appears to have used this information for his original edition of "The Measure of Man" (there are three editions 1959, 1960, and 1967) [DREY67]. The NASA data used here is the most recent and appears to be more consistent.

information an average probability of viewing can be calculated. This weighted average probability as a figure of merit for the overall visibility performance is the topic of the next subsection. The remainder of this section is concerned with finding y_i for some specified $P_{desired}$.

The nature of the random variables for the seated head height and seated eye height are known. These variables have a Gaussian distribution.

A rewriting of 2.9 yields

$$\begin{aligned} P_{desired} &= P\left\{ y_i - y_{i-1}\left(\frac{i\,h+b}{(i-1)h+b}\right) - \frac{(s-a)h}{(i-1)h+b} \geq L_{head}\left(\frac{i\,h+b}{(i-1)h+b}\right) - L_{eye} \right\} \\ &= P\left\{ y_i - y_{i-1}\left(\frac{i\,h+b}{(i-1)h+b}\right) - \frac{(s-a)h}{(i-1)h+b} \geq L_i \right\}. \end{aligned} \tag{2.10}$$

Where L_i is the random variable whose mean is μ_i and whose variance is $\sigma^2_{\ i}$. These parameters are functions of the geometry and so are different for each row (Figure 2.3 and Figure 2.4).

$$\mu_i = \mu_{head}\left(\frac{i\,h+b}{(i-1)h+b}\right) - \mu_{eye}\ \text{, and } \sigma^2_{\ i} = \sigma^2_{head}\left(\frac{i\,h+b}{(i-1)h+b}\right)^2 + \sigma^2_{eye}.$$

Using these parameters, the normalized form of the Gaussian random variable whose value is Z_i can be expressed.

$$Z_i = \frac{Y_i - \mu_i}{\sigma_i}.$$

$$P_{desired} = P\left\{ y_i - y_{i-1}\left(\frac{i\,h+b}{(i-1)h+b}\right) - \frac{(s-a)h}{(i-1)h+b} \geq \left(Z_i\sigma_i + \mu_i\right) \right\}. \tag{2.11}$$

The value of Z_i can be obtained by use of the inverse to the Gaussian distributed random variable and the $P_{desired}$ which the designer has selected. The only unknown is y_i. The minimum value of y_i occurs for the grazing line of sight which results in Equation 2.12.

$$y_i = y_{i-1}\left(\frac{i\,h+b}{(i-1)h+b}\right) + \frac{(s-a)h}{(i-1)h+b} + Z_i\sigma_i + \mu_i\,. \tag{2.12}$$

Using the same equation, for given y_i and y_{i-1}, the probability $P(Z_i)$ can be derived. The value will be a function of the row number, the sighting point, and the gender case. A weighted average of the four probabilities will be the resultant value which will be equal to $P_{desired}$ by setting the value of y_i.

The influence of the genders of the occupants can be seen in Figures 2.3, 2.4 and 2.5.

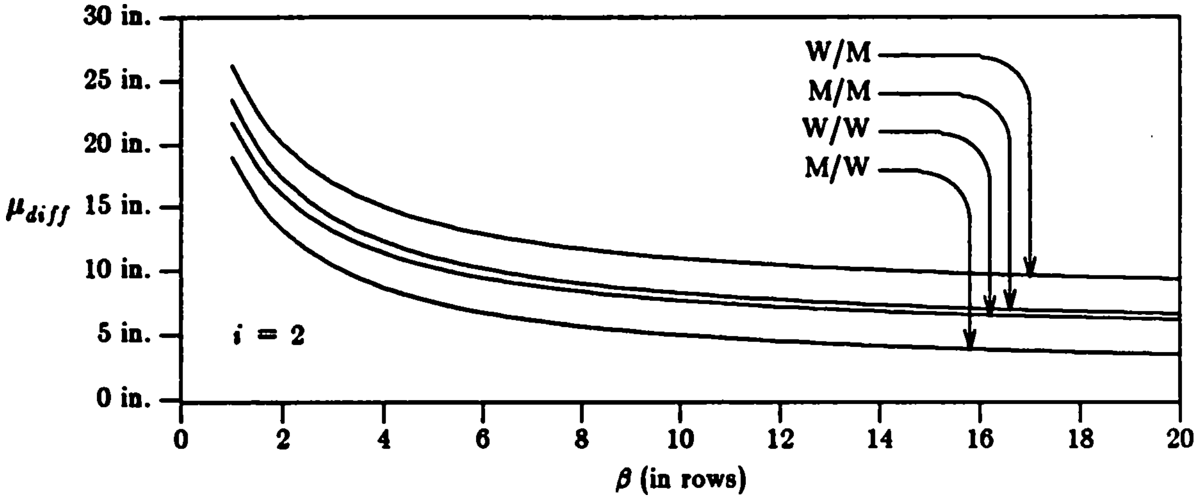


Figure 2.3

μ_{diff} as a function of β

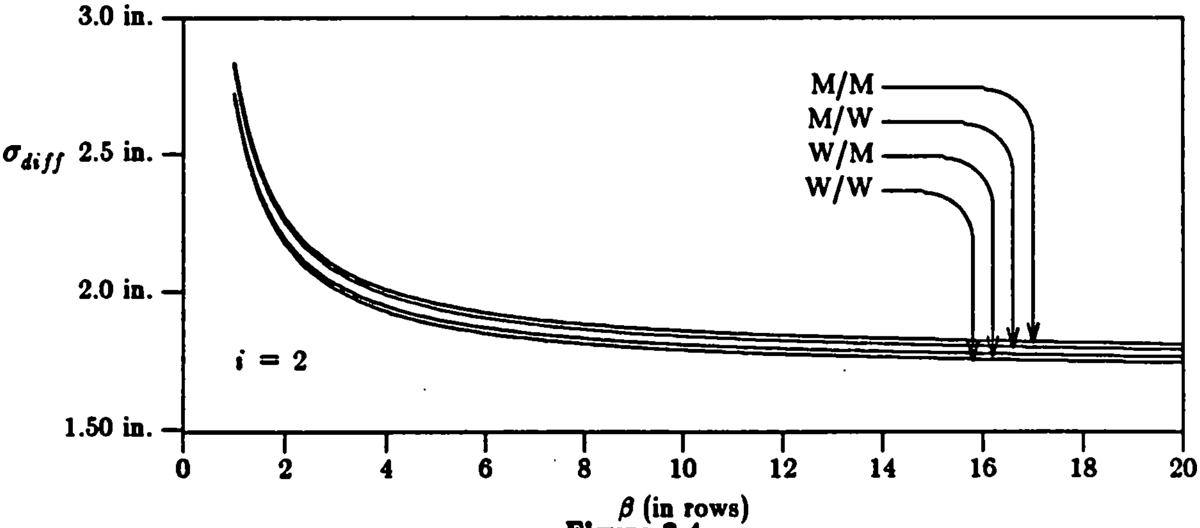


Figure 2.4

σ_{diff} as a function of β

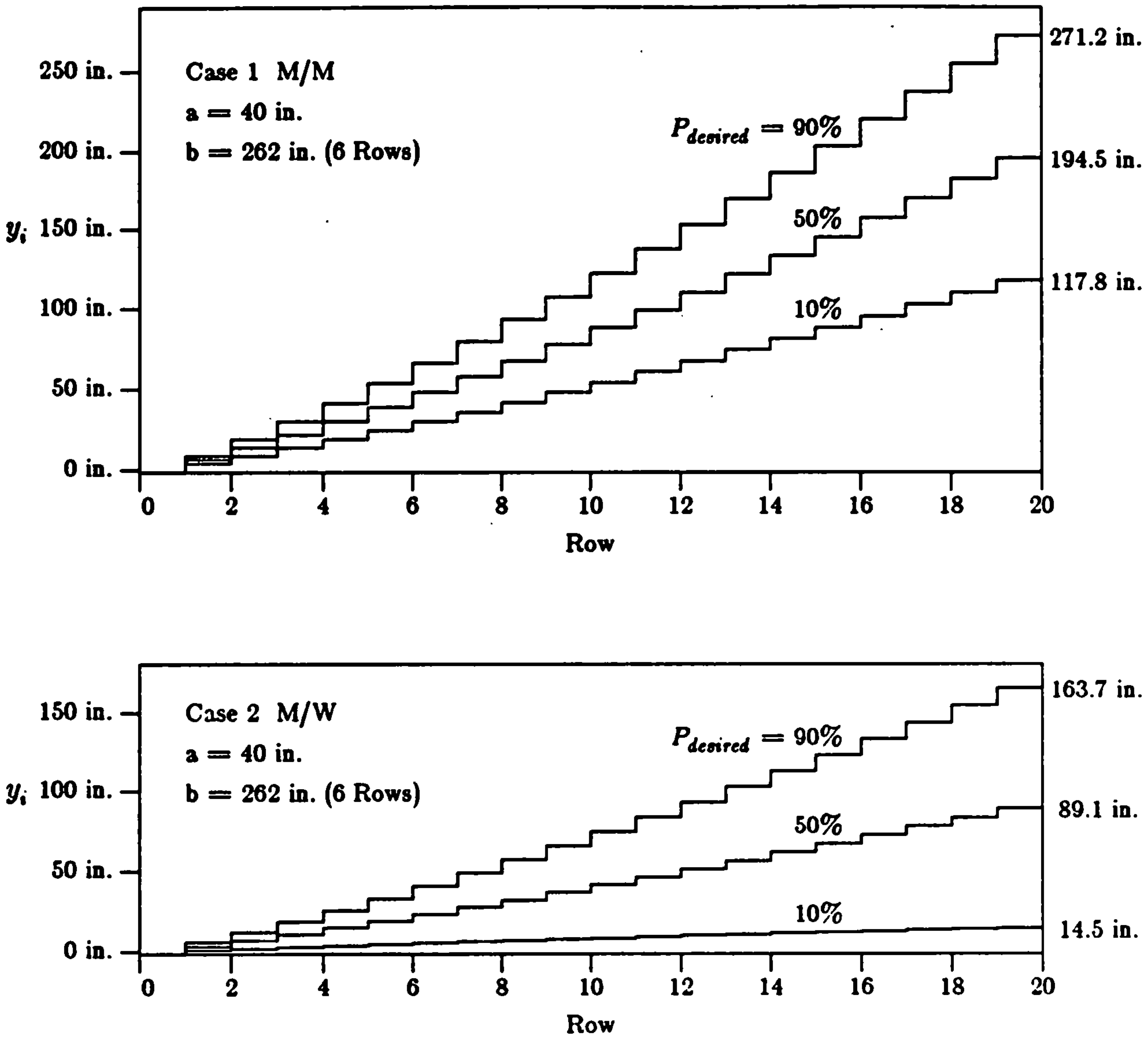
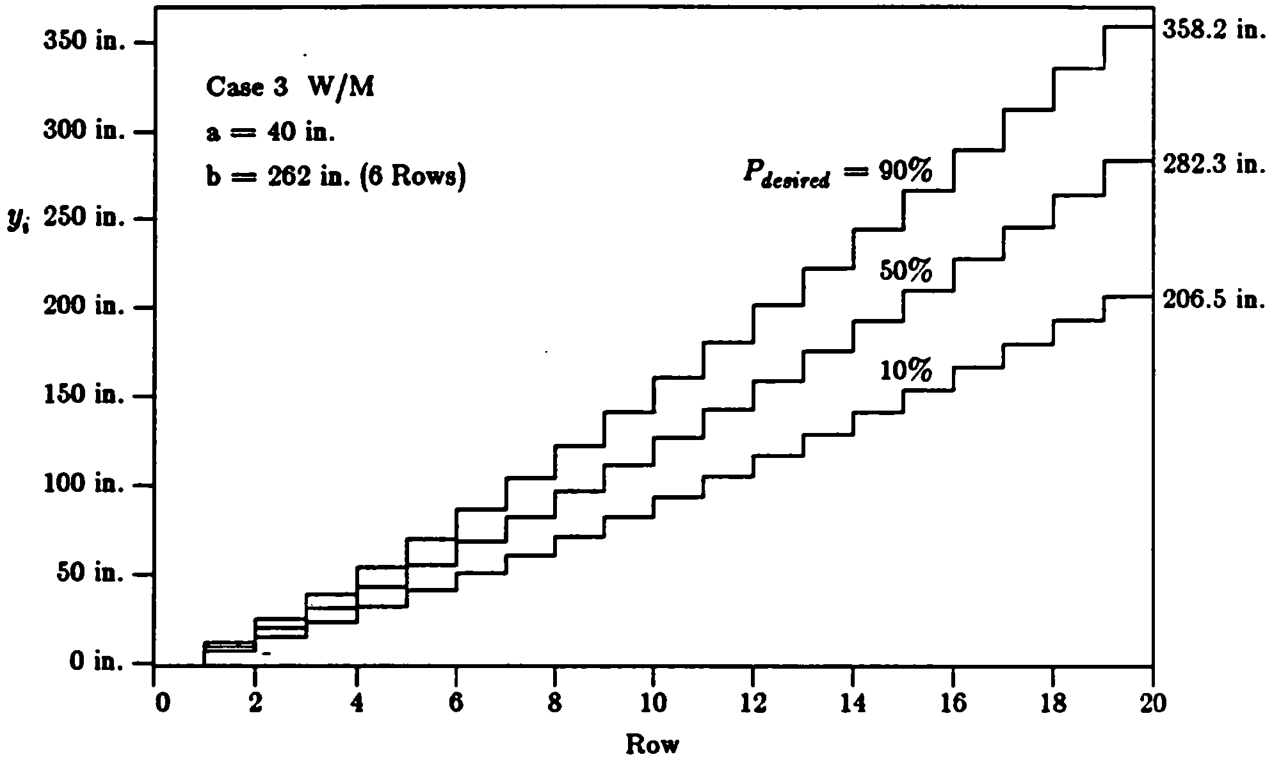


Figure 2.5

The Seating Floor Slope as a Function of the Gender Case



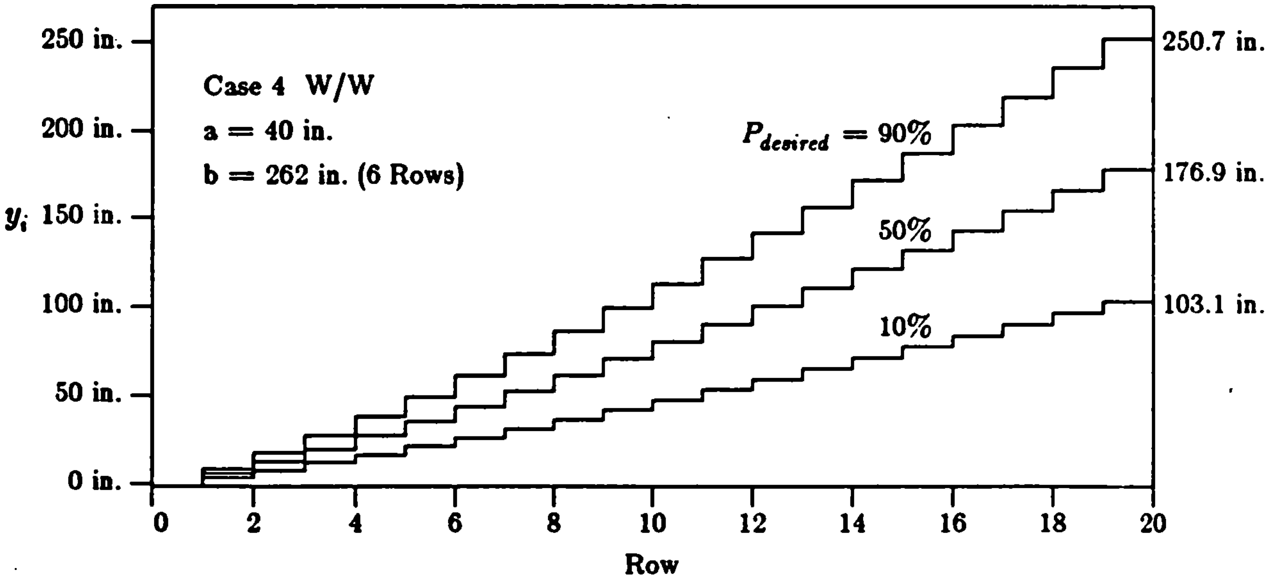


Figure 2.5 **Continued**

The Seating Floor Slope as a Function of the Gender Case

The Weighted Average Probability

At first glance it might seem that the head height or eye height of the male and female populations could be averaged into a single composite random variable. But it is not possible to combine the male and female populations in this way. It is easy to show that such averaging is an incorrect use of the notion of a weighted random variable.

Assume that there is an audience with a composition of an equal number of men and of women. As half of the audience is male and half of them are taller than the mean, twenty-five percent of the total audience will be men with a seated head height of 36.9 inches or more.

Using the weighted averaging discussed in Section 2.2 the male and female populations can be combined to produce a single Gaussian random variable with a mean μ_{ave} of 34.5 inches and a standard deviation σ_{ave} of 0.88 inches. A value of x of 36.9 inches will be $2.72\sigma_{ave}$ above the mean. The corresponding probability for a member of the composite population being taller is only 0.3%. But we know that 25% of the audience is at least 36.9 inches tall. The contradiction refutes the assumption that the male and female populations can be averaged together to form a composite. The "model" is in error because men and women are being mixed together to form a hybrid patron, rather than a man or a woman being placed in each of the critical seats. While there is only one gender pair at each pair of seats, the probability of viewing for a large number of such pairs can be expressed as a weighted average value.

