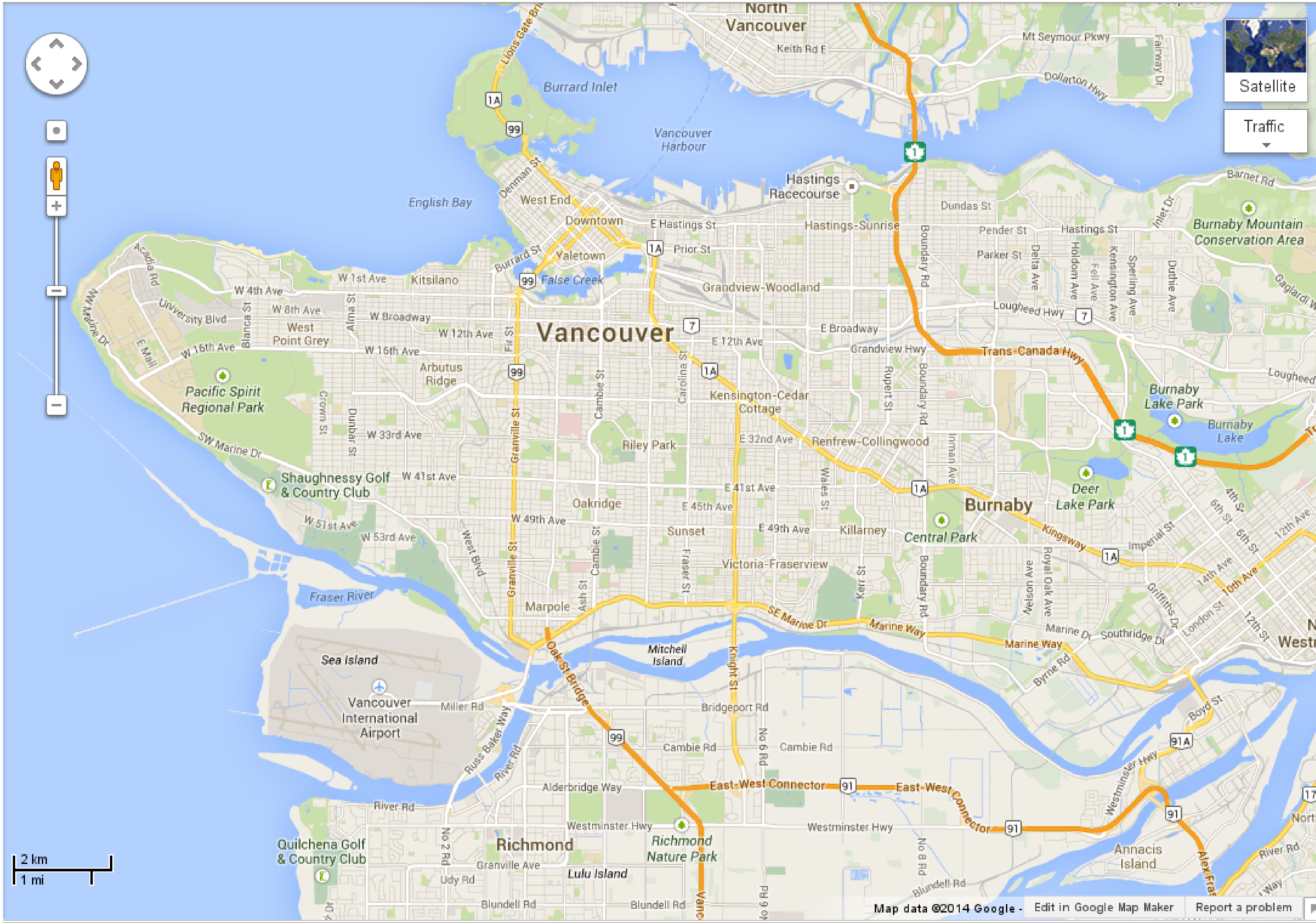
North Vancouver
Mt Seymour Pkwy
Keith Rd E
Dollarton Hwy
Fairway Dr
Satellite
Traffic
Burrard Inlet
Vancouver Harbour
English Bay
Lions Gate Br
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99
Denman St
West End
Downtown
E Hastings St
Hastings Racecourse
Hastings-Sunrise
Dundas St
Pender St
Parker St
Barnet Rd
Inlet Dr
Burnaby Mountain Conservation Area
Gaglardi Way
Hastings St
Boundary Rd
Delta Ave
Holdom Ave
Fell Ave
Kensington Ave
Sperling Ave
Duthie Ave
Prior St
Yaletown
Burrard St
False Creek
Grandview-Woodland
Lougheed Hwy
7
Acadia Rd
W 4th Ave
W 1st Ave
Kitsilano
University Blvd
W 8th Ave
Blanca St
Alma St
West Point Grey
W Broadway
W 12th Ave
Fir St
Vancouver
E Broadway
E 12th Ave
Grandview Hwy
Trans-Canada Hwy
Lougheed H
NW Marine Dr
E Mall
W 16th Ave
Arbutus Ridge
Carolina St
Kensington-Cedar Cottage
Rupert St
Burnaby Lake Park
Burnaby Lake
Pacific Spirit Regional Park
Crown St
Dunbar St
W 33rd Ave
Granville St
Cambie St
Riley Park
E 32nd Ave
Renfrew-Collingwood
Inman Ave
SW Marine Dr
Shaughnessy Golf & Country Club
W 41st Ave
E 41st Ave
Wales St
Burnaby
Deer Lake Park
4th St
6th St
12th Ave
W 51st Ave
W 53rd Ave
W 49th Ave
Oakridge
E 45th Ave
Sunset
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Killarney
Central Park
Kingsway
Imperial St
West Blvd
Ash St
Fraser St
Victoria-Fraserview
Kerr St
Nelson Ave
Royal Oak Ave
14th Ave
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Griffiths Dr
London St
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Marpole
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Southridge Dr
12th St
New Westm
Mitchell Island
Knight St
Oak St Bridge
Byrne Rd
Sea Island
Vancouver International Airport
Miller Rd
Russ Baker Way
River Rd
Bridgeport Rd
No 6 Rd
Cambie Rd
Boyd St
91A
Westminster Hwy
Alderbridge Way
East-West Connector
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17
North
River Rd
Quilchena Golf & Country Club
No 2 Rd
Richmond
Richmond Nature Park
No 8 Rd
Annacis Island
Alex Fraser
Granville Ave
Lulu Island
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2 km
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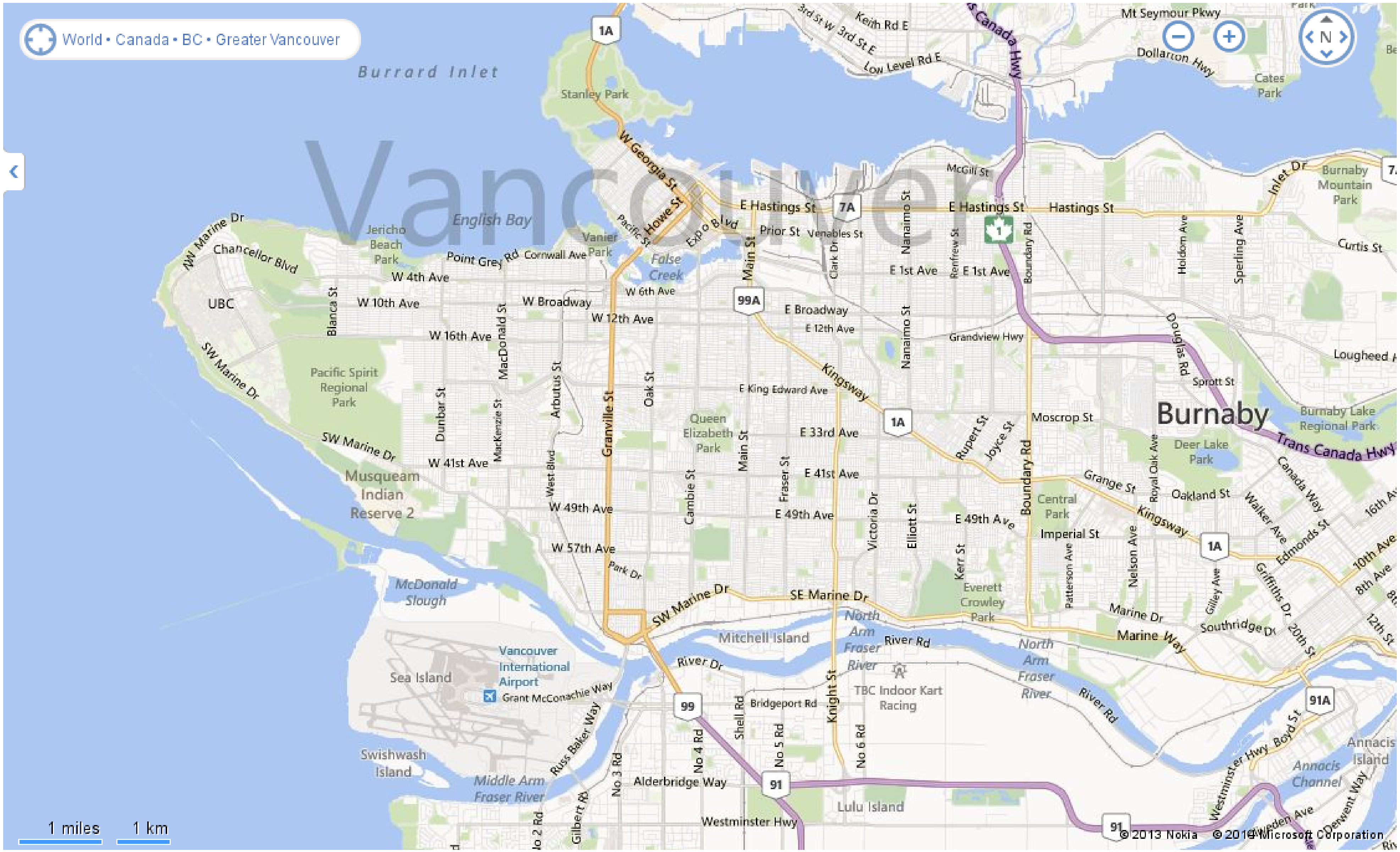