The correct model is to chose the gender of the patron in each seat by a random process so that on average α fraction of the time the occupant is male and $1-\alpha$ fraction of the time the occupant is female. The resultant probability of viewing is the weighted sum of the probability for each of the four cases multiplied by chance that case occurs.

$$P_{ave} = \alpha^2 P_{M/M} + \alpha(1-\alpha)P_{M/W} + \alpha(1-\alpha)P_{W/M} + (1-\alpha)^2 P_{W/W} \tag{2.13}$$

α = fraction of audience that is male.

This model takes into account only the differences between men and women. There are other forms of variabilty in the audience population. The most conspicuous omission is that no consideration has been given to children or adolescents. Changes in the anthropometric data due to age, environment or nutrition also have been left out. To use additional differentiation of the audience population would require information about the intended use of the theatre. The issues are too complex and too specialized to be considered here. The appropriate factors can be included when it seems necessary by generalizing the equation for P_{ave} to include them.

Standard Parameters Used in the Examples

In this part of the discussion, several examples will show the influence of various parameters on the shape of the floor slope. For simplicity of illustration, only one parameter will be varied at a time. The remaining values will be taken from a "standard" set of values presented here.

α	0.40*	Fraction of audience that is male.
a	40 in.	Sighting point above the elevation of row one.
b	252 in.	Sighting point in front of row one.
h	42 in.	Row to row spacing (Nominal "Continental Seating" spacing).
s	16.5 in.	Height of the seat above the floor.
$P_{desired}$	50%	Probability of seeing from a seat.

The Calculation of Row Heights for Iseidomal Viewing

The algorithm which is used here is:

1. Get a set of four initial values for y_i, one for each of the gender cases. Each of these are calculated using the desired probability of iseidomal viewing, the value of the height of the previous row y_{i-1}, the row to row spacing, and the sighting point.
2. Use each of the four initial values of y_i to calculate an average probability of iseidomal visibility. Depending on the α being used, $P_{desired}$ will fall somewhere in one of the three intervals which separate the gender case average probabilities.
3. Pick the two y_i values whose average probabilities straddle $P_{desired}$ and are closest to it. Use these two values of y_i as the initial points of the secant method fixed point iteration to determine the value of y_i which produces a P_{ave} close enough to $P_{desired}$ to be acceptable.

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* The usual assumption when designing a theatre or concert hall is that the audience is about 40% men and 60% women. Private communication, Wallace Russell, President, Theatre Projects Inc., Los Angeles, CA.

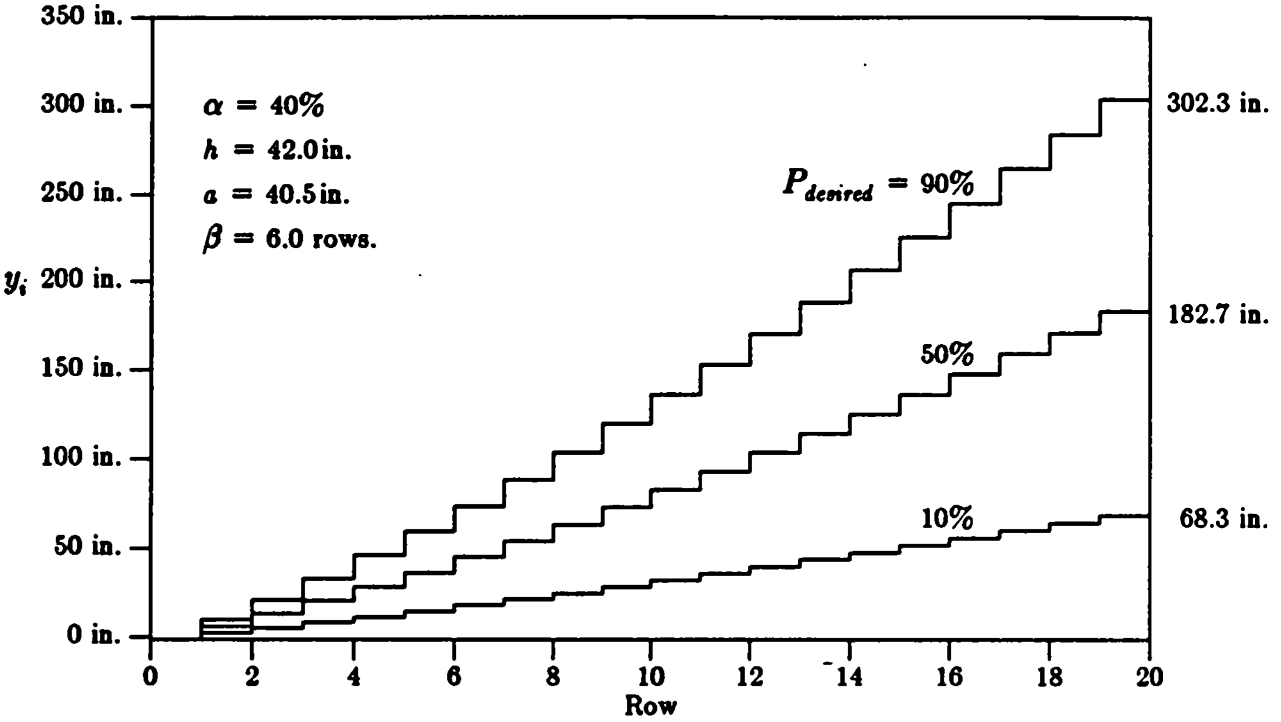


Figure 2.8

Weighted Average Composite Seating Floor Slope

There is some y_i that will have a weighted average probability equal to $P_{desired}$. The computational problem is to find the y_i which will satisfy $P_{ave}(y_i) \approx P_{desired}$. The iterative use of the secant method finds a value of y_i such that

$$\left| \frac{P_{ave}(y_i)}{P_{desired}} - 1 \right| \leq tol. \tag{2.14}$$

For the examples in this discussion, a tolerance value tol = 0.001 has been used.

The starting point for the iteration process is to find the y_i for each gender case using the composite y_{i-1} resulting from calculating the previous row. By the use of an approximation to the Gaussian function the probability of success for each of the four gender cases can be found for each of the four values of y_i. A weighted average of iseidomal viewing is then calculated for each of the four gender cases using one of the y_i values. This produces four weighted average values, one for each of the y_is.

For speed and stability the two values of y_i which have $P(y_i)$ closest to $P_{desired}$ are used as the starting points for the secant method fixed point iteration until Equation 2.14 is satisfied. The fixed point calculation used to produce Figure 2.6 is shown in Appendix 2.

The impact of the audience composition can be seen in Figure 2.7.

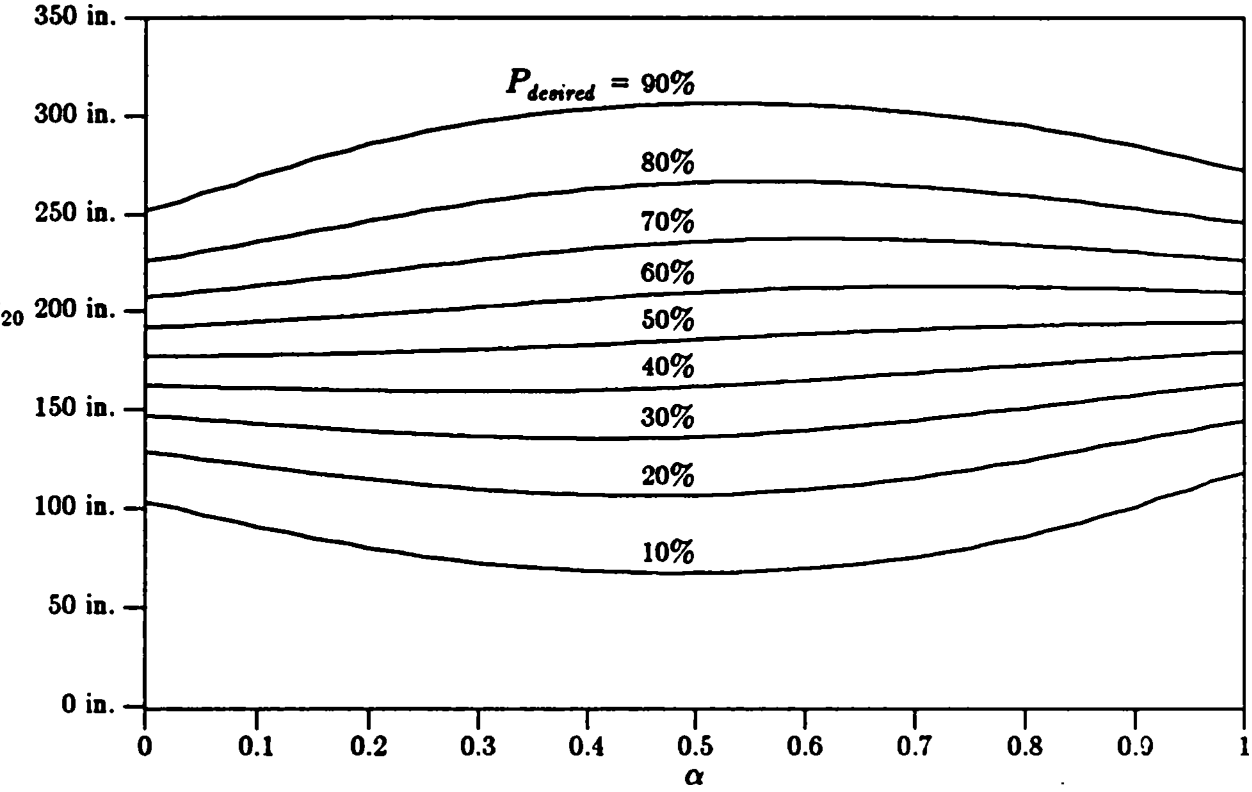


Figure 2.7

y_{20} **as a Function of** α

The Quality of the Recommendations for l_{diff}

As mentioned earlier, the prescribed l_{diff} value found in the literature of theatre design is usually given as 4 to 5 inches [BURR49][BURR64][FRNK68] [HAMM72][IZEN77][SOUL81]. With the tools that have been described in this essay it is possible to analyze the result of using one of the "rule of thumb" values for l_{diff} and confirm the weighted average $P(l_{diff})$ produced. While the values recommended are not justified in a quantitative way, they are made by individuals with considerable experience in theatre design and it would be surprising if these values produced results which were at the low end of the probability range. The results shown in Figure 2.8 shows that "rule of thumb" constants will produce probabilities that are near 50%.

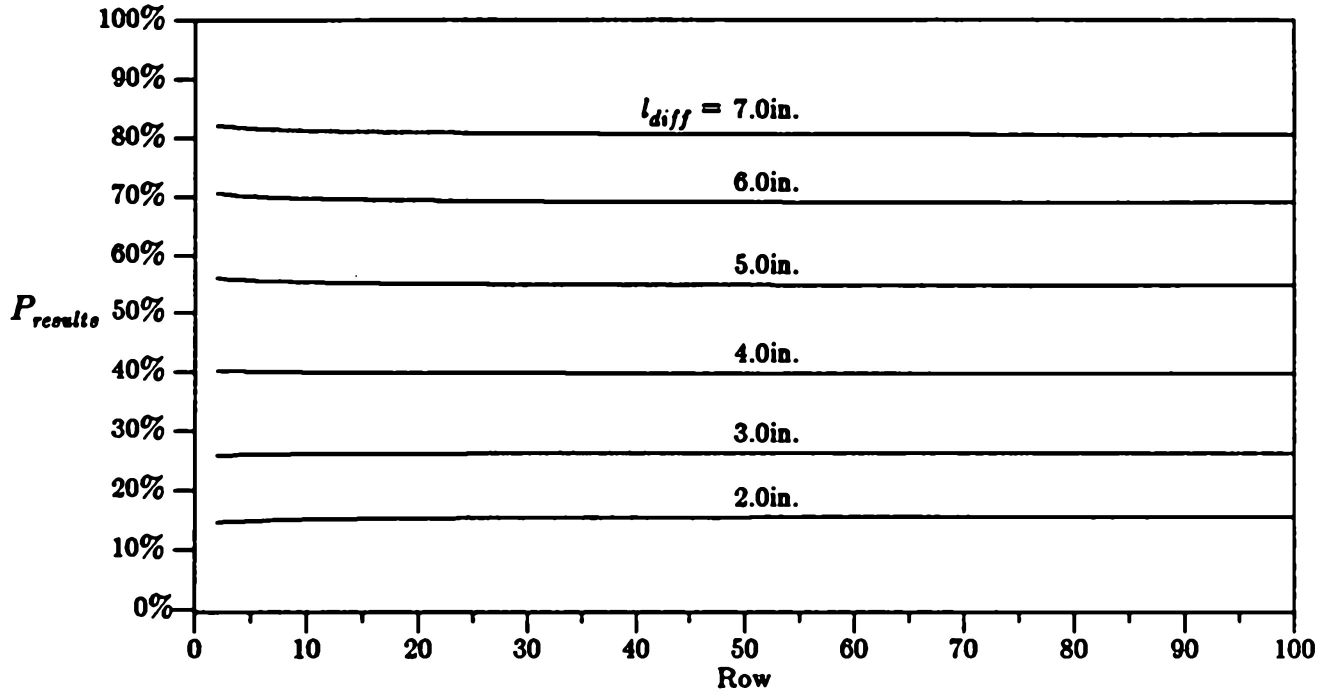


Figure 2.8

The Probability of Seeing Using Russell's Model

The Impact of Parameter Variation

All the parameters in Equation 2.1 have some impact on the form of the audience floor surface.

The lower the sighting point, the steeper the floor contour (Figure 2.9). Note that if the sighting point is high enough, the floor contour forms a dish. This style of design is sometimes used for inexpensive motion picture theatres where there is no stage behind the screen. The dish shape means that the front rows of the audience will be looking up at the screen (Figure 2.9). A patron seated in the second row, being below the front row, is looking upwards at the screen. From such a seat he would not be able to see the stage floor. Nor would that seat location be satisfactory for viewing anything which occurs below the sighting point (which might be the bottom edge of the screen). For theatres where dance is a part of the program, the sighting point is usually set so the audience can see the dancer's feet.

The wider the row-to-row spacing, the more volume will be enclosed by the audience surface. The back row will rise as the row-to-row spacing increases. It will move further from the stage at the same time. Both of these effects will cause the enclosed volume to increase.

Figure 2.10 shows the impact of changing the row to row spacing. The values of h are those for conventional Aisle-row seating and the limits of the range of sizes for Continental seating prescribed by the Uniform Building Code [UBC82]. (The Province of Ontario Theaters Act specifies that 32 inches be used for h in aisle-row seating and an h of 42 inches be used for Continental seating [PROV80].) The closer the sighting point is to the front row the steeper the rake of the floor (Figure 2.11).

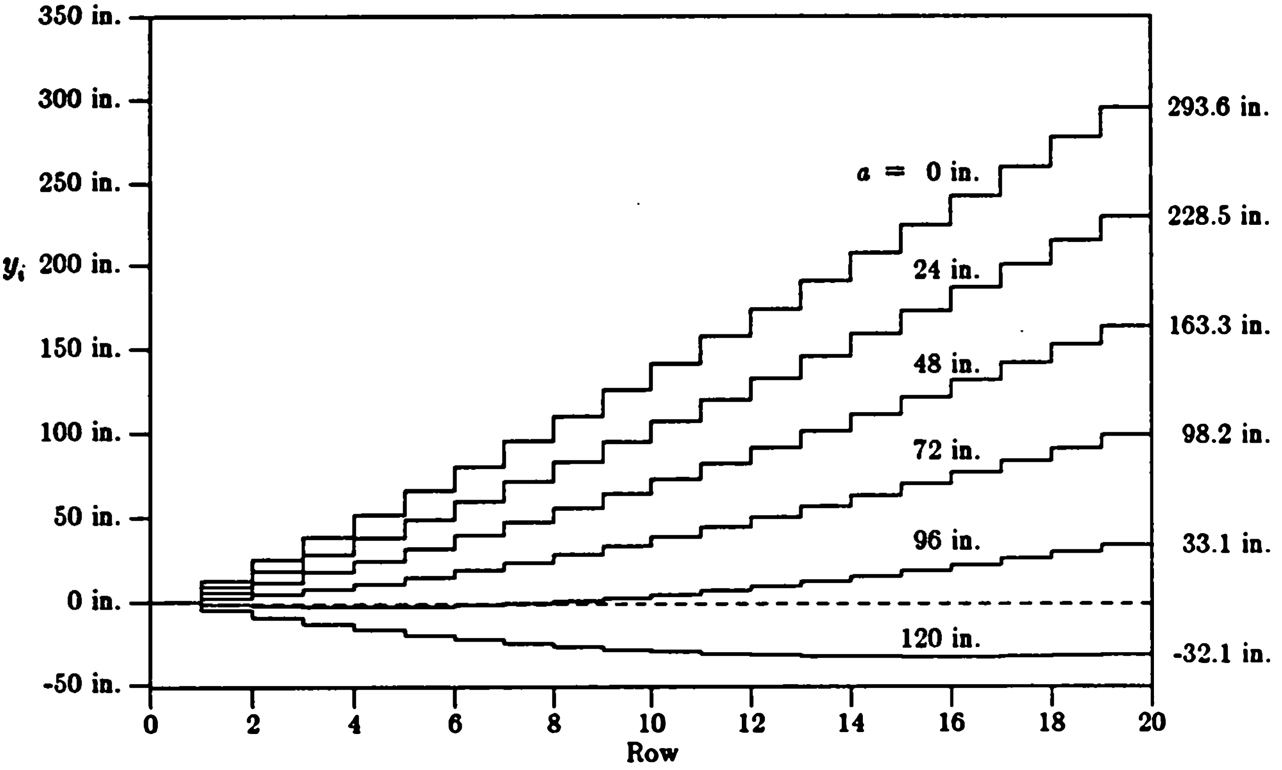


Figure 2.9

The Impact on y_n of the Sighting Point Height

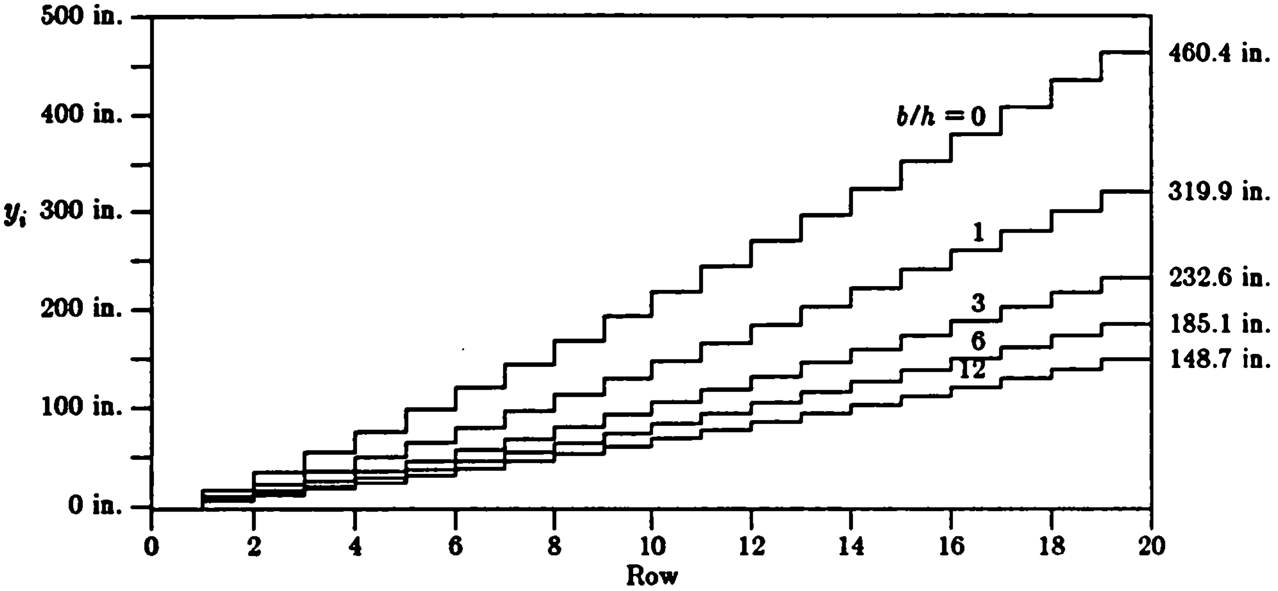


Figure 2.10

The Impact on y_n of the Sighting Point Distance

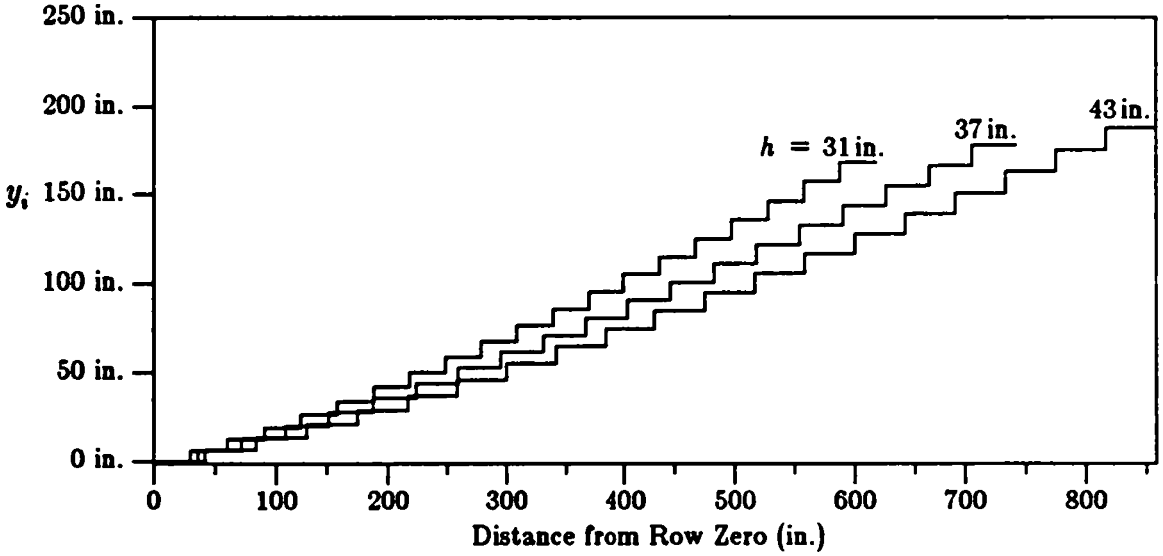


Figure 2.11

The Impact on y_n of the of Row to Row Spacing on y_n

"Cheating" the Design

In addition to altering the row spacing or moving the sighting point, another design strategy is to "cheat" by ignoring the patrons in one or more of the front rows. Leaving the floor flat will, in general, produce a very low probability of seeing in those rows, but the height of the back row will be correspondingly lower. This gives the designer the ability to raise the viewing probability for the remaining rows of seats and/or reduce the building's cubage and cost. The impact of "cheating" one or more rows is greater than just the loss of height caused by one riser step. Because of the geometry, each previous y_i is multiplied by a number greater than one. The loss of one row reduces the size of the multiplier in addition to removing one riser step.

For a room where a balcony is to be employed, the reduction of height of the main floor also reduces the steepness of the slope of the balcony floor.

A justification for this seemingly callous attitude is that patrons who purchase seats in the very front rows are often there to be seen, rather than to see. If the plan of the audience area is rectangular the negative impact of this design tradeoff is greater than if the seating is arranged in a fan shape pattern with relatively few seats in the front rows. Figure 2.12 illustrates the effect of "cheating" one to five rows.

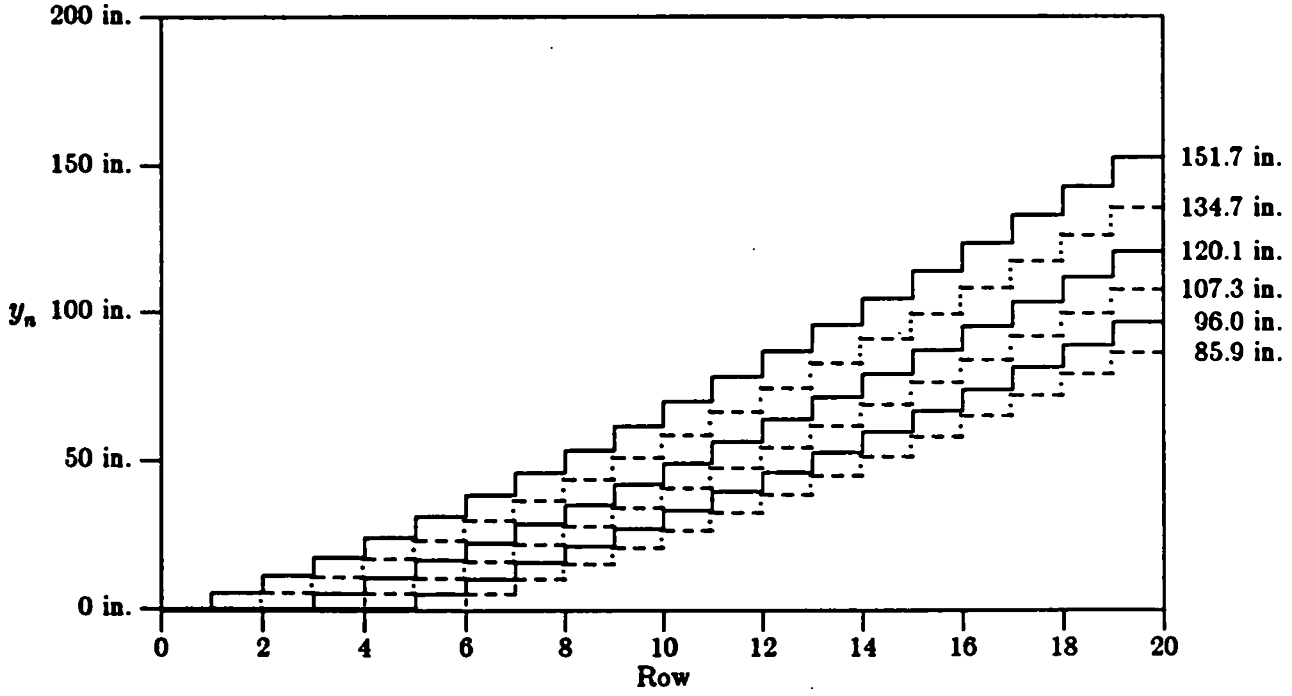


Figure 2.12

The Impact on y_n of "Cheating" One to Five Rows

Summary

By using the procedures shown in this section, it is possible to find, by numeric computation, the audience row heights which will result in the desired probability of viewing with the minimum volume of the audience surface and thus, all other things being equal, a minimization of construction cost. The probabilistic model described here is itself a simple model as it assumes that the viewers sit upright and do not slump in their seats. Also, this model makes no estimate of the effect of hats and/or hair styles. Nor does it consider the effects of the age distribution of the patrons.

Both the probabilistic method used here and Russell's technique use a single sighting point. A more comprehensive measure of successful seeing would use a number of different sighting points.

Using Russell's method and the "rule of thumb" criteria recommended in the literature has been shown to produce a high probability that the theatre patrons will have iseidomal viewing. But what Russell's method does not provide is an explicit measure of performance. The probabilistic model used here is a method of design which can provide an assessment of the impact of alterations to the building plan, and thus provide theatre architects with a better means of evaluating the impact of their design decisions.

3.0 Interactive System Design.

In addition to the solution of the iseidomal criterion with the anthropometric variability already considered, the designer's use of the Seating Planner must be considered. For the system to be of much use, it must allow easy access to the results and be convenient to control.

3.1 The Typical User of the Seating Planner

It is the function of the Seating Planner to provide the architect with a means of finding the centerline seating contour which he feels best meets the criteria set for the theatre building. A theatre is a highly specialized structure and its design is not the common assignment for most architects. The proper design of the audience seating surface is a basic concern. Because of the complexity of theatre building and its fittings, the architect and his consultants will be required to spend much of their thought and effort on other issues and topics. As a consequence, most architects will never become experienced with the details of the solution of the critical viewing problem.

The Seating Planner is a tool which can help the architect and/or his consultants to do this critical part of the theatre design. With it he can quickly examine alternative designs. For it to be useful, it will have to meet the requirements of the architect and his consultants without becoming a burden to learn or to use. Thus, the system needs to accommodate those who are interested in solutions to the visibility problem but who are not experienced users of the Seating Planner. These users will be making mistakes in the use of the system because they are using a tool whose controls are unfamiliar to them, on a task which they rarely perform.

3.2 Task Analysis.

Since this project is concerned with the design of an interactive system for a particular function, an appropriate beginning is an analysis of the task to be performed by the user. The purpose of the Seating Planner is to allow the designer of an audience space to concentrate on making the design decisions based on performance, space used, and cost, while leaving the mathematics and bookkeeping to the computer.

In its simplest form, the sequence of tasks is:

Turn the system on;
Run the problem until the user is satisfied;
Turn it off.

The Seating Planner requires a set of parameters for the calculation of the results for each trial. For example, the number of rows, the location of the sighting point and the seating style are always required. The user must enter his choice for these values and settings to establish the parameters for the trial design.

Thus, when using Seating Planner the basic design cycle is:

Set parameters;
Get the result;
Evaluate the result and take the appropriate action.

At the next level of detail the problem decomposes into:

Initialize the system;
Establish a set of parameters (Use the defaults, alter the existing set, or load a previous stored set);
Select how the parameters are to be manipulated (the design mode);
Calculate the result;
Based on the results
  (a) Change the parameters and/or the mode and continue;
  (b) Wrap up current session.

3.3 Communication with the User.

The very nature of an interactive system focuses attention on the narrow channel of the interface between the user's sensory and perceptual apparatus and the computer system's input and output devices. Both the user and the system must provide the other with appropriate information. In this section the means of communication to and from the user will be explored.

3.3.1 Communication by the User.

The user needs to indicate his choice for the parameter values and to select the mode of calculation for the floor contour. In addition, he will need to control a set of utilities which will allow him to store the current results and retrieve his previous designs. Before the design of the interactive aspect of the system can proceed, a definition is needed of the types and character of the input data and what commands will be recognized.

Modes of Operation.

For each trial of a design, the user selects one of three modes of developing the floor slope or one which analyzes an existing design.

These four modes of operation are:

1. *Uniform Probability.*

   Develop the floor slope with a fixed uniform probability of seeing from every location.
2. *Back Row Height.*

   Develop the floor slope, using a given back row height, with best average probability of seeing.
3. *Probabilities by Row.*

   Develop the floor slope where each row has been given its own specified probability of seeing.
4. *Analyze Plan.*

   Analyze an existing floor contour for the probability of seeing from each row.

User Controlled Parameters.

The constraint parameters which are common to all modes of operation of the Seating Planner are:

1. The units used for input and display: Imperial or Metric.
2. The composition of the audience: what fraction is male and what fraction is female?
3. The location of the sighting point: the vertical and horizontal coordinates, relative to the first row of seating, of that point which must be seen from every seat.
4. The size of the seating surface: this term can be specified one of two ways:
   (a) The total number of rows.
   (b) The total number of seats and the shape of the audience surface.
5. The style of seating (Continental or Aisle-Row) that is to be used.
6. The arrangement of the rows of seats, either in-line (the back of one seat directly behind another) or staggered (the back of one seat centered on the arm rest of the seat in front).
7. The presence of transverse aisles within the seating surface. If there are aisles, where are they located and how wide are they?
8. The governing jurisdiction in which the building is to built and the applicable building codes.

Most of these parameters will remain fixed for a number of runs, but all of them must be specifiable by the user of the system.

Mode Dependent Parameters.

When the user selects one of the modes of calculating the floor slope, additional information may be needed by the system. This will depend on the particular mode requested.

1. In the "Uniform probability" mode, the value for the desired probability of viewing is required.
2. The "Back Row Height" mode requires a value to be set for the height of the back (last row) row to produce the "best" uniform probability of iseidomal viewing.
3. The "Probabilities by Row" mode needs a value for the *probability of viewing* to be set for each row (or group of rows).
4. For the "Analyze Plan" mode, the elevation and width data for each row and the sighting point coordinates are required.

Utility Functions.

In addition to the controls which relate directly to the design, there are a number of utility functions which are necessary if the Seating Planner is to have much value.

1. The user must be able to save the results of his current work.
2. Previous work must be retrievable for review or for further work.
3. To create a "summary" of the design effort, the user must be able to append any particular seating plan, with its parameter values, to a designated "review" file (to do this, a means for "editing" the selected designs into a single file is required).
4. The user requires a list of the saved files.
5. The user must be able to delete a named file.

3.3.2 Communication to the User.

Even though the typical user has a number of sensory channels which could receive information from a computer system, the use of the visual channel with its wide bandwidth will be the only one which will be considered for communication to the user. The use of sound signals, synthetic speech, tactile pressure or motion, or smell would require additional equipment that is not conveniently available.

The information produced by the calculation of a floor contour, the set of parameter values and system status messages all need to be presented to the user in a timely fashion. There are three choices available as to how to present this information:

1. The presentation of messages and numeric values;
2. The presentation of line drawings or other schematic views (with or without hidden line removal);
3. A solid model representation of the data, using either gray scale shading or color for clarity.

The ultimate output from the Seating Planer is a report which shows for each design:

1. The set of values for the parameters used for the design and the "figure of merit" value produced;
2. The numeric values specifying the vertical and horizontal dimensions for each of the elevations for inclusion in the set of plans and specifications.

While the system is actually being used to develop seating contours, a more easily comprehended view of the results is desirable. A schematic or representational image will give the operator a better appreciation of the whole room being designed and presents the changes in a more understandable manner than streams of numbers. Thus, the results of a design trial are best shown as some form of graphic display.

3.3.3 Idiomatic Forms of Specification.

The designer of an interactive system must be aware not only of the data items that are needed and the range of values acceptable for each item, but must also be aware of the various "expected" or "usual" ways of describing them. (In the same spirit, Wixon et al. discusses the use of "synonyms" for commands [WIXO83].)