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Jericho Beach Park
Vanier Park
False Creek
UBC
Pacific Spirit Regional Park
Musqueam Indian Reserve 2
Queen Elizabeth Park
McDonald Slough
Sea Island
Vancouver International Airport
Swishwash Island
Middle Arm Fraser River
North Arm Fraser River
Mitchell Island
TBC Indoor Kart Racing
Lulu Island
Annacis Island
Annacis Channel
Central Park
Deer Lake Park
Burnaby Lake Regional Park
Burnaby Mountain Park
Everett Crowley Park
Cates Park
Mt Seymour Pkwy
Dollarton Hwy
Keith Rd E
Low Level Rd E
Trans Canada Hwy
W Georgia St
Howe St
Pacific St
Expo Blvd
E Hastings St
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Prior St
Venables St
Main St
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Inlet Dr
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Park Dr
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Patterson Ave
Nelson Ave
Gilley Ave
Griffiths Dr
SW Marine Dr
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Marine Dr
Marine Way
Southridge Dr
20th St
River Rd
River Dr
Knight St
Grant McConachie Way
Russ Baker Way
Bridgeport Rd
Shell Rd
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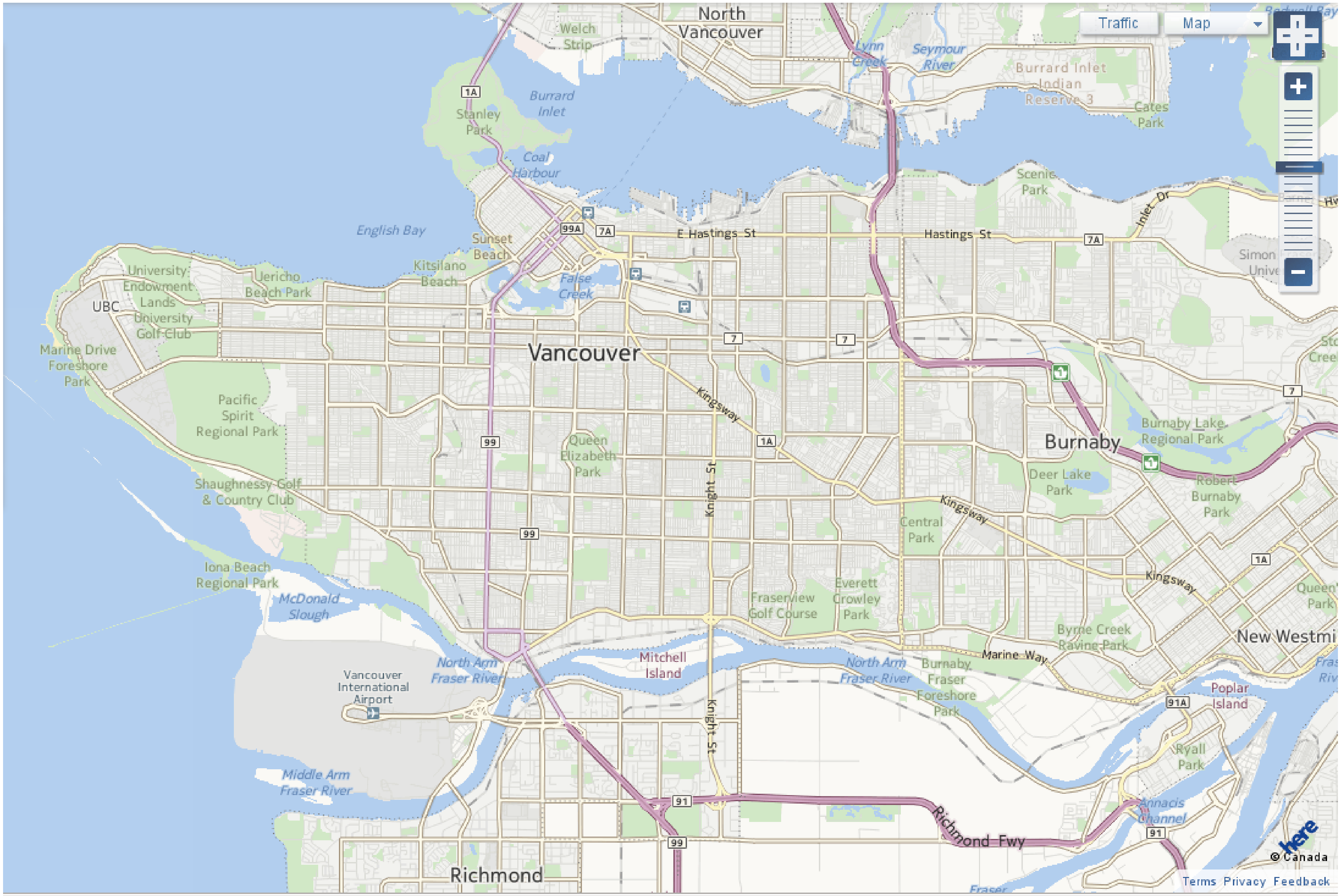
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Map
North Vancouver
Welch Strip
Lynn Creek
Seymour River
Burrard Inlet Indian Reserve 3
Cates Park
Stanley Park
Burrard Inlet
Coal Harbour
Scenic Park
English Bay
Sunset Beach
E Hastings St
Hastings St
Inlet Dr
Simon
Kitsilano Beach
False Creek
University Endowment Lands
Jericho Beach Park
UBC
University Golf Club
Marine Drive Foreshore Park
Vancouver
Pacific Spirit Regional Park
Kingsway
Burnaby
Burnaby Lake Regional Park
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Deer Lake Park
Robert Burnaby Park
Knight St
Shaughnessy Golf & Country Club
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Iona Beach Regional Park
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Fraserview Golf Course
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Byrne Creek Ravine Park
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Vancouver International Airport
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Mitchell Island
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Poplar Island
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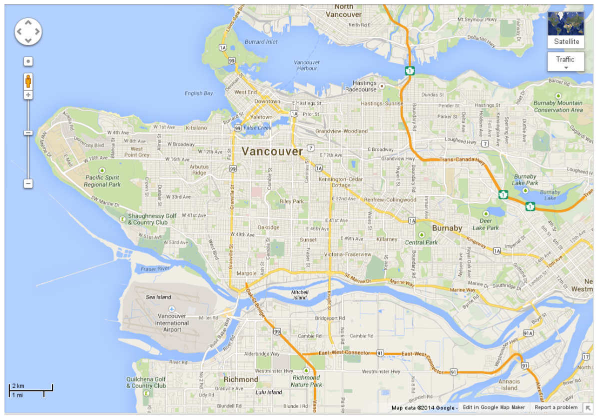
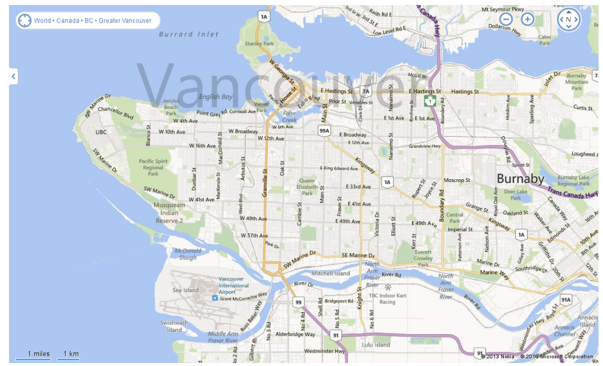
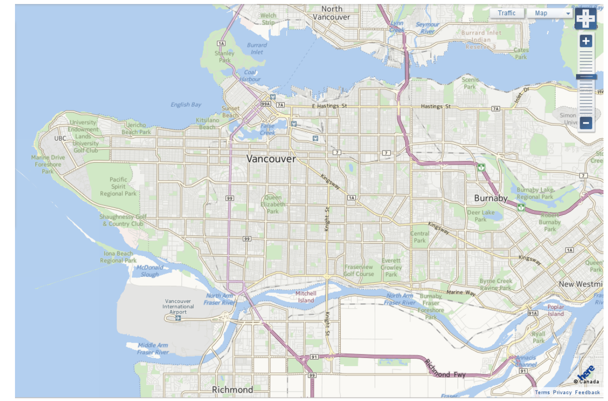




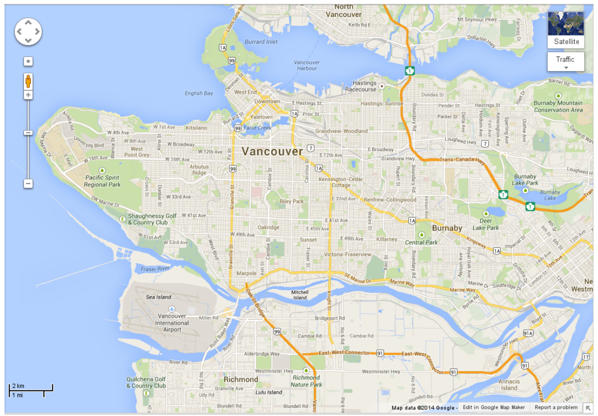


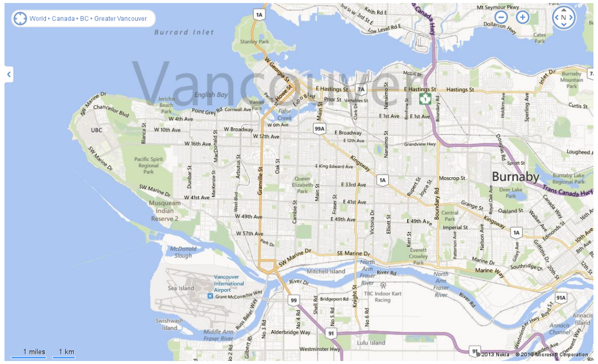
158 labels

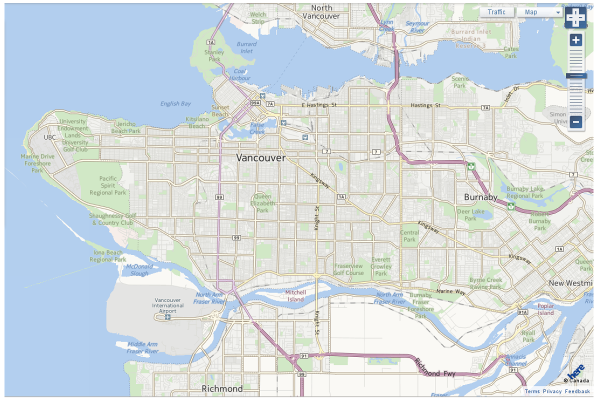
148 labels

60 labels

What makes a good labeling?