For example, using the Seating Planner:

> The system should be able to accept data in any of the "usual" forms that the user might choose (i.e. rows, feet, inches, feet and inches, meters, or centimeters). Probabilities might be expressed either as a decimal fraction or as a percentage.

3.3.4 A Breadboard Implementation

The current implementation of the Seating Planner is a "first cut". For various reasons (time, stupidity etc.), the current version does not exhibit all of the desired user interface objectives.

1. The utility functions have not been installed. Each of them represent a specialized interface to the set of the UNIX file utility commands.
2. The entering of parameter values is not independent of units being used for display. Thus, if the system is set to display Imperial units, it will not correctly accept values in metric units. Nor will

the system accept dimensions in the "feet and inches" form (this a particular nuisance when entering the sighting point values). Entry of values in units of rows was also not implemented in the first development of the parameter reader.

3. Describing the characteristics of the seating surface by total number of seats and room shape has not been installed. The tailoring of the row spacing to meet the requirements of various jurisdictions is yet to be done. The handling of cross-aisles and staggered rows are not developed in this preliminary implementation.

3.4 An Approach to the Interface Design

The question now is how to build an interface to the Seating Planner which will aid and support the community of interested users who are inexperienced with this system. The central idea in the development of this type of interface is to follow the notion of presenting to the user a "tool-like" view as proposed by Nakatani and Rohrlich [NAKA83]. They define their notion of "tool-like" as:

1. The use of the "tool" is learned mostly by exploration and "playing around" rather than by formal training or extensive consultations with instruction manuals;

2. What has been learned on one model is easily applicable to another model (for example: using a new model of a copier or typewriter);

3. Controls are specialized and optimized for efficient performance of the tool's function. That is, the choice of actions open to the user should be clear and how to perform each action ought to be obvious. This approach can reduce the cognitive burden placed on the user and simplify the learning process. It also views the computer as part of the mechanism but not a thing with which the user need be concerned.

What are the basic principles of such an approach?

1. The user has a task he wants to accomplish. His focus is that task, not the details of the commands. The visual presentation should supply to the user all he needs in order to use the Seating Planner. The mechanics of the system interface should be as unobtrusive as can be arranged.
2. The user always needs to be aware of the "mode" of the system.
3. In order to minimize the cognitive load, command actions should be as simple as possible. The control sequence should be: "do this action", "get that result".

Ideally, a tool-like system will have almost no external written instructions. Rather, the means of its use becomes evident to the user as the usage proceeds. Wright has pointed out that often users deliberately avoid using the documentation until they have a need for information [WRIG83]. Instead, people will "experiment" with the device until they either understand its workings or they ask someone else who may have the required knowledge. The tool-like character of the system depends on the user's employment of the recognition of what to do, rather than expecting the user to remember the required procedures. The manufacturers of cars recognize this. Every new car comes with an instruction book which specifies all the various pressures and quantities which the owner will need in order to maintain the car. But the critical the tire pressures and other basic data are also is printed on the glove compartment door or on the gas tank cover as the manual may never be read by the owner.

It is possible to set down some criteria to guide the design of a tool-like interactive system.

1. The "mode" status is always displayed to the user.
2. The user is "allowed" to use only the commands which are currently appropriate [SING82].
3. All commands are initiated by a single physical action. Commands with a potential for serious loss of previous work are an exception. This class of command requires the user to confirm his intention.
4. There is a direct association between a control device and a command action.

For example:	joystick	<==>	movement
	switch	<==>	selection
	keyboard	<==>	parameter input

5. The results should be displayed in a form that is familiar and convenient for the user.

3.5 The Selection of Input and Output Devices.

The Seating Planner has been conceived as an interactive CAD tool to aid in the design of theatres. While the system will need to store the floor contour data in numeric form, a schematic display of the room cross-section would seem more useful to the designer using the system. This type of display would show the location of the risers and steps which form the floor contour. Additional displays of the room showing a perspective view from a selected seat location would be useful, especially if the effects of changing the parameters could be illustrated.

There are, within the Computer Graphics Laboratory, three major devices for interactive display.

1. The CRT text display terminal, with an alphanumeric keyboard plus a set of "program function" keys. A cathode ray tube is used for the presentation of alphanumeric characters.
2. The frame buffer display system can provide an image in color and with animation.
3. The calligraphic vector display system provides a black and white image without hidden line removal. A close to real time control of the image is possible because of the internal display processing.

The choice made for this project was influenced by a desire to develop some demonstration programs for the calligraphic vector display system. The MPS, unlike the frame buffer system, was readily available.

The MPS system has a wide variety of input devices:

a light pen;
a tablet and stylus;
a joystick;
thirty-two illuminated push buttons;
and eight rotary potentiometers.

By comparison, the frame buffer has only a tablet and puck which contains four selector buttons.

For this project it was decided to produce a version of the Seating Planner which would use the Evans and Sutherland MPS calligraphic display system. The first approach, for the prototype system, was to show only the axial cross-section view of the room. Since each such representation in a is vertical plane, the control of the display by the user is limited to rotation about a vertical axis in the plane of the image and translation of the image left, right, up and down.

As mentioned before, the MPS has a number of interactive input devices so that there is the potential for flexible and varied techniques of control and display. Many of the input devices are less than ideal. The tablet or the light pen could be used for the control of the translation of the image. The control of rotation would require the use of some sort of quasi-circular motions by the user and special software would be needed to recognize them. Another way of using the wand or light pen to control all three motions would be to use it to move the "slider" of the pseudo valuator. The status of the cursor for each valuator would be displayed on the calligraphic display. For the Seating Planner there are two objections to this approach.

1. The pattern of control would then be: move cursor with the wand (or pen) to the desired slider

on the screen. Then move slider to change the rate or position of the image. This type of control would violate the "do this action", "get that result" style of command execution.

2. The screen area used to present the image of the sliders reduces the amount which is available for displaying the floor contour.

The dangling cable from either the lightpen or the stylus is a nuisance and a bother. As will be discussed later, the joystick allows all three types of control by the operator in a more natural manner.

The present MPS potentiometers are neither rate controls nor position controls. To be a true rate control, the result of turning the control knob ought to be a value indicating the amount of rotation in some time interval. On the MPS units, this is the output until the rotation is enough to cross the end of the element in the "pot". At this time a large value is emitted. This anomaly most probably is the result of using a potentiometer with a varying output voltage rather than a digital resolver. As the "pot" rotates past the end of the resistance material there is a jump in its output voltage which is converted by the A/D converter and sent to the MPS controller. By contrast, the digital resolver outputs a pulse for each element of rotation. By the use of the timing of a pair of pulses the direction of rotation also can be found.

Even if the output of the MPS "pots" were acceptable for use as valuators, there were other serious failings. The set of potentiometers were mounted in a cramped physical layout. There was no provision for legends to label the function of the controls for the user assistance. The potentiometers themselves were not provided with logging scales nor was there any indication of a "home position". The designers of the MPS seemed to intend that the potentiometers were to be thought of as treadmills to indicate rate information rather than as valuators [FOLE74]. But in fact they are neither.

There was a trackball available, but the ball required a relatively large amount of force because of its high resistance to movement, this made it seem sluggish and hard to control.

The joystick, which provided three axes of control, appeared to be adequate. It also had the attribute that, when the joystick was released, it returned to a center (off) position. If the user releases the joystick, its output goes to zero and consequently the rate of movement will also go to zero, thus stopping the motion of the displayed object. This feature made the joystick a convenient device for use as a rate control input device.

So in summary, for the Seating Planner the devices chosen for use as the interface between the user and the computer system were:

1. The MPS calligraphic 1000 line by 1000 line vector display device;
2. The three axis joystick and the set of thirty two illuminated pushbutton switches associated with the MPS display;
3. A CRT terminal with its keyboard.

3.6 Implementation of the Interactive Control System.

Once the decisions are made as to which of the particular interactive subsystems that are going to be employed, the process of implementation takes on form. The next step in the development of the Seating Planner was to implement the tasks within a framework of the interactive design principles and the ideas of Section 3.4. The problem now is to allocate the sub-tasks to the most suitable device. It is not really proper to discuss the implementation as a separate step because most of the allocation occurs together with the selection of the hardware.

What was needed was a definition of the major task elements and how to assign them to the interactive process. The questions at this point centered about what the system was to display in response to the user's input. That is, what actions of the user and what data provided by the user would cause a response and what should be the form of that response.

System Sign On.

After the inevitable rassle with the Unix operating system, the first action of the Seating Planner is to confirm to the user that the system is active and awaiting a command. On both the MPS display and the CRT terminal screen, messages are presented that announce that the Seating Planner is ready for business. In addition, the CRT display also indicates the operational modes. Thus, in the first response, the user has confirmation that the Seating Planner is operational and has instruction as to what he might choose to do next.

Response to the User Request.

When the user selects a mode, the immediate reaction of the Seating Planner is to display the appropriate parameter values which are current (Figure 3.8). Each mode has its own particular set and only those are presented to the user. In addition, instruction on how to proceed is also indicated.

Update of the parameter Values.

If the parameter values need modification, the user is presented with an instructional menu which shows all the available parameters and indicates which are the allowed entries for those requiring a choice, and indicates a specification for those where numeric data is required (Figure 3.11). Entering a value on the keyboard causes a redisplay of the parameter status menu confirming that the user's request has been executed. The experienced user, who remembers a parameter name, or enough characters to form a prefix which is unique, is free to enter the command without display of the instructional menu.

For example: to select imperial units, the user could enter "units imperial" or "u i". However, to change the "shape" parameter, the user must enter at least "sh f" of the "shape fan" command (Figure 3.11).

To Run a Trial.

The next step for the normal use of the Planner is to run the system to produce a seating contour. The command "go" is the termination of the setting of the parameters activity. A lone carriage return causes the instructional "help" menu to appear. The "go CR" sequence acts as a protection against accidental runs by requiring a more complex mental and tactile activity.

Emergency Procedures.

Whenever the Seating Planner is not active in a particular mode, the user can exit by the use of pushbutton number 16 on the MPS. The use of the "BREAK" key on the CRT, at any time, causes the program to terminate. In this breadboard system, terminating the current session results in the loss of any trial data. With the implementation of the file utility functions, the normal termination of a session would be less catastrophic. With the "normal" exit, the results would be stored in a predetermined file or the exiting task could request confirmation from the user that he intended to abandon the results of the session.

Once a mode is selected, the user can abort the mode selected by typing "cancel", or any prefix of it, followed by a carriage return.

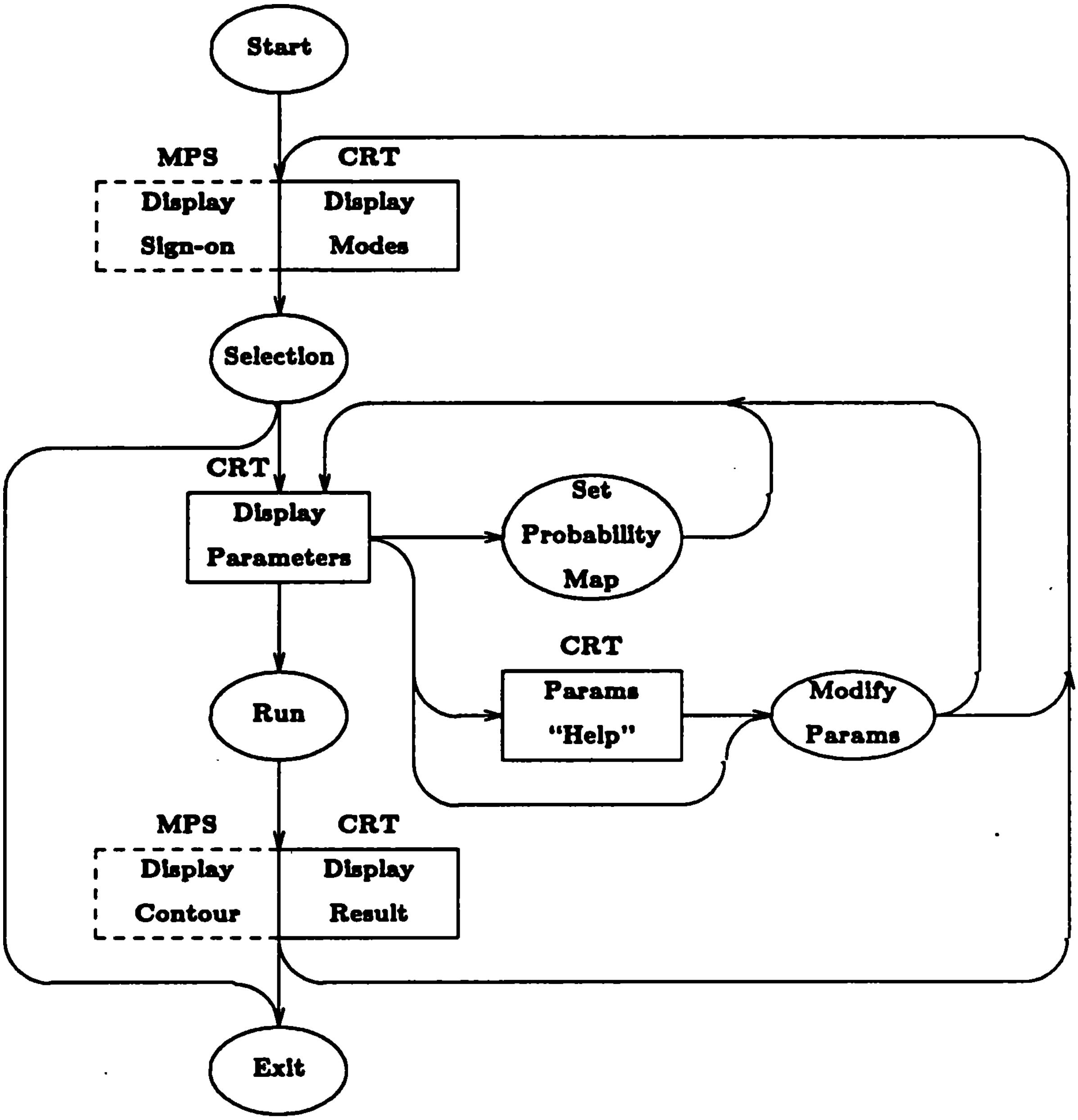


Figure 3.1

Allocation of Tasks to Hardware

3.7 Device Dependent Issues.

As Buxton points out, it is inevitable that the particulars of the underlying electronics impact the form and functions of the interactive interface [BUXT82]. In the case of the MPS, the most conspicuous difficulty was the I/O interrrupt problem. The MPS can generate a interrupt in response to an I/O action (such as a pushbutton switch closure), but the software to allow this signal to be properly recognized by an application program is not yet in place.

Polling the I/O.

At this time, for any interactive application of the MPS, the only way an application program can recognize that the user has activated one of the input devices is by continuous polling of the I/O ports. This is not a permanent state of affairs but at this time the necessary driver has not yet been written. Not only is this scheme wasteful of cpu cycles, but it is constricting in that it limits the use of a prioritized command structure. The polling of I/O status is analogous to waiting for a package to be delivered while preparing dinner when the doorbell is broken. There is a continuing set of interruptions to the task at hand to check the front door. So with polling, each mode must have its own polling mechanism to check for the "exit" or other overriding system commands. In some cases it is not easy to share control between the command reader and the working process. The result is unduly complex software.

Pushbuttons.

Because of the polling mode for sampling the I/O devices, it was possible for the program to interrogate the pushbuttons more than once during the time the user had the button depressed. For some actions multiple requests are benign. But for commands which cause a toggling action with the resultant rapid switching, back and forth, between the two states, the user cannot control the the selection. When the release of the pushbutton is recognized by the software, the last state selected by the toggling may be the one desired by the user but that it is not is equally likely.

An example of this problem is:

> When the user is setting the probability of each row individually, he has the choice of setting only the particular row indicated by the cursor location, or that row and all the subsequent rows together (Section 3.6, Figures 3.6 and 3.7).
>
> At all times the user can switch between these two editing modes. The change is made by

> depressing switch ten of the set of pushbuttons. If the host computer is lightly loaded, the time elapsed to poll all of the MPS interactive devices is so short that the edit mode could toggle between the two states several times before the user could remove his finger from the button. The resulting editing mode seemed to be a random event beyond the user's convenient control.

To make each action unambiguous, a software latch, or "debouncer", was installed. The action of this latch is analogous to that of the R-S flip-flop. The button that was pushed and all other buttons must be "off" before the first button pushed will be acknowledged and acted upon. This prevents the same button "push" from being read more than once. In a sense, the user is confirming his intention by releasing the selected button. This notion of debounce is in addition to the hardware switch debounce which is used to eliminate contact noise and mechanical bouncing of the switch contact. An objection to the form of "debounce" used in Seating Planner is that it does not provide any type of "key rollover". Here, the first switch depressed is the one which the causes the response. By contrast, many keyboards use a "two key rollover" where the last switch depressed is the one that causes an action. Another style of response is "n-key rollover" where every switch depression causes an action as the keystrokes are queued up, and then the queue is completely emptied. The use of a n-key rollover allows for a type-ahead operation.

In the Paint picture creation system of Plebon and Booth, a form of debounce was used because the tablet data output was not reliable [PLEB82].
There were two problems.