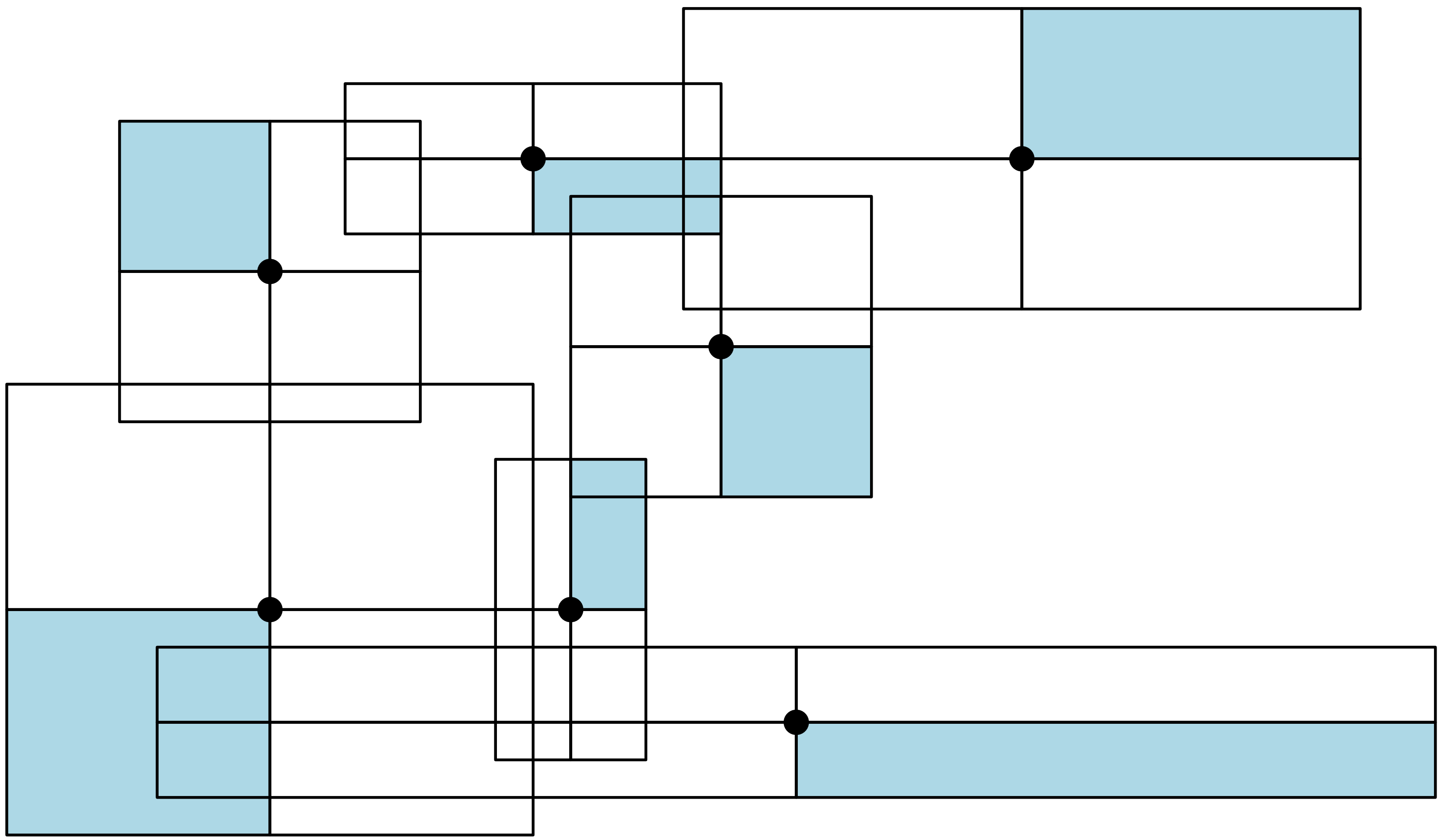




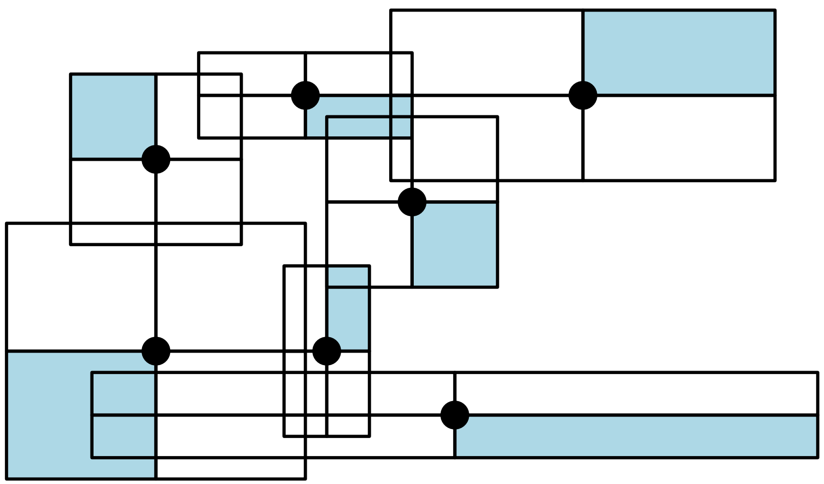
158 labels	148 labels	60 labels
Google	Bing	Yahoo

What makes a good labeling?

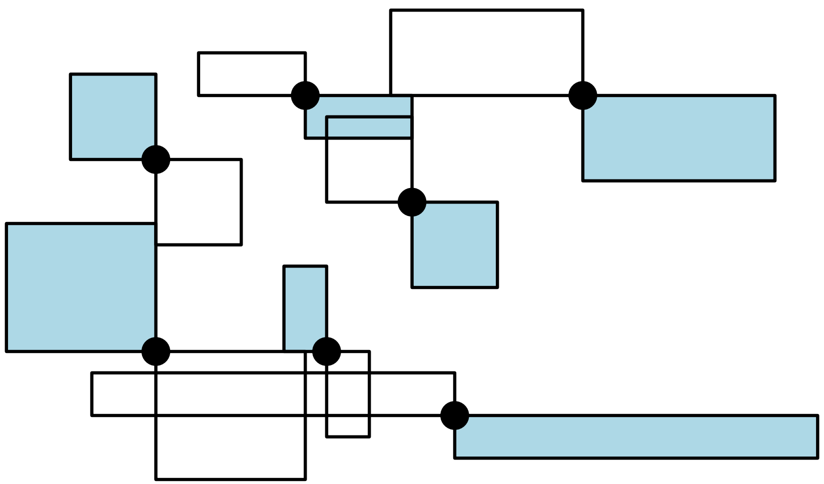
Point labels



Given n distinct points in the plane each with an associated rectangle, is it possible to place every rectangle (axis-aligned) with a corner on its point so that no rectangles overlap?



NP-complete



Solvable using 2SAT

[Formann & Wagner 91]

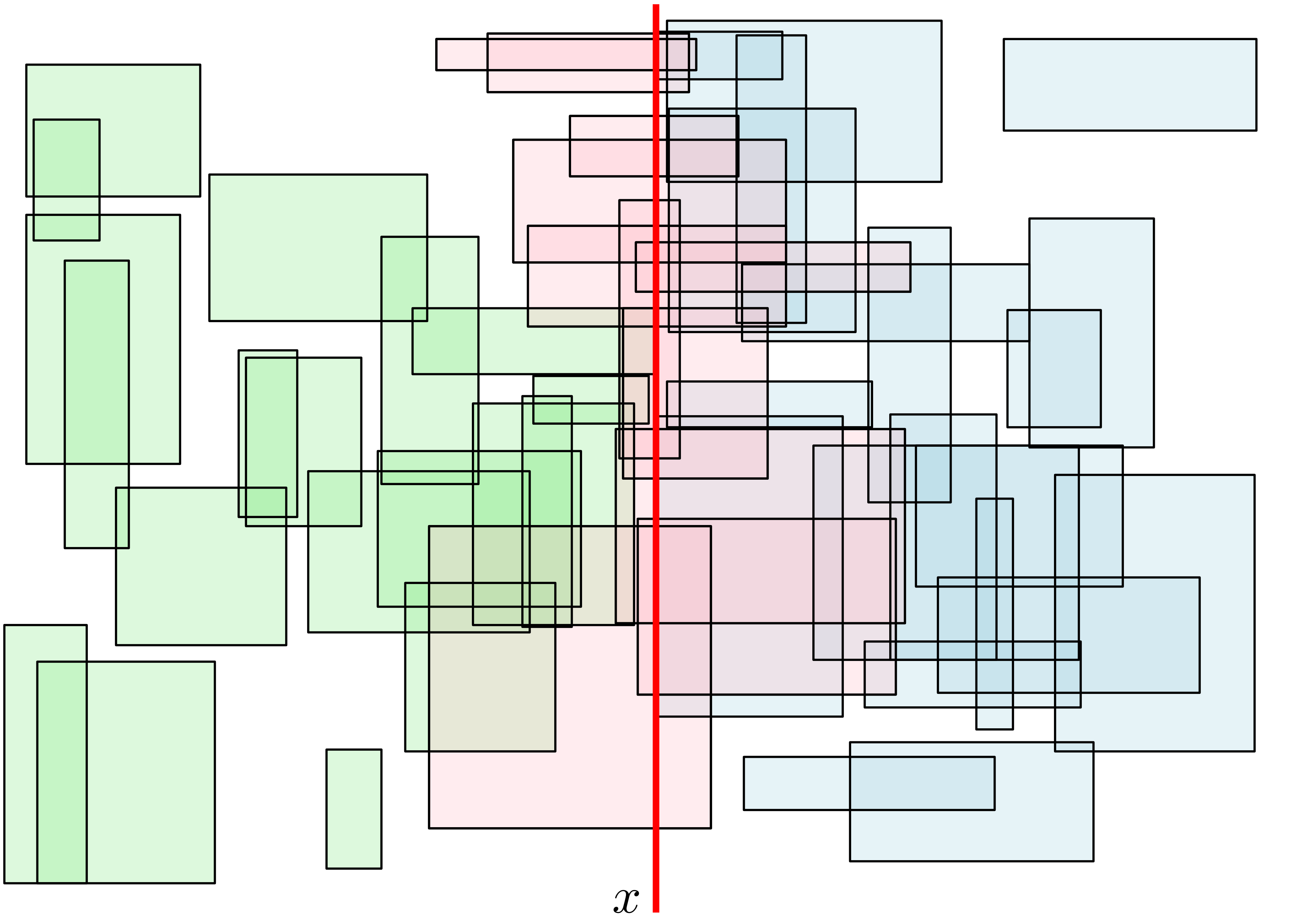
Approximately optimal number of labels

[Agarwal, van Kreveld, Suri 98]

Find a large* independent set in a set $\mathcal{R}$ of n rect's.

1. Let x be the median x-coordinate of $\mathcal{R}$.
2. Partition $\mathcal{R}$ into $\mathcal{R}_{<x}$, $\mathcal{R}_x$, and $\mathcal{R}_{>x}$.
3. Compute I_x, the max indep. set of $\mathcal{R}_x$.
4. Recursively compute $I_{<x}$ and $I_{>x}$, the approx. max indep. sets of $\mathcal{R}_{<x}$ and $\mathcal{R}_{>x}$.
5. If $|I_x| \geq |I_{<x}| + |I_{>x}|$ return I_x else return $I_{<x} \cup I_{>x}$.