> The tablet, with the puck at rest, produced a stream of small but nonzero readings (background noise) and the tablet occasionally emitted very large readings (spike noise). In this case the software ignored very small readings and very large ones. If the tablet had been used as a rate (velocity) controller rather than a position controller, probably the spike noise could have been ignored as it was of a short duration.

The Unhidden Line.

The MPS hardware contains a "Central Graphics Processor" which contains an arithmetic processor, a picture memory and a line generator which creates the vectors that are displayed on the screen. The rotation and translation of the objects on the display list is carried out within the MPS unit which has no provision for hidden line elimination. The hidden line removal would have to be done before the object was sent to the MPS. Thus, each change of position would require the host VAX to do all the work that the MPS internal processor was intended to do. There is no high speed way of doing the necessary work to produce an image with the hidden lines removed. If the image has sufficient coherence which the viewer can resolve as the representation of a three dimensional object, then this limitation is not serious. A demonstration of the sort of ambiguity caused by the unhidden line can be seeen in Figure 3.2. In the simple cubic figure it is not clear whether corner A is in front of corner B or vice versa. With the use of line weights and dashed lines there is a stronger impression that corner C is in front of corner D.

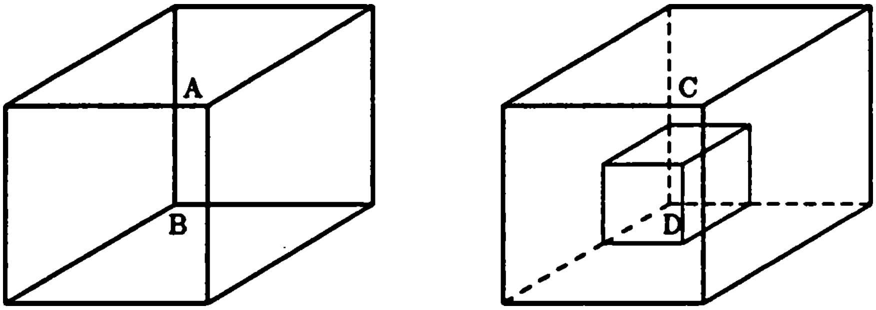


Figure 3.2

An Example of Ambiguity in Line Figures

The most natural display format for the Seating Planner was the centerline cross-section that the architect would normally use. In the breadboard version of the system only this display format is used. The problem is to display the results of the various runs for comparison purposes. The use of rotation of the image about a centered vertical axis seemed to provide some additional visual resolution which was an aid to the viewer.

Two different display formats were tried.

1. A step back view. The latest run is displayed at the z location zero. That is, it is located at the front clipping plane. As each new result is produced, all the older displays are stepped back in z. In the resulting display, it is very difficult to compare the results of several runs. Rotation of the step back display does appear to give the user additional visual discrimination. But it is difficult to evaluate the various design representations.
2. A "pie wedge" display. This display technique also displays the latest result at the front of the the image, at the near clipping plane. The difference is that in this mode of display the older images pivot about a vertical line through the sighting point. The rotation of the "pie wedge", so that the apex appears pointing away from the viewer, allowed for easier comparisons of the results of the various runs.

The Joystick.

For the Seating Planner, the three degrees of freedom of the joystick provided easy access to vertical and horizontal motion of the displayed image by the use of the two directions of movement of the joystick and the rotation of the image about a vertical axis. The rotation is controlled by the use of the rotary potentiometer which is the end of the joystick itself. Unlike a mouse or a puck, the joystick housing does not shift its location while the joystick is in use. The arm moves in response to the user's efforts but the unit remains in the same location on the table. As the unit physically remains fixed, the user has only to learn the one action of placing his hand on the joystick control. Nor is the user hampered by a trailing electrical cord. The unit's fixed position allows the cable to be dressed out of the way. The joystick on the MPS has a known "at rest" position. The MPS joystick contains springs within the unit which returns the joystick to its center position when the handle is released. For rate control this feature means that when the handle is released the motion stops. But it is this feature of the joystick that makes it unsuitable for situations which require position control. (Most joysticks have a built-in centering mechanism, but there are some built for positioning applications and for these, the joystick remains at whatever position it was last set.)

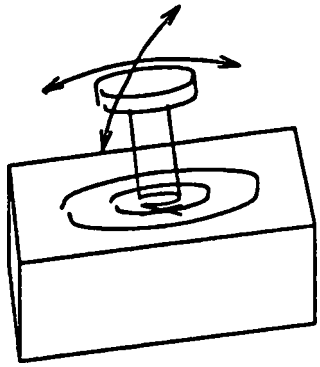


Figure 3.3
A Three Axis Joystick

The use of the joystick in a rate control mode still requires some care. The user will want to use the joystick for two different types of movement. The careful positioning of the controlled object, "docking", which requires a low rate of motion of the image is one type of useage. The other type is the rapid movement of an object about the display space, "slewing". The rate of movement of the selected object caused by deflection of the joystick can be a function of the displacement. Thus, for a large movement of the joystick, the object "slews" rapidly across the display at high speed; for smaller movements the control is more critical, slowly and precisely moving the object to particular position.

To accomplish both slewing and docking with same control, the joystick response to a small displacement from the "null" position causes only a small change in velocity, while the same displacement at some "off null" position results in a much larger change in velocity. An easy way to do this is to apply a nonlinear function to the displacement value from the joystick. It was not clear what the proper control function would be. A number of different combinations of a linear term, and either a quadratic or cubic term were tried.

For example: one such function using a cubic plus linear term is cubic plus linear function is $f(\theta) = \theta^3/8 + \theta$.

Experimentation with different functions to find those which provided both good Experimentation with these functions to find those which provided both good docking and satisfactory slewing action showed that the cubic function did not provide adequate control for docking. Also it saturated, with no further increase in rate, in about the center of the joystick deflection range. The saturation was the result of all of the operations being done in sixteen bit integer arithmetic with a test which limited the output of the nonlinearization to the maximum value that the joystick controller would supply 2^{15}. The linear control was very good for the precise control needed for docking but the slewing for large traverses took too long. Of the three remaining control curves (θ^2, $\theta^2/4+\theta$ and $\theta^3/8+\theta$), it was hard to pick a clear favorite. For the Seating Planner, the $\theta^2/4+\theta$ function seemed to answer the application's needs. It had enough control for small movements and enough speed for the slewing kinds of activity.

In addition to the control of the rate function of the joystick, the resolution of the control needs consideration. There is a D/A converter which is the output of each of the joystick's three potentiometers. The range of values it outputs is a number between -2^{15} and $2^{15}-1$. The rate control is accomplished in adding the scaled value of the joystick controller each time the polling cycle occurs. The raw values from the controller are too large for convenient rate control of a displayed object. To make the rate more manageable, the output number from the joystick is scaled to a range of 0 to 22. Because the MPS uses integer arithmetic the rate of motion does not change smoothly but rather jumps as the calculated scaled rate changes by integer steps. For the scale factor of 22, the jumps occur every 4.545% of the total travel of the joystick along one of its axes (Figure

3.5). This quantization of the rate provides one real and important benefit. When the control is released, the zeroing is not critical because any position that is within ±4.5% of the null point has a zero rate.

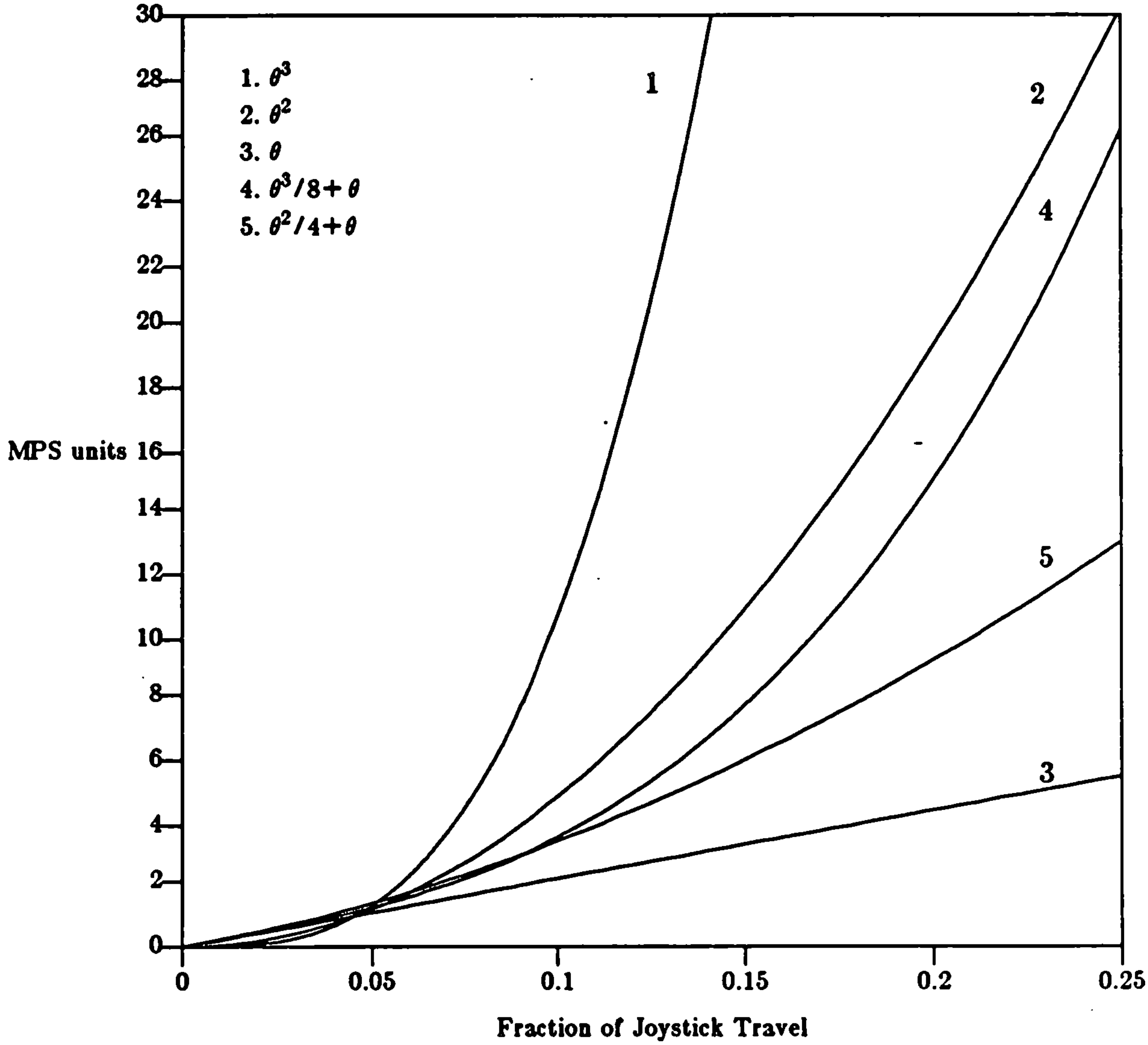


Figure 3.4

Control Functions for Mapping Joystick Movement to Rate Control

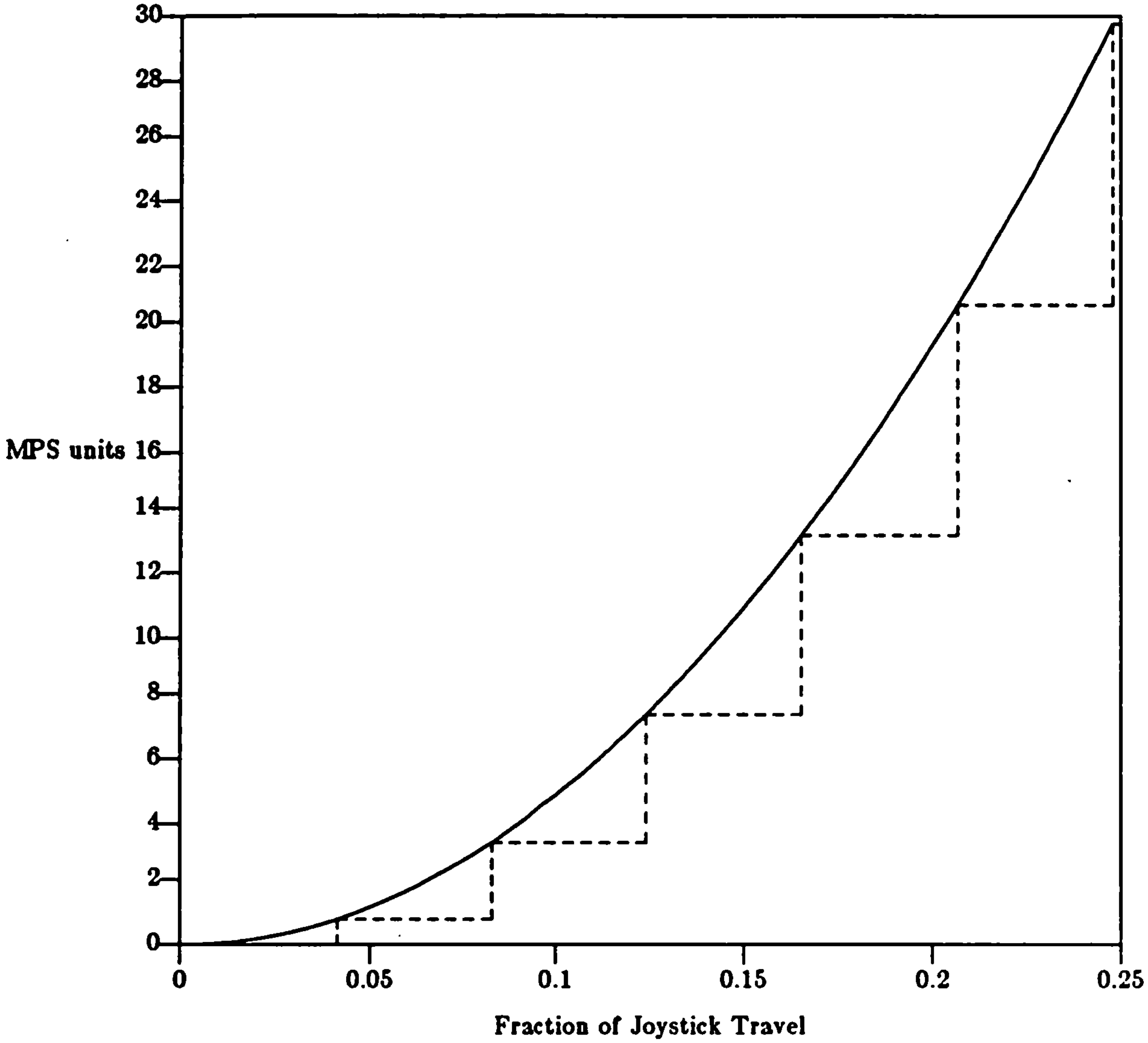


Figure 3.5

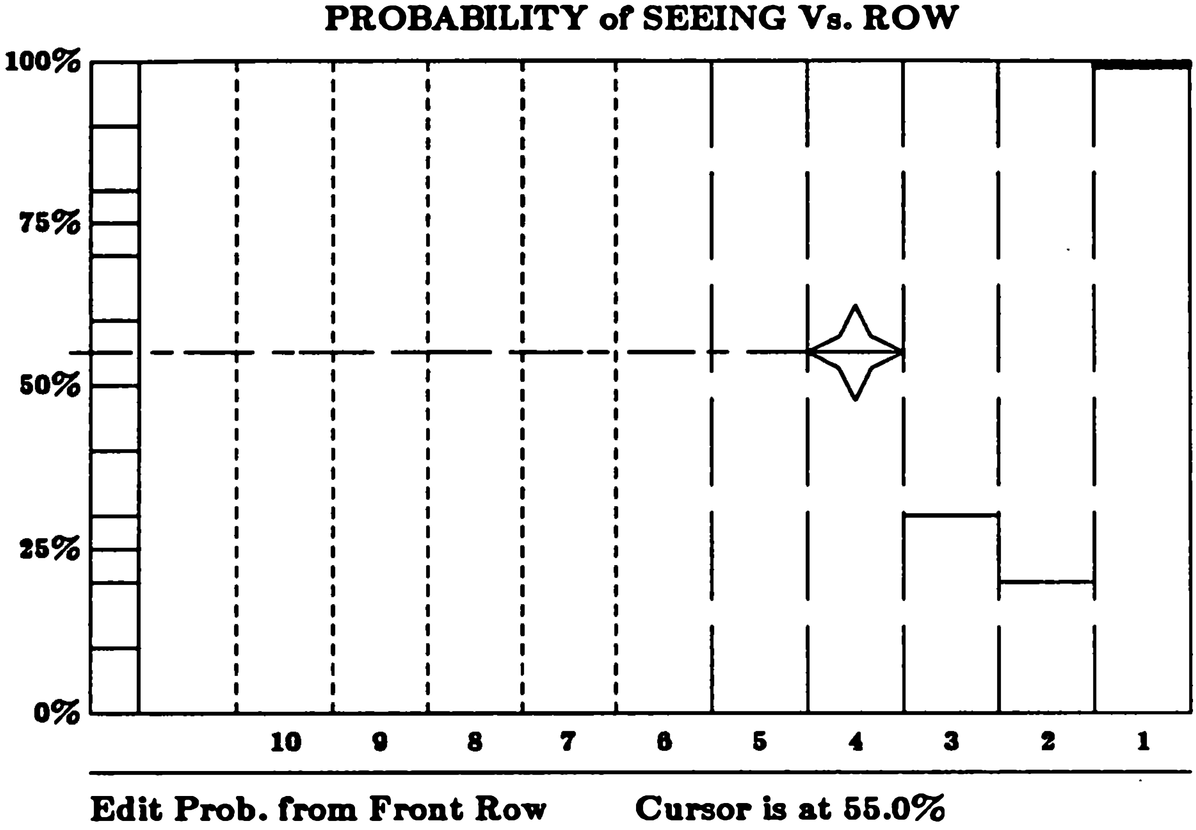
Quantization of Joystick Control

3.8 Utilization of the Joystick

The Seating Planner uses the joystick in two different ways. The control of the display position of the various seating contours has been discussed before (Section 3.7). When in the *Probability by Row* mode, where the user sets the probability of viewing for each row, the joystick is used to set the desired value. The cursor indicates the probability value is which controlled by the joystick. The forward-backward motion controls the rate at which the cursor increases or decreases the row's probability of viewing. The left-right motion of the joystick causes the cursor to jump, in the selected direction, to an adjacent row. The cursor object has a shape which is formed by a star shaped octagon (Figure 3.6). A horizontal line through the center indicates the probability value. As the number of rows of seating are changed, the number and width of the columns in the display also are adjusted. The cursor size is scaled to the spacing of the columns.

Equations 2.2 and 2.12 indicate that the y_i values set for the first few rows, determined by the probability of viewing set for those rows, has the largest influence on the height of the back row (y_n). For this reason, the first five or six rows of seats are always displayed, When more than twenty rows are requested, the succeeding rows are grouped into sets of two, five or ten rows to a group. (The Seating Planner will accept designs with up to two hundred rows.) As the first few rows are those which have the most impact on y_n, the cursor position is set to row one when entering the *Probability by Row* mode. It is expected that the user will most likely want to set uniquely the probabilities of the first few rows and then set all the succeeding rows to a common value. The default editing submode is for this style of editing (Figure 3.6). A dashed line on the display indicates that the setting of all rows to the left of the cursor are set to the cursor indicated value.

A second editing mode provides the ability to change only the value in the row indicated by the cursor. The cursor position is also marked on the probability scale on the left end. When the user slews the cursor to another row, it automatically sets to the existing value (Figure 3.7). Thus, the user does not need to adjust the cursor to the value in the column as it is set automatically.



Edit Prob. from Front Row Cursor is at 55.0%

Figure 3.6

Joystick Controlled Cursor Used for Setting Row Probabilities

The probability value indicated by the cursor position is indicated by the line across the vertical scale on the left and by the numeric value indicated near the bottom of the display.

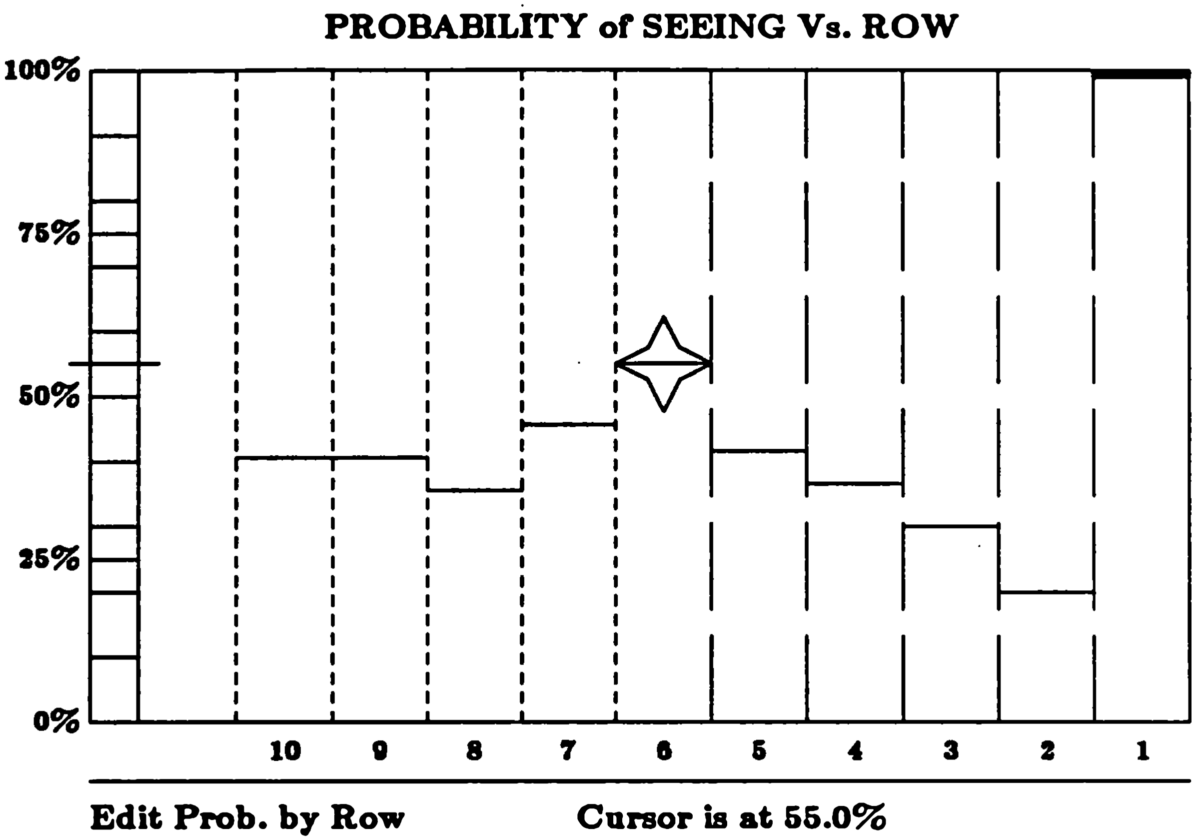


Figure 3.7

Joystick Controlled Cursor Used for Setting Probabilities by Row

Note that the cursor is only controlling the row six probability value. An indicator of the cursor position is displayed on the probability scale.

3.9 Status, Mode, Parameter and Result Displays

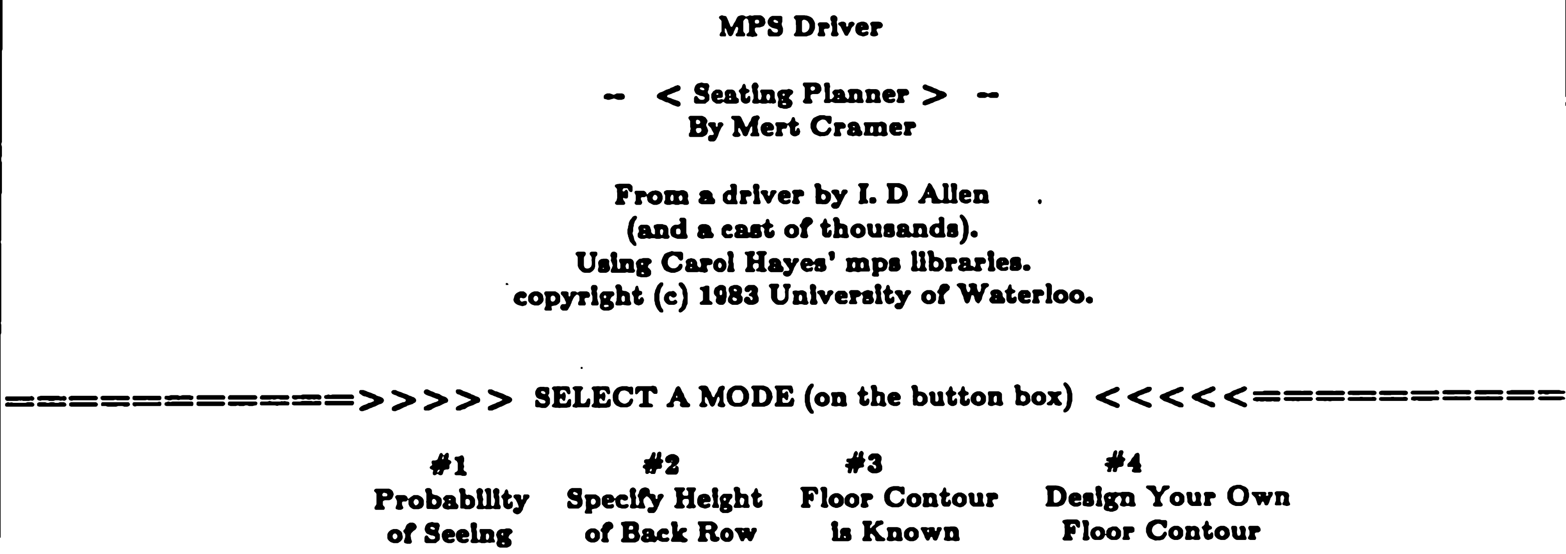
The use of two display devices, one for textual material and another for pictorial representations, is somewhat controversial. Plebon states that she feels that "to shift one's focus of attention from one screen to another during an operation causes a loss of visual continuity" [PLEB82]. This may a point with highly interactive systems such as her Paint program. For the Seating Planner or other "run oriented" CAD operations, the change in focus may not be as critical nor as disturbing. In this prototype of the Seating Planner, the choice was made to leave the calligraphic display free from information that was textual in nature and which could be as easily displayed on CRT. This left the MPS screen free to display the largest practicable image of the floor contour with almost no useful area being absorbed by menu or status displays. Figures 3.6 and 3.7 show the display for setting individual row probability values. In this case only ten rows were being considered. When a larger number of rows needs to be displayed (say 40 or 60 rows) the width of the row column is much smaller and the size of the cursor shrinks correspondingly.

It is the system designer's decision to allocate the area of the calligraphic display to the different visual requirements. The size of the area of the calligraphic display surrendered to data that can be presented just as well on a ordinary CRT must be weighed against the the size of the drawn image. Because the Seating Planner is a tool for a task which uses discrete runs rather than continual interactive input and because of the desire to create as large a cross-section representation of the floor contour as possible, the parameter data was presented on the CRT.

The CRT displays have four purposes:

1. Show the state of the system and current choices;
2. Display the current status of the parameters;
3. Report the results of the last trial;
4. Supply a "help" for reminding the user of the system modes, command, and parameter value designations.

Examples of the some common CRT displays illustrating these goals follow.



```
                              MPS Driver

                      --  < Seating Planner >  --
                            By Mert Cramer

                       From a driver by I. D Allen
                       (and a cast of thousands).
                     Using Carol Hayes' mps libraries.
                 copyright (c) 1983 University of Waterloo.

============>>>>> SELECT A MODE (on the button box) <<<<<===========

         #1              #2               #3                #4
    Probability    Specify Height    Floor Contour    Design Your Own
     of Seeing      of Back Row        is Known        Floor Contour
```

Figure 3.8

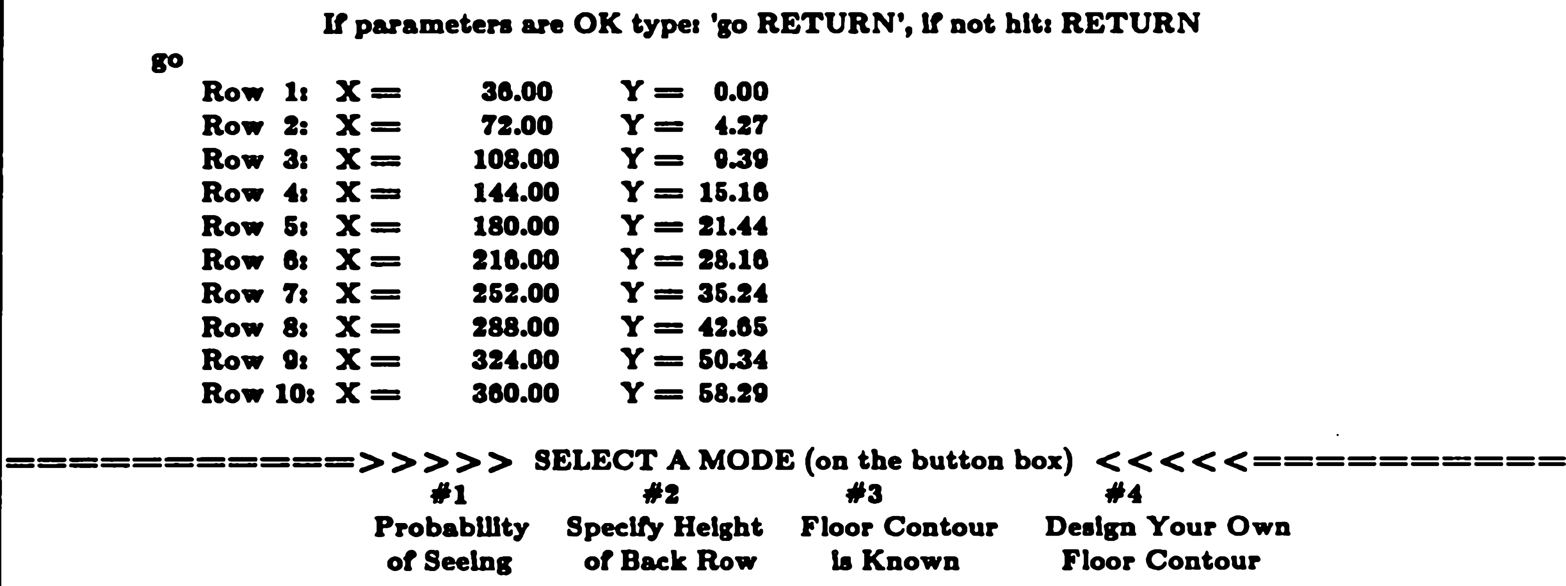
Sign-on Message with Mode Menu

```
                         << SEATING PLANNER >>

                                MODE

                       Specify Seeing Probability
=============================================================================

      Input and Display are in IMPERIAL units.

      For this plan, of 10 rows,
      the form is  AISLE ROW style with a RECTAGULAR shape,
      the seats are INLINE with a row to row spacing of 36.00 inches.

      Probability of seeing is  50.00 %.

      The Sighting Point is:  Length is 3 ft. 0 in.
                              Height is 4 ft. 0 in.

      The Composition of the Audience is: 50% MEN, 50% WOMEN.

           If parameters are OK type: 'go RETURN', if not hit: RETURN
```

Figure 3.9

Constant Probability of Seeing Default Display

The command "go RETURN" will cause the contour to be generated. "RETURN" presents the "help" menu to aid the user in making changes to the set of parameters (figure 3.11).



```
                If parameters are OK type: 'go RETURN', if not hit: RETURN
go
   Row  1:  X =     36.00     Y =  0.00
   Row  2:  X =     72.00     Y =  4.27
   Row  3:  X =    108.00     Y =  9.39
   Row  4:  X =    144.00     Y = 15.16
   Row  5:  X =    180.00     Y = 21.44
   Row  6:  X =    216.00     Y = 28.16
   Row  7:  X =    252.00     Y = 35.24
   Row  8:  X =    288.00     Y = 42.65
   Row  9:  X =    324.00     Y = 50.34
   Row 10:  X =    360.00     Y = 58.29
===========>>>>> SELECT A MODE (on the button box) <<<<<===========
                #1            #2              #3              #4
           Probability   Specify Height   Floor Contour   Design Your Own
            of Seeing     of Back Row       is Known      Floor Contour
```

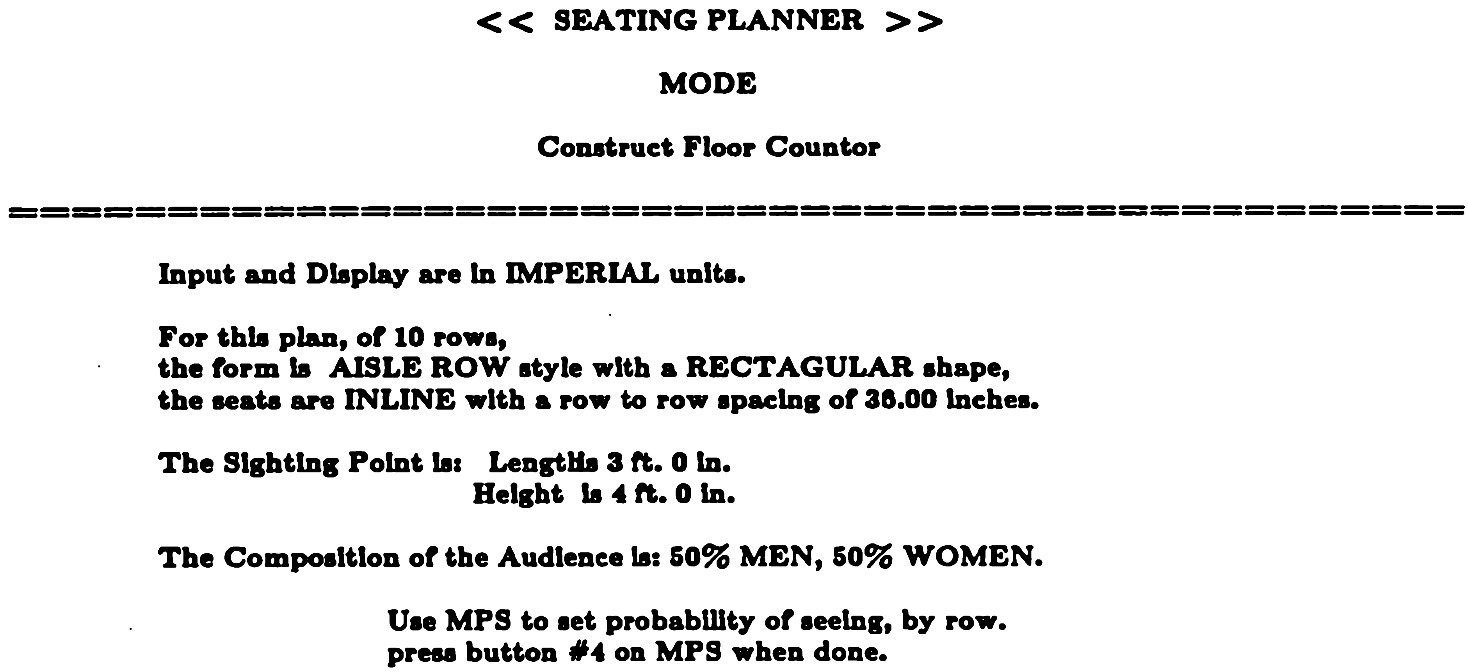
Figure 3.10

Constant Probability of Seeing Results Display

```
If parameters are OK type: 'go RETURN', if not hit: RETURN



To change a value, enter the name and value.

Units        :    IMPERIAL or METRIC
Style        :    AISLE ROW or CONTINENTAL
Pattern      :    INLINE or STAGGERED
Shape        :    RECTANGULAR or FAN
Rows         :    < number of rows >
Probability  :    < percentage >
Sight Point  :    < distance from row 0 >
                  < point above or below row 0 floor >
```