$*$ at least OPT$/\log n$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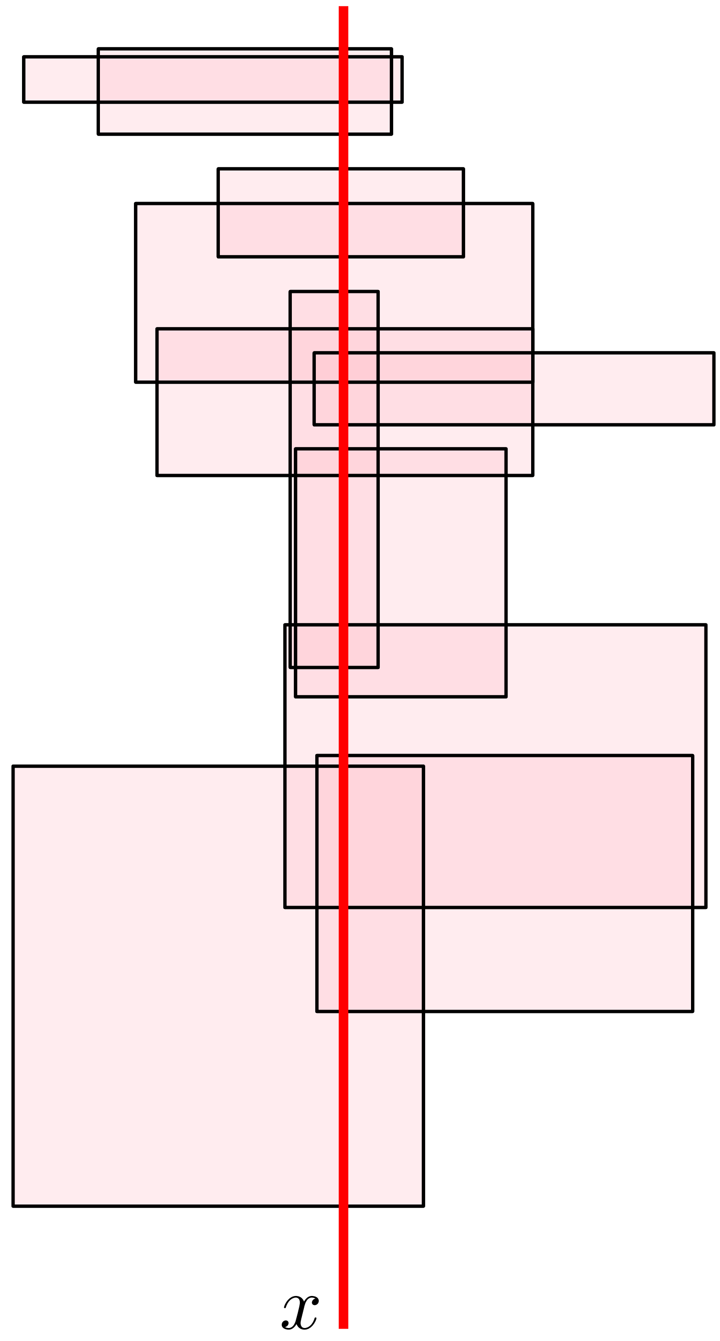

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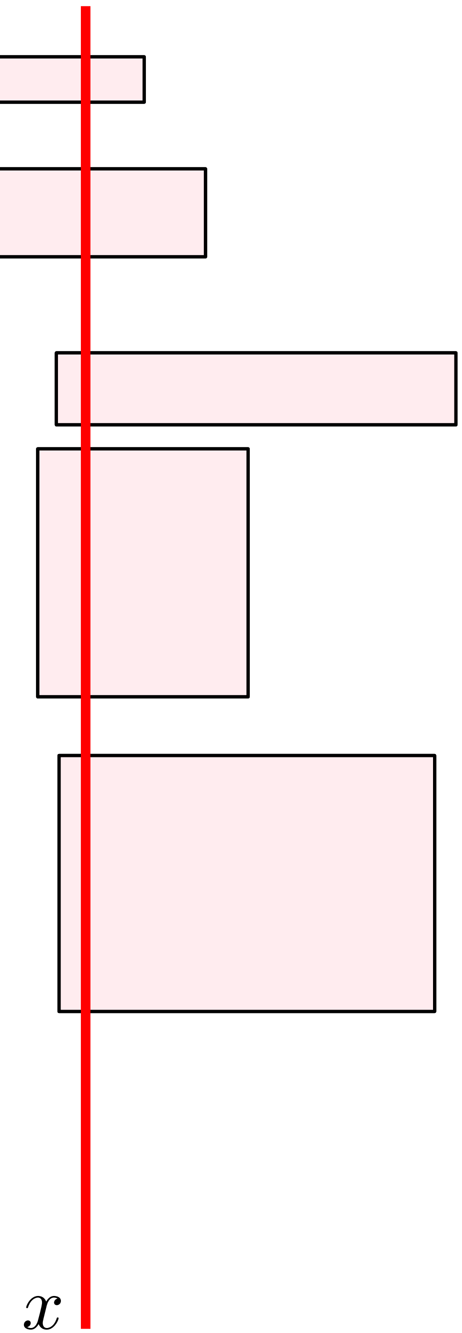
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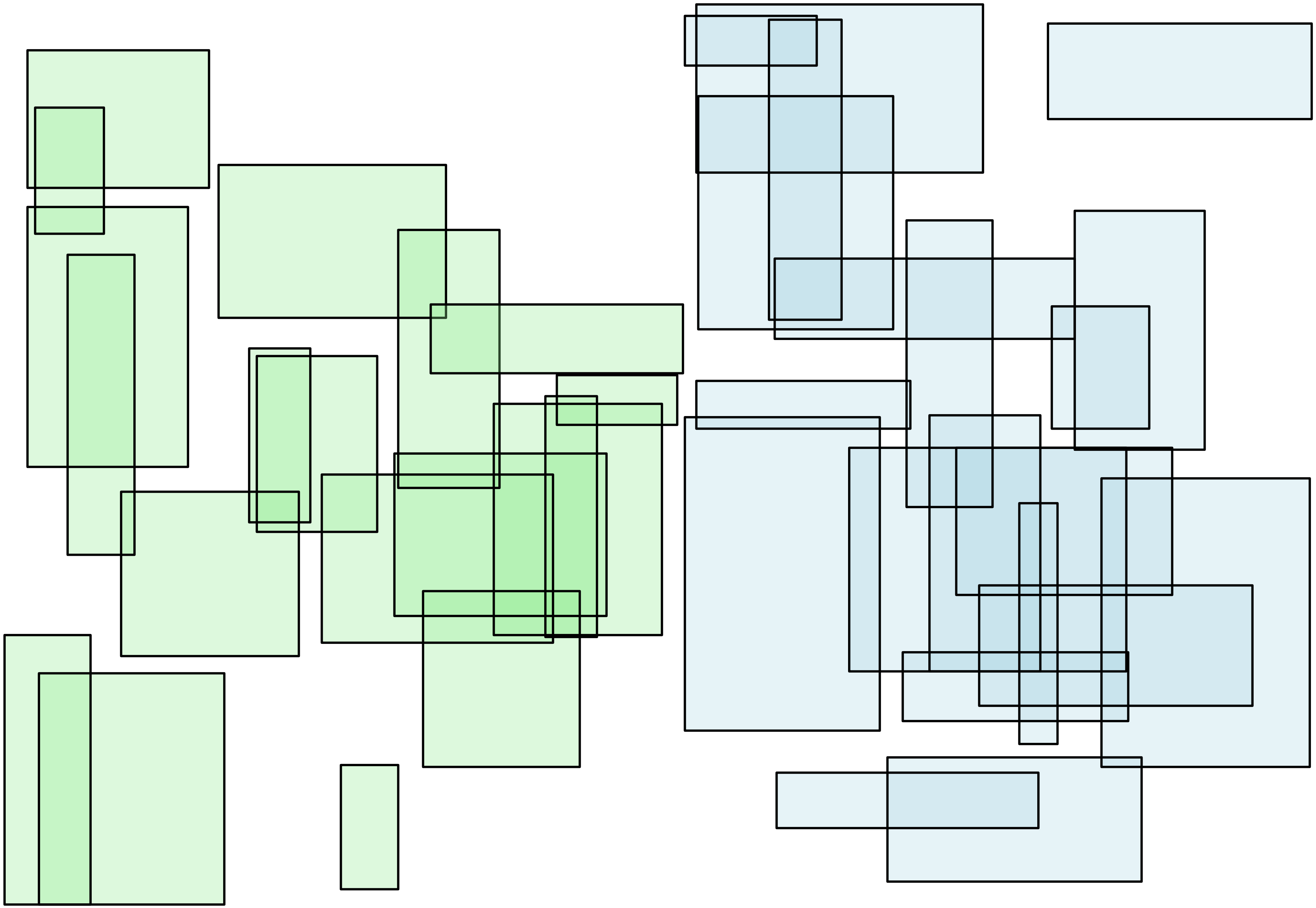
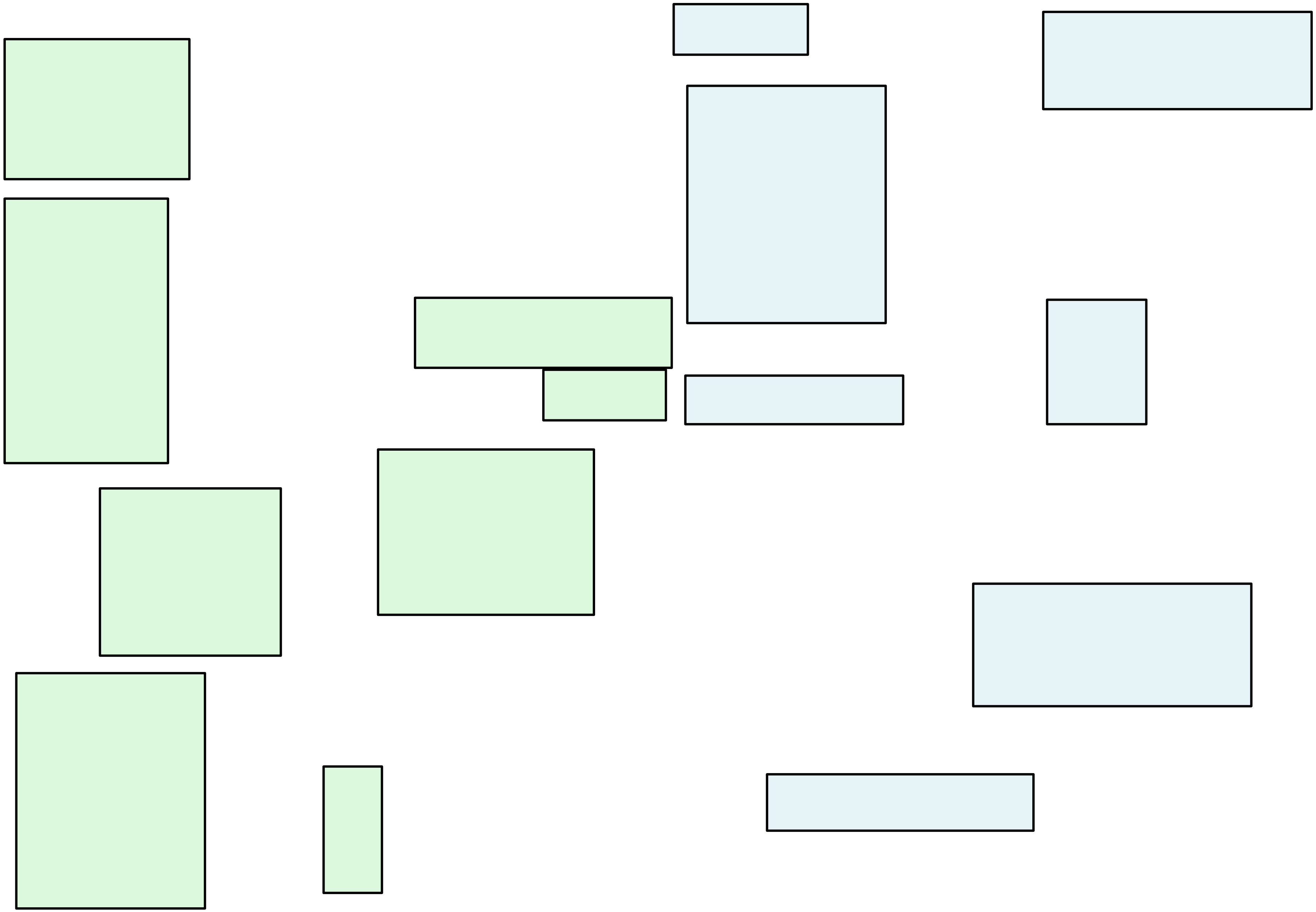