Figure 3.11

Constant Probability of Seeing Help Menu



```
                      << SEATING PLANNER >>

                              MODE

                     Construct Floor Countor
================================================================

        Input and Display are in IMPERIAL units.

        For this plan, of 10 rows,
        the form is  AISLE ROW style with a RECTAGULAR shape,
        the seats are INLINE with a row to row spacing of 36.00 inches.

        The Sighting Point is:  Lengthis 3 ft. 0 in.
                               Height  is 4 ft. 0 in.

        The Composition of the Audience is: 50% MEN, 50% WOMEN.

                       Use MPS to set probability of seeing, by row.
                       press button #4 on MPS when done.
```

Figure 3.12

Variable Probability by Row

```
If parameters are OK type: 'go RETURN', if not hit: RETURN

For this contour, the probability of seeing is:


Row 1  -----  99.90
Row 2  -----   9.89
Row 3  -----  25.11
Row 4  -----  35.00
Row 5  -----  39.89
Row 6  -----  59.89
Row 7  -----  59.89
Row 8  -----  59.89
Row 9  -----  59.89
Row 10 ----  59.89
```

Figure 3.13

Confirmation of the Setting of Row-by-Row Probability

3.10 Summary

The present version of the Seating Planner was designed with the user's convenience as first priority.

1. By use of the "Reset" pushbutton, the user can always exit from the top level of the system.
2. When in one of the modes, the user can always step back to the command level by use of a "cancel" command from the keyboard.
3. The current selected mode and the status of the parameters are always presented except when results are being displayed. After the results are fully displayed, the parameter status reappears automatically.
4. The controls have a "natural" coupling to the object being changed. The joystick controls motion of the image and the parameter values are set by use of the keyboard.
5. The display of the seating profiles gives the user a view with which he is familiar for the convenient comparison of his designs.

4.0 Future Developments.

The basic form and function of the Seating Planner has been built and tested. There are a number of necessary features that were not implemented because of time limitations and because of the project's focus on the interactive design and on the determination of the optimum y_i values.

1. A better technique of display of the results is needed. The use of the available MPS screen area for menu items and for virtual valuators should be expanded.
   There are alternative displays to use when setting probability values for individual rows. A dynamic display which shows the range of allowable values at the current row is one such possibility.

2. Interactive control of the sighting point should be included in the Seating Planner. The display of the floor contour should follow the changes made by the user.

3. The various Canadian jurisdictions, outside of the province of Ontario, have not been factored into the system.

4. The influence of row length, for Continental Seating, on the row-to-row spacing has not been installed.

5. All of the idiomatic data input forms are not yet accepted by the system. For example: the sighting point constraints now can be input only in units of inches or meters. Other idiomatic forms are feet and inches, feet with decimal fractions, and centimeters.

6. The calculation of the number of rows and their spacing from a requested total number of seats needs to be done.

7. The system lacks a set of archiving functions. There is no way to save and reuse the results developed in a session, nor is there any hardcopy output for future reference.

8. The pie wedge display is acceptable but it is still a long way from being adequate for day-to-day usage. The use of color and/or a display editor which would show only two or three results at any one time, should improve visual clarity.

9. When an user initiated interrupt is available on the MPS unit, the whole input control subsystem should be redone to take advantage of interrupt driven I/O handling.

10. More work is needed on the docking vs. slewing question for both rate and position sensitive controllers.

All of these areas need further work to produce a production tool.

Despite the variety of input devices available for use with the MPS system, there are a number which are lacking. A touch screen device should be installed. A more responsive trackball would also be desirable. A set of linear, rather than rotary potentiometers, would likely prove useful. A treadmill style of linear control which has mouse-like properties might be another adjunct input device. An upgrade of the current potentiometers and joystick would make it easier to evaluate interactive input designs. Better valuators with logging scales and provision for legends, in addition to a joystick with a switch at the end of the wand and free from calibration pots, would allow more variety of testing of interactive designs.

5.0 Conclusion.

As a consequence of the work on the Seating Planner, the nearly 145 year old problem of how to find the height of row y_i needed to meet a desired figure of merit value has been given a convenient numeric solution. This technique for finding the floor contour for the iseidomal calculation can be used wherever predetermined viewing performance is required. The related isoacoustic floor contour can be produced by use of the same technique of calculation when "line of sight" audibility is needed by replacing the eye height statistics with ear height data.

The planning of public places of assembly can be designed using a figure merit rather than by "a rule of thumb". The viewing performance of the space now can be calculated as a factor in the building costing and changes to the plan can be assessed as to their impact.

The Seating Planner has been given a preliminary development to explore the design of an interactive system. The ideas shown in this project are only some of the issues that still trouble the designers of interactive computer applications. With time and in succeeding generations of the Seating Planner, better answers for a number of them can be found.

Appendix 1. Approximation of the Gaussian Distribution Function

The physical attributes of the human population are examples of variables which exhibit a Gaussian distribution (also called the Normal distribution). This function is well understood, and its properties make certain mathematical manipulations very simple [CROW60][PARZ60].

For a Gaussian random variable x, with a mean of zero and standard deviation of unity, the probability that the random variable has a value equal to or greater than x is

$$Q(x) = \frac{1}{\sqrt{2\pi}} \int_{x}^{\infty} e^{-\frac{t^2}{2}} \, dt \tag{A.1}$$

and the probability that the variable is at most x is

$$P(x) = 1.0 - Q(x). \tag{A.2}$$

In general, an exact solution of the integral cannot be computed. Direct numerical integration is a laborious calculation and not convenient to use. The integral and its inverse can be approximated by a technique which was proposed by Hastings [HAST55]. An improved form is used here [HEWL81].

The approximation is

$$Q(x) = f(x)(b_1 t + b_2 t^2 + b_3 t^3 + b_4 t^4 + b_5 t^5) + \epsilon(x) \qquad \text{for } x \geq 0 \tag{A.3}$$

$$\text{where } f(x) = \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}} \tag{A.4}$$

$$\text{and } t = \frac{1}{1.0 + rx} \qquad \text{for } r = 0.2316419.$$

Because the probability density function $f(x)$ is symmetric

$$Q(x) = 1.0 - Q(|x|), \qquad \text{if } x < 0.$$

The polynomial factor is computed using the coefficients

$b_1 = 0.31938153$

$b_2 = -0.356563782$

$b_3 = 1.781477937$

$b_4 = -1.81255978$

$b_5 = 1.330274429.$

For $x \geq 0$ the error term is bounded by $|\epsilon(x)| < 7.5 \times 10^{-8}$.

The inverse of the calculation of the Gaussian integral is that given a probability Q, for the random variable x, calculate a value such that x will be at least that value with a probability Q. The inverse does not have such an accurate approximation, but the following is adequate for our purposes. As in the calculation of $Q(x)$, the evaluation of the inverse will take into account the symmetry of $f(x)$. This is reflected in the expressions for t and x.

$$t = \begin{cases} \left(\ln \dfrac{1}{Q^2}\right)^{1/2} & \text{if } 0.0 < Q \leq 0.5 \\ \left(\ln \dfrac{1}{(1-Q)^2}\right)^{1/2} & \text{if } 0.5 < Q < 1.0 \end{cases}$$

$$y = t - \frac{c_0 + c_1 t + c_2 t^2}{1 + d_1 t + d_2 t^2 + d_3 t^3} + \epsilon(Q) \tag{A.5}$$

$$x = \begin{cases} y & \text{if } 0.0 < Q \leq 0.5 \\ -y & \text{if } 0.5 < Q < 1.0 \end{cases}$$

The polynomial factors are computed using the following coefficients.

$c_0 = 2.515517$ $\quad d_1 = 1.432788$

$c_1 = 0.802853$ $\quad d_2 = 0.189269$

$c_2 = 0.010328$ $\quad d_3 = 0.001308.$

The error term is bounded by $|\epsilon(Q)| < 4.5 \times 10^{-4}$.

The random variable treated here is the normalized form. A random variable is normalized by translating its value to the origin (subtracting the mean μ) and scaling (dividing by the standard deviation σ). An example of this transformation is shown in Section 2.3.