Approximation factor

3. Compute I_x, the max indep. set of $\mathcal{R}_x$.

4. Recursively compute $I_{<x}$ and $I_{>x}$,
the approx. max indep. sets of $\mathcal{R}_{<x}$ and $\mathcal{R}_{>x}$.

5. If $|I_x| \geq |I_{<x}| + |I_{>x}|$ return I_x
else return $I_{<x} \cup I_{>x}$.

$$|I_x| \geq |I^* \cap \mathcal{R}_x| \qquad |I_{<x}| \geq \frac{|I^*_{<x}|}{\log(n/2)} \geq \frac{|I^* \cap \mathcal{R}_{<x}|}{\log n - 1}$$

$$\begin{aligned} |I| &= \max\{|I_x|, |I_{<x}| + |I_{>x}|\} \\ &\geq \max\left\{|I^* \cap \mathcal{R}_x|, \frac{|I^* \cap \mathcal{R}_{<x}| + |I^* \cap \mathcal{R}_{>x}|}{\log n - 1}\right\} \end{aligned}$$

$$
\begin{aligned}
|I| &= \max\{|I_x|, |I_{<x}| + |I_{>x}|\} \\
&\geq \max\left\{|I^* \cap \mathcal{R}_x|, \frac{|I^* \cap \mathcal{R}_{<x}| + |I^* \cap \mathcal{R}_{>x}|}{\log n - 1}\right\} \\
&\geq \max\left\{|I^* \cap \mathcal{R}_x|, \frac{|I^*| - |I^* \cap \mathcal{R}_x|}{\log n - 1}\right\}
\end{aligned}
$$

If $|I^* \cap \mathcal{R}_x| \geq |I^*| / \log n$ then done.

Otherwise $\dfrac{|I^*| - |I^* \cap \mathcal{R}_x|}{\log n - 1} \geq \dfrac{|I^*| - |I^*| / \log n}{\log n - 1} = \dfrac{|I^*|}{\log n}$

Approx. optimal number of labels - unit height

[Agarwal, van Kreveld, Suri 98]

2-approximation

1. Let $\ell_0, \ell_1, \ldots, \ell_{m-1}$ be horizontal lines spaced > 1 apart that intersect all $\mathcal{R}$.
2. Let $\mathcal{R}_i$ be rects that intersect ℓ_i.

Approx. optimal number of labels - unit height

[Agarwal, van Kreveld, Suri 98]

2-approximation

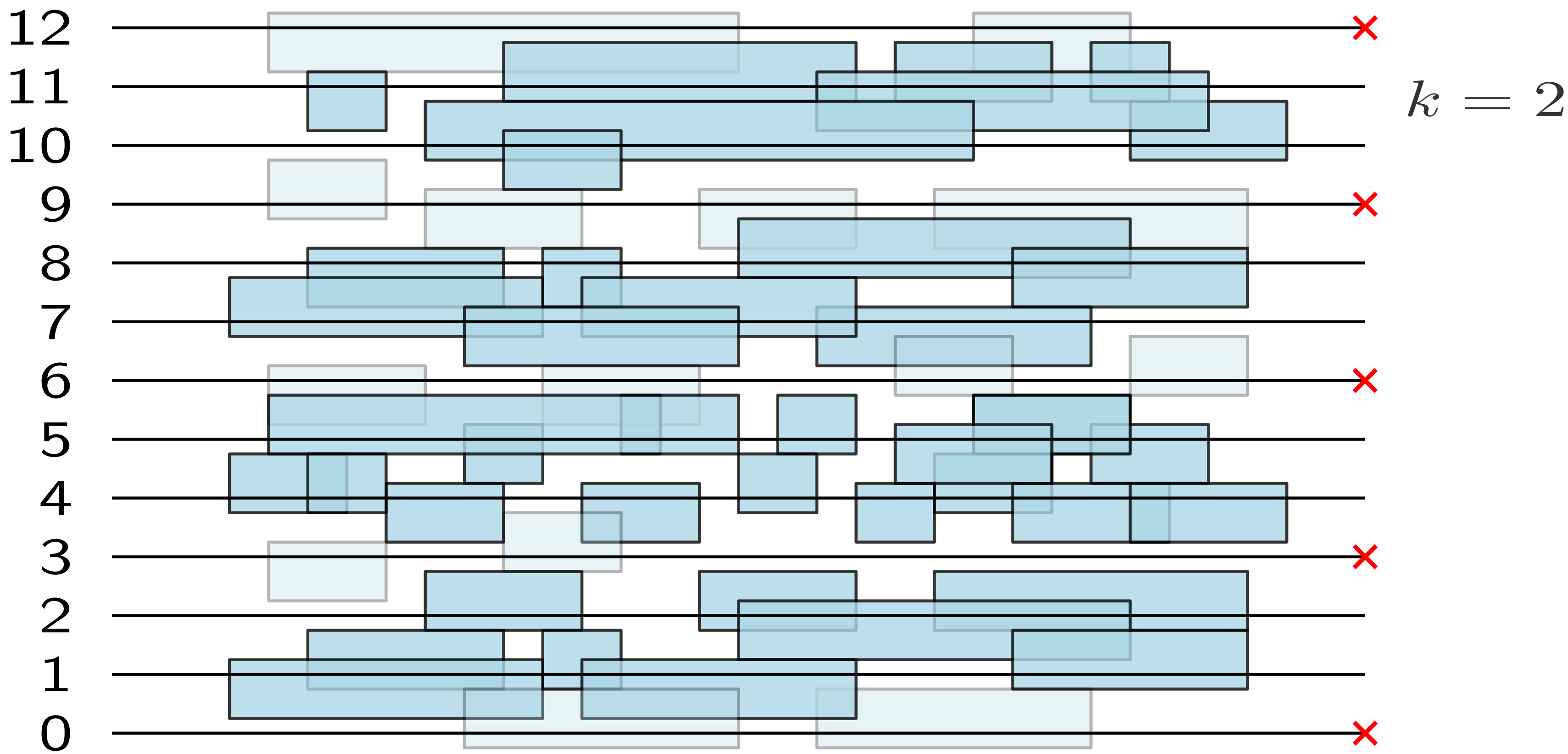
1. Let $\ell_0, \ell_1, \ldots, \ell_{m-1}$ be horizontal lines spaced > 1 apart that intersect all $\mathcal{R}$.
2. Let $\mathcal{R}_i$ be rects that intersect ℓ_i.
3. Let I_i be max indep. set in $\mathcal{R}_i$.
4. Return the larger of $I_0 \cup I_2 \cup \cdots \cup I_{m-1}$ and $I_1 \cup I_3 \cup \cdots \cup I_m$ (assuming m odd).

Approx. optimal number of labels - unit height

[Agarwal, van Kreveld, Suri 98]

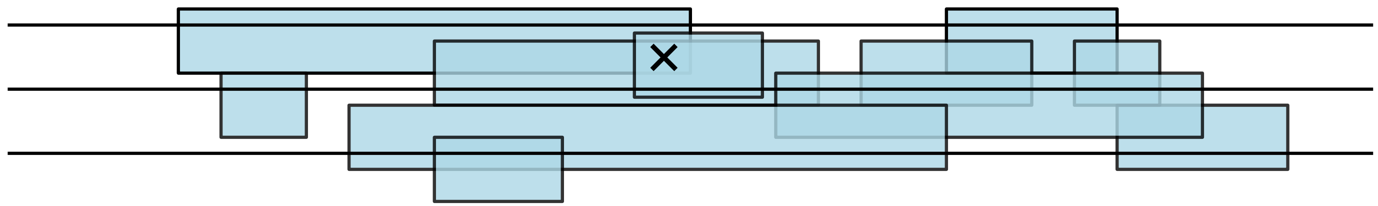
$(1 + 1/k)$-approximation

Idea: Use dynamic programming to optimally solve subproblems $\mathcal{R}_i \cup \mathcal{R}_{i+1} \cup \cdots \cup \mathcal{R}_{i+k-1}$.

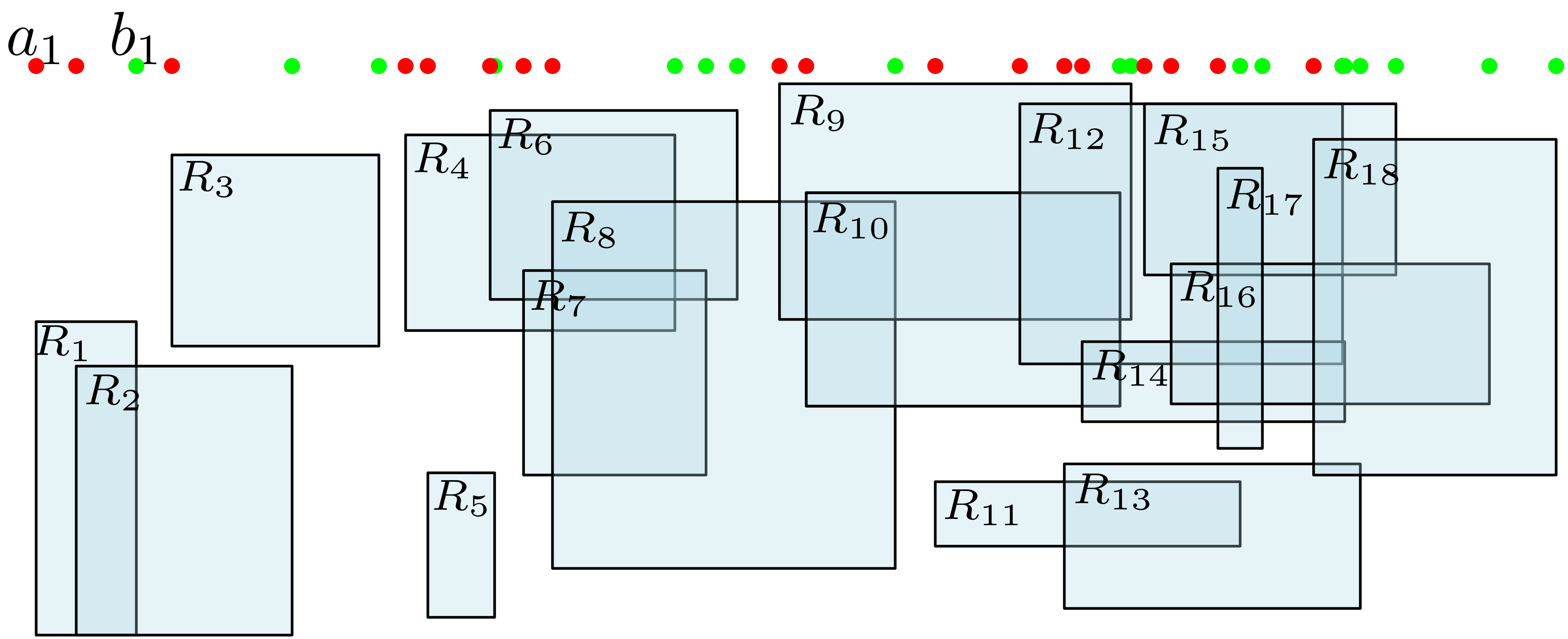


Dynamic programming subroutine [Chan 04]

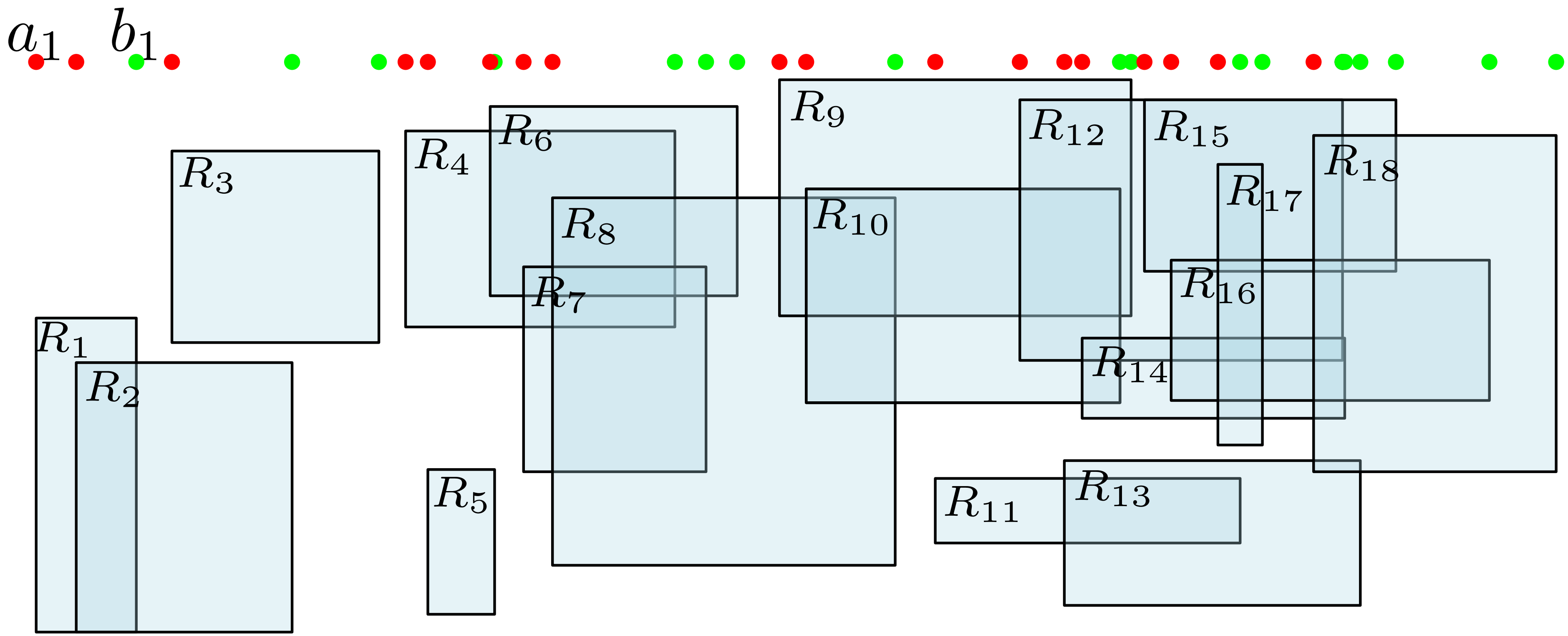
Theorem: If $\mathcal{R}$ is stabbed by k horizontal lines, we can find a max indep. set in $O(n \log n + n\Delta^{k-1})$ time, where Δ is the max number of rects a point can be in.



Dynamic Programming (works for general rects)

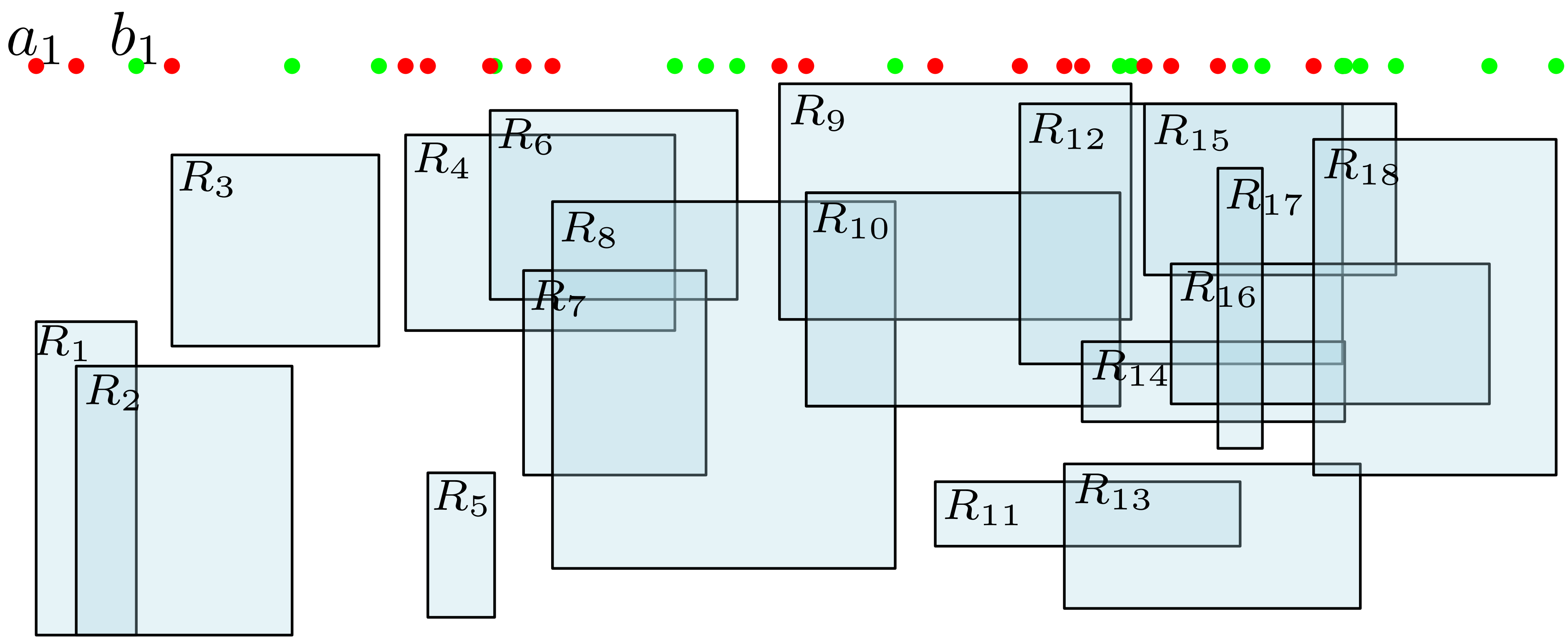


Sort rectangles by left coordinate.



Let $next[j]$ = smallest i with $a_i > b_j$.

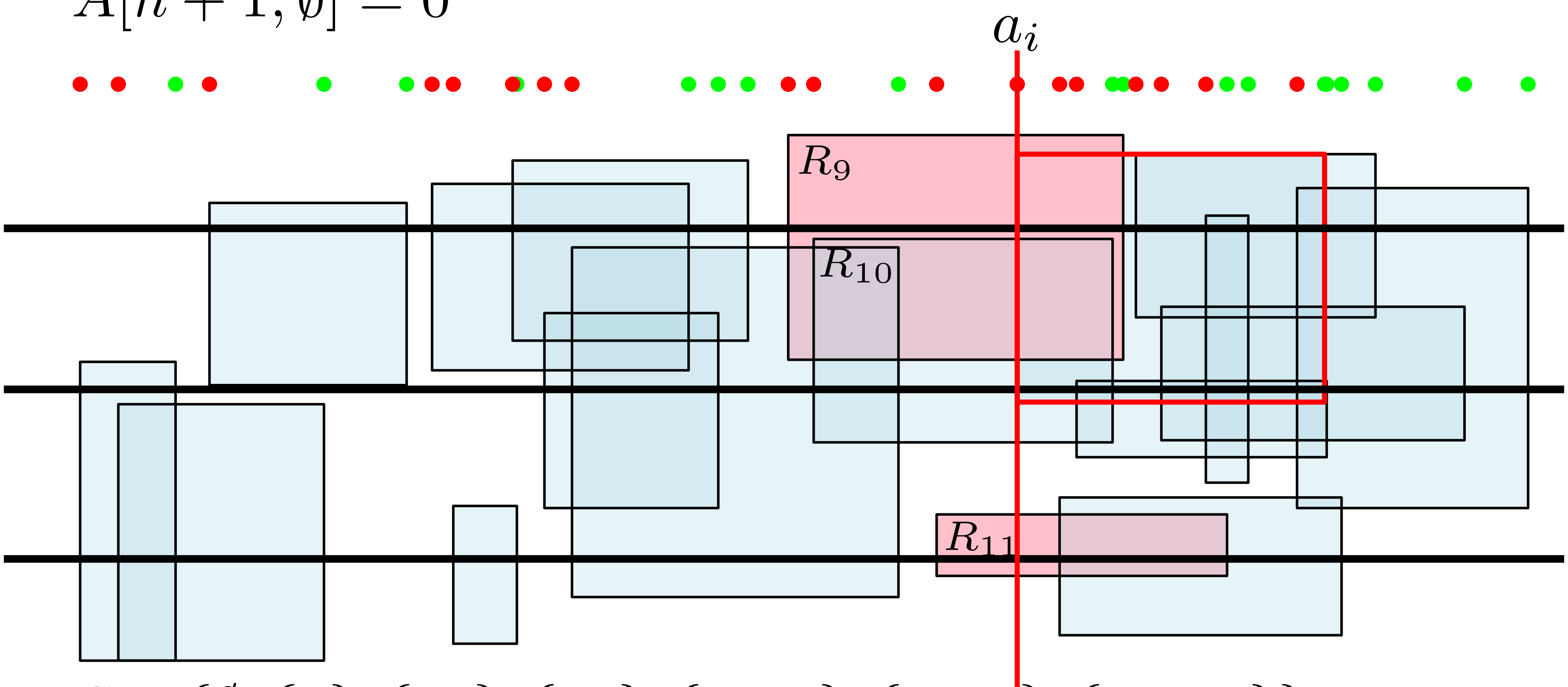
$$next[9] = 15$$



Let $A[i, S]$ be the maximum number of disjoint rectangles from $R_i \dots R_n$ that do not intersect the rectangles in S.