Appendix 2. A "C" Program to Produce the Composite Floor Slope

the program that produced the "Weighted Average Composite Seating Floor Slopes" in Figure 2.4 is shown here. The output of this code is a set of commands to the troff text formatter and the PIC preprocessor.

```
/*
 *      Find the Composite yN[i]s over all the rows.
 */

#include    <stdio.h>
#include    <sys/types.h>
#include    <math.h>
#include    <graphics/mps.h>
#include    "../src/mpsProj.h"

float
main()
{                                                       S
    int  flag, i, j, k, l, rowMax, rowNum;
    float   bottomScale;
    float   len;
    float   normDist();
    float   topOfScale;
    double  a, alpha, alphaSq;
    double  b;
    double  delta;
    double  fndFlr();
    double  getZ();
    double  aGamma;
    double  h[201];
    double  limit;
    double  max, min, maxHead, maxEye, minHead, minEye, muDiff;
    double  mean[2][2];
    double  probA, probB, probLower, pTarg, probTst, probOld, probRatio, probUpper, probWork;
    double  rr;
    double  s, scale, sigmaDiff, sigma[2][2];
    double  test, tol, tstA, tstB, testEye, testHead, trialVarY;
    double  xDist[201], xEye, xHead;
    double  y, yDiff, yLower, yN[201][5], yTst, yWork[200], yUpper;
    double  z[4];
```

```
/* Revised 7 Jan. 84 */

mean[0][0]      =       36.90;   /* Men seated head height mean. (inches.)       */
mean[0][1]      =       33.90;   /* Women seated head height mean. (inches.)     */
mean[1][0]      =       32.00;   /* Men seated eye height mean. (inches.)        */
mean[1][1]      =       29.30;   /* Women seated eye height mean (inches.)       */

sigma[0][0]     =       1.277;   /* Men seated head hgt. std. dev. (inches.)     */
sigma[0][1]     =       1.216;   /* Women seated head hgt. std. dev. (inches.)   */
sigma[1][0]     =       1.216;   /* Men seated eye hgt. std. dev. (inches.)      */
sigma[1][1]     =       1.186;   /* Women seated eye hgt. std. dev. (inches.)    */

printf(“.EQ\ndelim \n.EN\n”);

a           =       40.0;       /* SightingPoint Height.                    */
b           =       262.0;      /* SightingPoint Length. 6 rows             */
tol         =       0.001;      /* Limit for cutoff of fixed point calc.    */
rr          =       42.0;       /* Continental seating row spacing.         */
s           =       16.5;       /* Seat height above the floor.             */
rowMax      =       21;         /* Limit on number of rows.                 */

for( i= 0; i < 5; i++ )          yN[i][1]  =  0.0;    /* Height of the first row.   */
for( i= 0; i < rowMax; i++ ) h[i]      =  rr;     /* Row to row spacing.        */

printf(“.PS\nalt = 0.0100i\nbase = 6.0i\noffset = 1.0i\nmajTick = 0.1i\n”);
printf(“.ps +2\n”);

max     =       -100.0;
min     =        100.0;
```

```
/* Outer loop on alphas. */
for( alpha = 0.4; alpha < 0.41; alpha += 0.10 )
{
    printf(“# alpha= %6.3f\n”, alpha );

    /* Loop on probs.          */
    for( pTarg = 10.0; pTarg < 91.0; pTarg += 40.0 )
    {
        printf(“# ======  pTarg= %g ======\0’, pTarg );

        xDist[1]          =       h[1];
        yWork[1]          =       0.0;

        /* Start of the fixed point process.   */

        for( i= 2; i < rowMax; i++ )
        {
            /* Find pUpper, pLower, and pWork.            */
            /* Find the limiting values of Y and P( Y ).   */

            alphaSq           =       alpha * alpha;
            xDist[i]          =       xDist[i-1] + h[i];
            probUpper         =       100.0;
            probLower         =       0.0;

            /* Calculate the yN values for each of the gender cases.   */

            for( k= 0; k < 4; k++ )
                yN[i][k] = fndFlr( pTarg, yWork[i-1], k, xDist[i-1], h[i], a, b, s);

            /* Using the yN[l][l] values get the weighted prob. for each      */
            /* case. Then use the yNs and probs which fit closet to pTarg.  */

            for( l= 0; l < 4; l++ )
            {
                for( k= 0; k < 4; k++)  z[k] = -getZ( yN[i][l], yWork[i-1], k, xDist[i-1], a, b, h[i], s);

                probTst    =
                        (double)(normDist( z[0], FORWARD ) ) * alphaSq +
                        (double)(normDist( z[1], FORWARD ) ) * ( alpha - alphaSq ) +
                        (double)(normDist( z[2], FORWARD ) ) * ( alpha - alphaSq ) +
                        (double)(normDist( z[3], FORWARD ) ) * ( ( 1.0 + alphaSq ) - 2.0 * alpha );
```

```
    if( ( probTst - pTarg > 0.0 ) && ( probTst < probUpper ) )
    {
        probUpper       =       probTst;
        yUpper          =       yN[i][l];
    }

    if( ( probTst - pTarg < 0.0 ) && ( probTst > probLower ) )
    {
        probLower       =       probTst;
        yLower          =       yN[i][l];
    }
}

yWork[i]            =       yLower;
limit               =       tol * pTarg;
test                =       2 * limit;

/* Now that the closest values to y[i] are known,           */
/* proceed to find fixed point of pTarg.                    */
/* Secant method used here.                                 */

while( test > limit )
{
    probA           =       probUpper - pTarg;
    probB           =       probLower - pTarg;

    /* Test probA and probB to set up secant to avoid stability */
    /* problems. (See [VAND83] for details.)                    */

    tstA    =       ( probA > 0.0 ) ? probA : -probA;
    tstB    =       ( probB > 0.0 ) ? probB : -probB;
    if( tstA > tstB )
    {
        probRatio   =       probB / probA;
        yDiff       =       yUpper - yLower;
        yTst        =       yLower;
    }
    else
    {
        probRatio   =       probA / probB;
        yDiff       =       yLower - yUpper;
        yTst        =       yUpper;
    }
```

```
                /* Actual Calculation Here! */

                yWork[i]    = yTst - yDiff * probRatio / ( 1.0 - probRatio );

                max         = ( yWork[i] > max ) ? yWork[i] : max;
                min         = ( yWork[i] < max ) ? yWork[i] : min;

                for( k= 0; k < 4; k++) z[k] = -getZ( yWork[i], yWork[i-1], k, xDist[i-1], a, b, h[i], s);

                probWork        =
                    (double)(normDist( z[0], FORWARD ) ) * alphaSq +
                    (double)(normDist( z[1], FORWARD ) ) * ( alpha - alphaSq ) +
                    (double)(normDist( z[2], FORWARD ) ) * ( alpha - alphaSq ) +
                    (double)(normDist( z[3], FORWARD ) ) * ( ( 1.0 + alphaSq ) - 2.0 * alpha );

                if( ( probWork - pTarg ) * probA  > 0.0 )
                {
                    yUpper      =       yWork[i];
                    probUpper   =       probWork;
                }
                else
                {
                    yLower      =       yWork[i];
                    probLower   =       probWork;
                }

                test = ( ( probWork - pTarg ) > 0.0 ) ? probWork - pTarg : pTarg - probWork;

            } /* End of while prob diff > tol. */

        } /* End of optimization by rows.   */

        scale           =       xDist[rowMax];
        printf("move to ( %6.2f*base+offset, %6.2f*alt )\n", xDist[1]/scale, yWork[1]);

        for( l= 2; l < rowMax; l++ )
        {
            printf("line to ( %6.2f*base+offset, %6.2f*alt )\n", xDist[l-1]/scale, yWork[l]);
            printf("line to ( %6.2f*base+offset, %6.2f*alt)\n",  xDist[l]/scale, yWork[l]);
        }

    } /* End of pTarg loop. */

} /* End of alpha loop. */
```

```
/*  Now draw the box around the curves.         */

    printf(“#\n.ps -2\n”);

    topOfScale          =       (float)( ( (int)(max+25.0)/50 + 1 ) * 50 );
    if( min < 0.0 ) bottomScale = (float)( ( (int)(min)/10 - 1 ) * 10 );
    else                bottomScale     =       0.0;

    printf(“move to ( offset, %6.2f*alt )\nBotLine : Here\n”, bottomScale );
    printf(“line to ( offset - majTick, %6.2f*alt )\n”, bottomScale );
    printf(“\“%1d in. \“ rjust\n”, (int)(bottomScale) );
    printf(“move to ( offset, %6.2f*alt )\n”, bottomScale );

    for( len = bottomScale; len < topOfScale; len += 50.0 )
    {
        if( len + 50.0 > topOfScale ) printf(“line to ( offset, %6.2f*alt )\n”, topOfScale );
        else
        {
            printf(“line to ( offset, %6.2f*alt )\n”, len + 50.0 );
            printf(“line to ( offset - majTick, %6.2f*alt )\n”, len + 50.0 );
            printf(“\“%3d in. \“ rjust\n”, (int)(len + 50.0) );
            printf(“move to ( offset, %6.2f*alt )\n”, len + 50.0 );
        }
    }

        /* Close the outline box. */

    printf(“move to ( offset, BotLine.y )\n“);
    printf(“line to ( offset + base, BotLine.y )\n“);
    printf(“line to ( offset + base, %6.2f*alt )\n“, topOfScale );
    printf(“line to ( offset, %6.2f*alt )\n“, topOfScale );

        /* Draw the horizontal Scale.    */

    for( len = 0.0; len <= 1.001; len += 0.10 )
    {
        printf(“move to ( offset + %6.2f*base, BotLine.y )\n”, len );
        printf(“line to ( offset + %6.2f*base, BotLine.y - majTick )\n”, len );
        printf(“\“%g\“ at ( offset + %6.2f*base,”, 20*len, len );
        printf(“ BotLine.y - majTick * 2.0 ) \n”);
    }
    printf(“.PE\n” );

} /* End of main */
```

```
/*
 *     NORMAL DISTRIBUTION CALCULATOR
 *
 *     THIS CALCULATION PROVIDES THE PROBABILITY THAT X IS GREATER THAN VALUE.
 *     FOR FINDING PROB. X LESS THAN VALUE, CALL WITH -VALUE.
 *
 *
 *     This function:  1. when given the variable returns the probability
 *                        of the normal error function;
 *                     2. When given the probability of an error function
 *                        returns the variable that would cause it.
 *
 *     The function returns a float result from a float value and
 *     int direction. The specifications for the direction params are
 *     in mpsProj.h.
 *
 *     The approximation formulas are from Abramowitz and Stegun
 *     "Handbook of Math. Functions", NBS 1964 by way of the HP 11
 *     Handbook.
 *
 */

#include    <math.h>
#include    <graphics/mps.h>
#include    "../src/mpsProj.h"

float
normDist( value, direction)
    int      direction;
    float    value;
{
    /* These array values are the coefficients spec'ed by the HP handbook.  */

    double   b_Const[6];
    double   c_Const[3];
    double   d_Const[4];

    int      i;
    double dblValue;
    double t, tmp, tmpTwo, tmpThree, result;
    double fX;                                 /* probability distribution funct. */
    double polynom;                            /* FORWARD local vars.             */
    double c_Sum, d_Sum;                       /* INVERSE local vars.     */

    double pi_Const     =      0.398942280;    /* (1/ sqrt(2 * pi))       */
    double r_Const      =      0.2316419;
```

```
/* FORWARD direction co-effs. */

b_Const[1]        =        0.31938153;
b_Const[2]        =       -0.356563782;
b_Const[3]        =        1.781477793;
b_Const[4]        =       -1.821255978;
b_Const[5]        =        1.330274429;

/* These array values are used in the INVERSE direction.              */

c_Const[0]        =       2.515517;
c_Const[1]        =       0.802853;
c_Const[2]        =       0.010328;

d_Const[1]        =       1.432788;
d_Const[2]        =       0.189269;
d_Const[3]        =       0.001308;

/*  Forward calculation */

if ( direction == FORWARD )
{
    polynom       =       0.0;
    tmp           =       (double) value;
    tmp           =       ( value < 0.0 ) ? -tmp : tmp;

    t             =       1.0 / ( 1.0 + r_Const * tmp);
    tmp           *=      tmp / 2.0;
    fX            =       pi_Const * exp( -tmp );

    tmp      =       t;        /* Use tmp as the t to nth  */

    for ( i = 1; i < 6 ; i++ )
    {
        polynom     +=      b__Const[i] * tmp;
        tmp         *=      t;                        /* next power of t. */
    }    /* end of for loop that calculates polynom. */

    result                =        fX * polynom * 100.0;          /* Probability in percentage. */

    return( ( value < 0.0 ) ? (float)(100.0 - result) : (float) result );

} /* end of the FORWARD calc. */
```

```
    /* The INVERSE calculation */

    if ( direction == INVERSE )
    {
        dblValue=       (double)(value) / 100.0;        /* From percentage form. */

        if ( value < 0.0 || value > 100.0 ) return( (float) ERROR_ND );

        tmp         =       ( dblValue < 0.5 ) ? dblValue : ( 1.0 - dblValue );
        tmp         =       sqrt( log( 1.0 / ( tmp * tmp ) ) );

        tmpTwo      =       tmp * tmp;
        tmpThree    =       tmp * tmpTwo;

        c_Sum       =       c_Const[0] + ( (c_Const[1] * tmp ) + ( c_Const[2] * tmpTwo ) );

        d_Sum       =       1.0 + ( d_Const[1] * tmp + ( ( tmpTwo * d_Const[2] ) + ( tmpThree * d_Const[3]

        result      =       tmp - ( c_Sum / d_Sum );

        return( ( value <= 50.0 ) ? (float) result : (float) -result );

    } /* end of the INVERSE calc. */

} /* end of normDist. */
```

```
/*
 *              Get the Z value for a given yN and gender case.
 */

#include    <stdio.h>
#include    <sys/types.h>
#include    <math.h>
#include    <graphics/mps.h>
#include    "../src/mpsProj.h"

double
getZ( yN, yNnOne, genderCase, xDist, a, b, h, s)
    int     genderCase;
    double  a;
    double  b;
    double  h;
    double  s;
    double  xDist;
    double  yN, yNnOne;

{
    double  delta;
    double  eyeMean, eyeSigma;
    double  aGamma;
    double  headMean, headSigma;
    double  muDiff;
    double  mean[2][2];
    double  sigmaDiff, sigma[2][2];
    double  yDiff;
    double  zDiff;
```

```
    mean[0][0]    =  36.90;  /* Men seated head height mean. (Inches.)         */
    mean[0][1]    =  33.90;  /* Women seated head height mean. (Inches.)       */
    mean[1][0]    =  32.00;  /* Men seated eye height mean. (Inches.)          */
    mean[1][1]    =  29.30;  /* Women seated eye height mean (Inches.)         */

    sigma[0][0]   =  1.277;  /* Men seated head hgt. std. dev. (Inches.)       */
    sigma[0][1]   =  1.216;  /* Women seated head hgt. std. dev. (Inches.)     */
    sigma[1][0]   =  1.216;  /* Men seated eye hgt. std. dev. (Inches.)        */
    sigma[1][1]   =  1.186;  /* Women seated eye hgt. std. dev. (Inches.)      */

    headMean      =     mean[ 0 ][ genderCase%2 ];
    eyeMean       =     mean[ 1 ][ genderCase/2 ];
    headSigma     =     sigma[ 0 ][ genderCase%2 ];
    eyeSigma      =     sigma[ 1 ][ genderCase/2 ];

    /* Set up new random variable. */

    aGamma        =     h / ( xDist + b );
    delta         =     1.0 + aGamma;
    muDiff        =     headMean * delta - eyeMean;
    sigmaDiff     =     sqrt( headSigma * headSigma * delta * delta + eyeSigma * eyeSigma );

    yDiff         =     yN - delta * yNnOne - ( s - a ) * aGamma;
    zDiff         =     ( yDiff - muDiff ) / sigmaDiff;
    return( zDiff );

}   /* End of getZ */
```

```
/*
 *      fndFlr.c
 *      Find the value yN for a given probability, yN-1,
 *      distance from row zero of yN-1, h (row to row spacing for that row,
 *      a, b, s, and gender case.
 */

#include    <stdio.h>
#include    <sys/types.h>
#include    <math.h>
#include    <graphics/mps.h>
#include    "../src/mpsProj.h"

double
fndFlr( prob, oldY, genderCase, oldXDist, h, a, b, s )
    int     genderCase;
    double  a;
    double  b;
    double  h;
    double  oldXDist, oldY;
    double  prob;
    double  s;

{
    int         i;
    float       normDist();
    double      delta;
    double      eyeMean, eyeSigma;
    double      findMin();
    double      aGamma;
    double      headMean, headSigma;
    double      muDiff;
    double      mean[2][2];
    double      sigmaDiff, sigma[2][2];
    double      trialVarY;
    double      yN;
```

```
    /* Revised 7 Jan. 84   */

    mean[0][0]    =  36.90;      /* Men seated head height mean. (Inches.)          */
    mean[0][1]    =  33.90;      /* Women seated head height mean. (Inches.)        */
    mean[1][0]    =  32.00;      /* Men seated eye height mean. (Inches.)           */
    mean[1][1]    =  29.30;      /* Women seated eye height mean (Inches.)          */

    sigma[0][0]   =  1.277;      /* Men seated head hgt. std. dev. (Inches.)        */
    sigma[0][1]   =  1.216;      /* Women seated head hgt. std. dev. (Inches.)      */
    sigma[1][0]   =  1.216;      /* Men seated eye hgt. std. dev. (Inches.)         */
    sigma[1][1]   =  1.186;      /* Women seated eye hgt. std. dev. (Inches.)       */

    i              =      genderCase;
    headMean       =      mean[ 0 ][ i%2 ];
    eyeMean        =      mean[ 1 ][ i/2 ];
    headSigma      =      sigma[ 0 ][ i%2 ];
    eyeSigma       =      sigma[ 1 ][ i/2 ];

    /* Set up new random variable. */

    aGamma         =      h / ( oldXDist + b );
    delta          =      1.0 + aGamma;
    muDiff         =      headMean * delta - eyeMean;
    sigmaDiff      =      sqrt( headSigma * headSigma * delta * delta + eyeSigma * eyeSigma );

    trialVarY      =      -(double)normDist( prob, INVERSE );
    yN             =      oldY * delta + (s - a) * aGamma + sigmaDiff * trialVarY + muDiff;

    return( yN );

}   /* End of fndFlr */
```

REFERENCES

BODY81 H. Bodycombe,
"The Design and Evaluation of Auditorium Seating Rakes",
Journal of Sound and Vibration,
Academic Press, London, 1981, Vol.78, No. 4, pp 598—602.

BURR49 H. Burris-Meyer, E. C. Cole,
Theatres and Auditoriums,
Reinhold, New York, 1949.

BURR64 H. Burris-Meyer, E. C. Cole,
Theatres and Auditoriums, 2nd Edition,
Reinhold, New York, 1964.

BUXT82 W. Buxton,
"Lexical and Pragmatic Considerations of Input Structures",
Computer Graphics, Vol.16, No.3, Fall 82.

CRAM68 M. Cramer,
"Generation of Audience Seating Arrangements by Digital Computer II",
Theatre Design and Technology, U. S. Institute for Theatre Technology,
New York, October, 1968, No. 14, pp 18—20, 32.
(One of two articles in this issue concerning CAD for theatre design.)

CROW60 E. D. Crow, F. A. Davis, M. W. Maxfield,
Statistics Manual,
Dover Publications, New York, 1960.
(Original title: The Care and Feeding of Pet Projects or What to do Till the Statistician Comes.)

DAMO66 A. Damon, H. W. Stoudt, R. A. McFarland
The Human Body in Equipment Design,
Harvard University Press, Cambridge, MA, 1966.

DREY57
H. Dreyfuss
Industrial Design, Volume 5,
Henry Dreyfuss and Associates, New York, 1957.
This book was published by Henry Dreyfuss and Associates apparently as a promotional presentation to potential clients. It contains anthropometric data which updates some values in the later [DREY67].

DREY67
H. Dreyfuss
The Measure of Man, Human Factors in Design,
Whitney Library of Design, New York, 1967.
Much of the data in the 1967 revision is from prior to 1960. Some of the anthropometric data is older than and is superseded by [DREY57].

EBER64
C. Ebert, G. Graubner, W. Unruh,
Unsere Theaterneubauten Nach 1945,
Deutschen Sektion des Internationen Theaterinstitutes E. V.,
Recklinghausen, Germany, Circa 1964.

FRNK68
P. H. Frink,
"Generation of Audience Seating Arrangements by Digital Computer I",
Theatre Design and Technology U. S. Institute for Theatre Technology,
New York, October 1968, No. 14, pp 11−17.
(One of two articles in this issue concerning CAD for theatre design.)

FOLE74
J. D. Foley, V. L. Wallace
"The Art of Natural Graphics Man-Machine Conversation",
Proc. of IEEE, IEEE, New York, Vol 62, No. 4 (April 1974), p 462.

HAMM72
R. Hamm,
Theatre Planning ABTT,
The Architectural Press, London, 1972.

HAST55 C. Hastings, J. T Hayward, J. P. Wong,
Approximations for Digital Computers,
Princeton University Press,
Princeton, New Jersey, 1955.

HAYE83 C. J. Hayes,
Software Support for an Evans & Sutherland Multi Picture System,
University of Waterloo, Department of Computer Science,
Masters Thesis,
Waterloo, Ontario, Canada. 1983.

HEWL81 Hewlett-Packard Staff,
HP-11 Solutions Handbook,
Hewlett-Packard Co., Corvallis, Oregon, 1981.

IZEN77 G. C. Izenour,
Theatre Design,
McGraw-Hill, New York, 1977.
Contains a copy of John Scott Russell's paper in Appendix Three.

KNUD50 V. O. Knudsen, C. M. Harris,
Acoustical Designing in Architecture,
John Wiley, New York, 1960.

NAKA83 L. Nakatani, J. A. Rohrlich,
Soft Machines: A Philosophy of User-Computer Interface Design,
Proc. of CHI'83 Human Factors in Computing Systems
Boston December 12—15, 1983, ACM, New York. pp19—23.

NASA78 National Aeronautics and Space Administration,
Anthropometric Source Book, Vol 1,
Anthropometry for Designers Series,
NASA Doc. No. 1024, 1978.

OED71	*Oxford English Dictionary — Compact Edition* Oxford University Press, Oxford, Great Britain, 1971 See page 505 for the definition of "Iseidomal".
PARZ60	E. Parzen, *Modern Probability Theory,* John Wiley, New York, 1960, Normal or Gaussian distributions are discussed in chapters 6 and 7.
PLEB82	D. A. Plebon, K. S. Booth, **"Interactive Picture Creation Systems",** *Techical Report CS-82-46* University of Waterloo, Department of Computer Science, Waterloo, Ontario, Canada.
PROV80	Province of Ontario, *Regulation 811 — Theatres Act,* Queen's Printer, Toronto, 1980.
RUSS38	J. S. Russell, "Elementary Considerations of Some Principles in the Construction of of Buildings Designed to Accommodate Spectators and Auditors.", *Edinburgh New Philosophical Journal,*, Vol. 27, Society of Arts, Edinburgh, 1838, (Reprinted in [IZEN77]).
SING82	B. Singh, **A Graphic Editor for Benesh Movement Notation,** University of Waterloo, Department of Computer Science, Masters Thesis, Waterloo, Ontario, Canada. 1982.
SOUL81	C. Soule, "Technical Quality Control of Motion Picture Theaters", *SMPTE*, New York, 1981, Vol.90, No. 4, pp 255—262, (Uses a head-to-eye spacing of only four inches).

UBC82
International Conference of Building Officals,
Uniform Building Code,
International Conference of Building Officals, Whittier, California, 1982.
Section 33.16 is concerned with public assembly seating.

VAND83
J. S. Vandergraft,
Introduction to Numerical Computations,
Academic Press, New York, 1983.
See page 265 for the stable form of the secant method.

WEVE68
K. Wever,
"Entwicklung der Sitzanordnung in Zuschaurerramen durch Isovituellen",
SCENA,
Instituts fur Technologie kultureller Einrichtungen, Berlin, DDR, 1969.
In German.

WIXO83
D Wixon, J. Whiteside, M. Good, S. Jones,
Building a User-Defined Interface,
Proc. of CHI'83 Human Factors in Computing Systems
Boston December 12–15, 1983, ACM, New York, pp24–27.

WOOD64
W. E. Woodson, D. W. Conover,
Human Engineering Guide for Equipment Designers,
University of California Press, Berkeley California, 1964.

WRIG83
P.Wright,
Manual Dexterity, a user-oriented approach to creating computer documentation,
Proc. of CHI'83 Human Factors in Computing Systems
Boston December 12–15, 1983, ACM, New York, pp11–18.