$S =$ any set of $\leq k - 1$ disjoint rects that intersect $x = a_i$.

$A[n + 1, \emptyset] = 0$



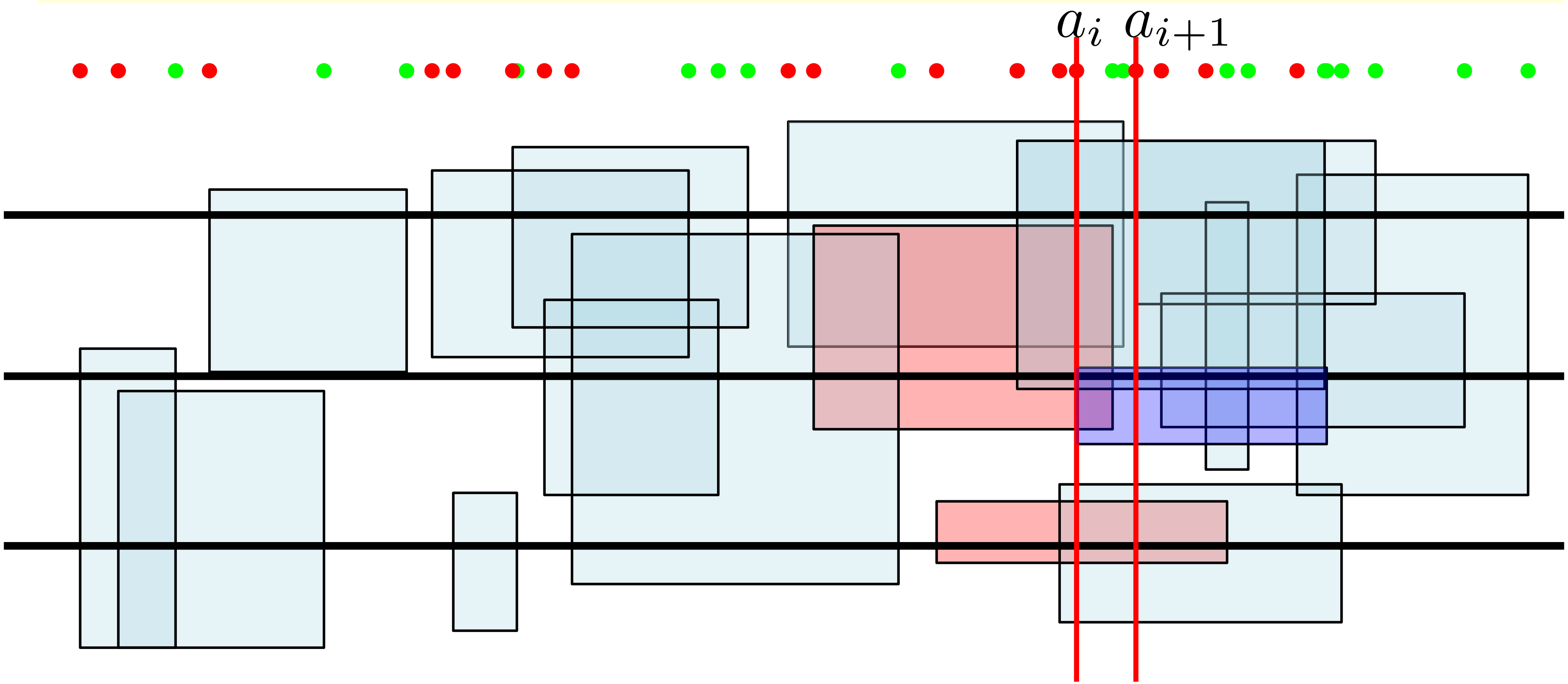
$S \in \{\emptyset, \{9\}, \{10\}, \{11\}, \{9, 10\}, \{9, 11\}, \{10, 11\}\}$

For $i = n$ to 1

For all sets S of $\leq k-1$ disjoint rects intersecting $x = a_i$

If R_i intersects some rect in S then

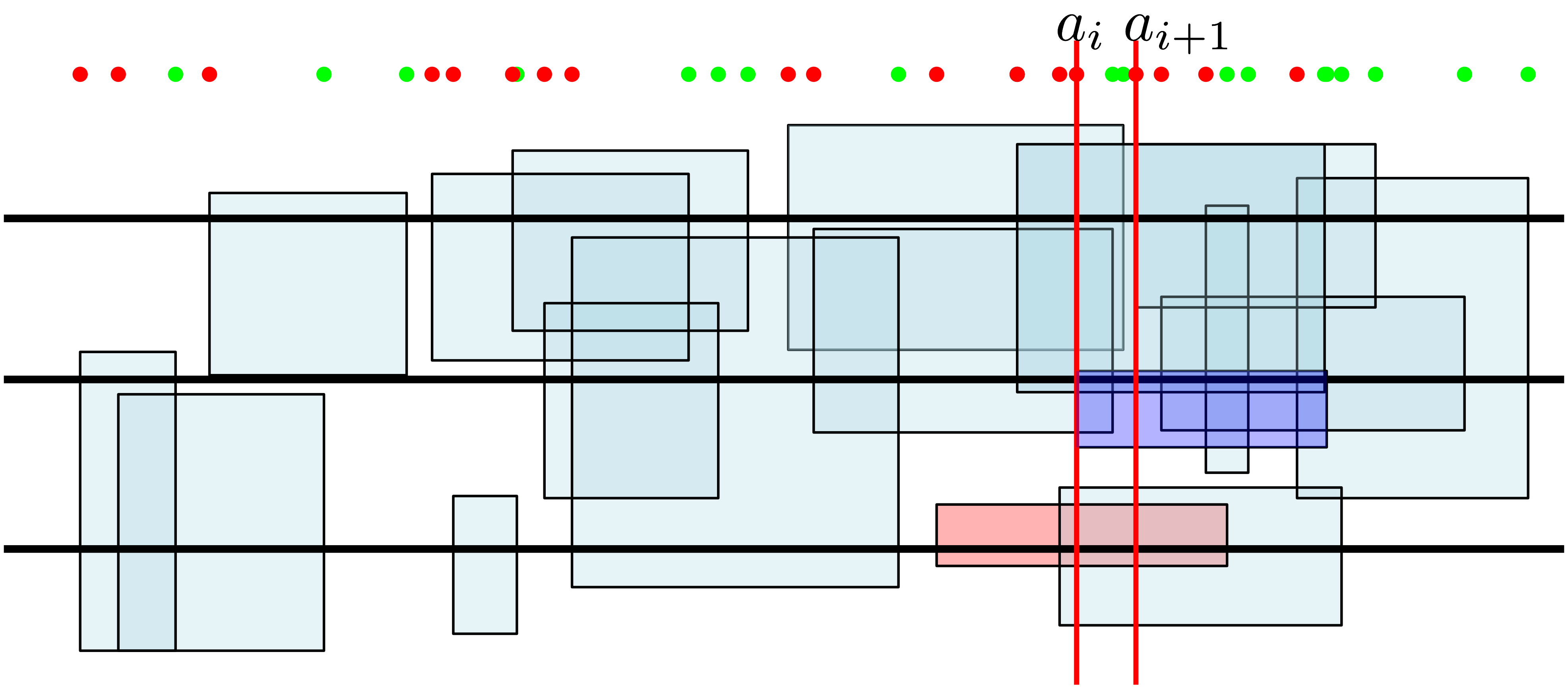
$$A[i, S] = A[i+1, S|_{i+1}].$$



R_i cannot be used for this subproblem.

else if $|S| < k - 1$ then

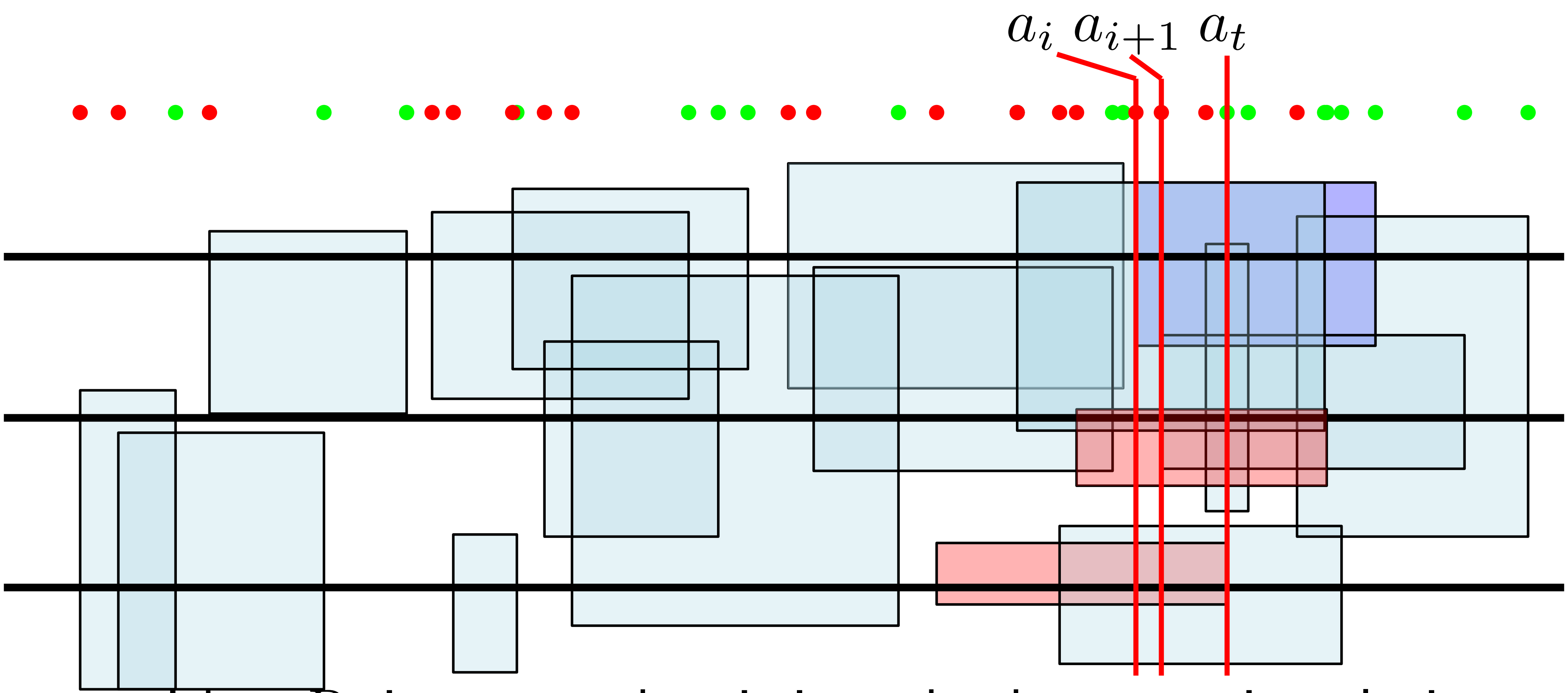
$$A[i, S] = \max\{A[i+1, S|_{i+1}], \\ 1 + A[i+1, (S \cup \{R_i\})|_{i+1}]\}.$$



R_i cannot be used for this subproblem.

else $(|S| = k-1)$ Let $t = \min_{R_j \in S \cup \{R_i\}} next[j]$

$$A[i,S] = \max\{A[i+1, S|_{i+1}], \\ 1 + A[t, (S \cup \{R_i\})|_t]\}.$$



either R_i is not used or it is and other rects in solution must be to the right of some rect in $S \cup \{R_i\}$.

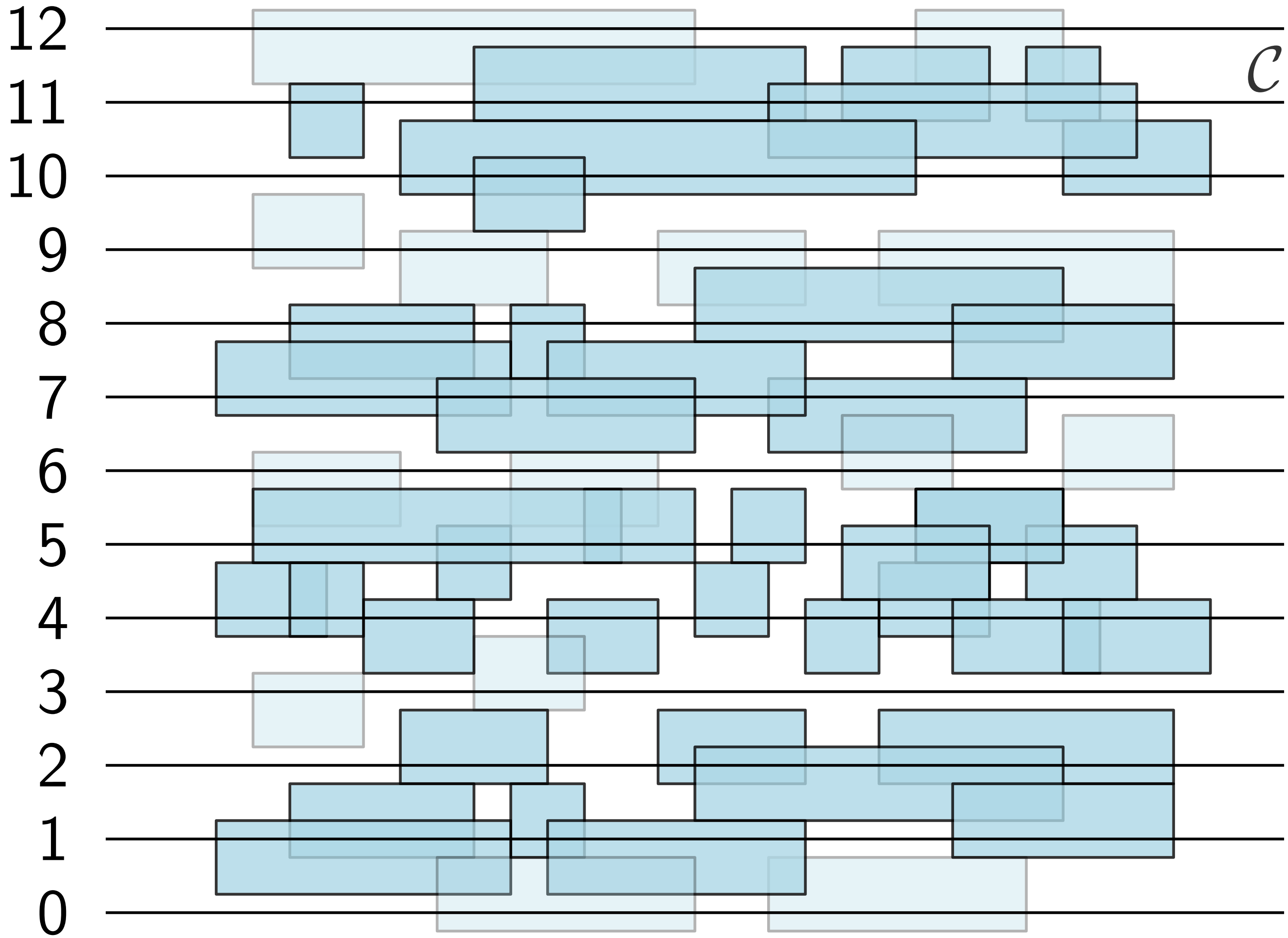
Running time:

Number of subproblems is $O(n\Delta^{k-1})$.

Size of array A is $\Theta(n^k)$ but can be reduced to $O(n\Delta^{k-1})$.

Running time $O(n\Delta^{k-1})$

For $i = 0, \ldots, k$, let $\mathcal{C}^{(i)}$ be the subset of rects that do not intersect line $y \equiv i \bmod (k+1)$.



$\mathcal{C}^{(0)}$ for $k = 2$

$(1+1/k)$-approximation in unit-height case

Find the optimal solution $\mathcal{S}^{(i)}$ for each $\mathcal{C}^{(i)}$.

Return the largest solution $\mathcal{S}$ from $\mathcal{S}^{(0)}, \ldots, \mathcal{S}^{(k)}$.

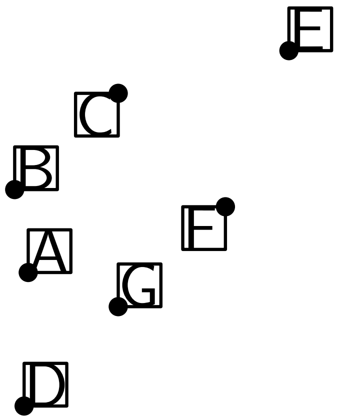
Each unit-height rectangle belongs to exactly k of the $k+1$ subsets $\mathcal{C}^{(i)}$.

$$k|\mathcal{S}^*| = \sum_{i=0}^{k} |\mathcal{S}^* \cap \mathcal{C}^{(i)}| \leq \sum_{i=0}^{k} |\mathcal{S}^{(i)}| \leq (k+1)|\mathcal{S}|$$

so $|\mathcal{S}^*| \leq (1+1/k)|\mathcal{S}|$.

What if we want to label *all* points?

How big can we make the labels without overlap?



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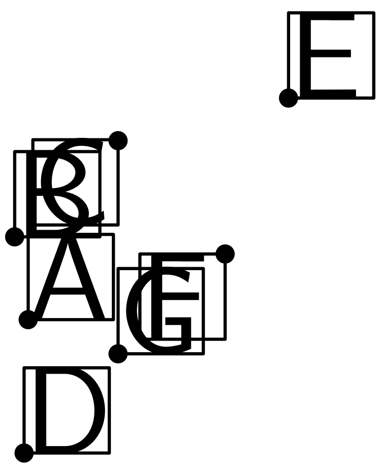

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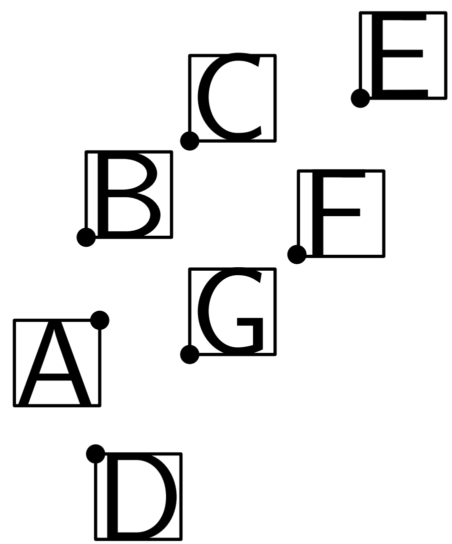
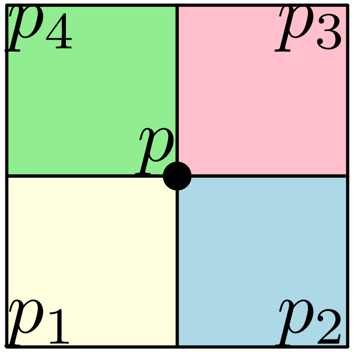


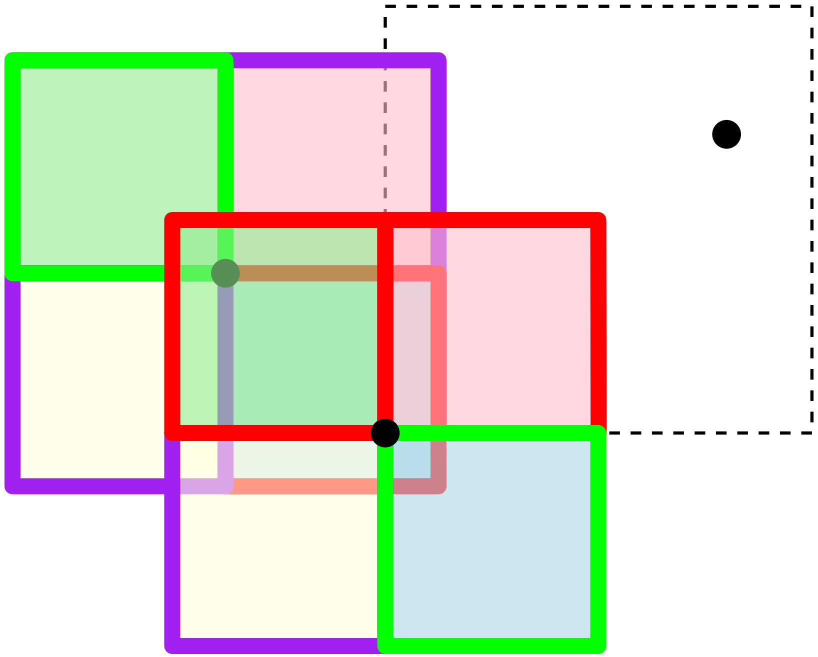
Approximate optimal label size [Formann & Wagner 91]

2-approximation for square labels:



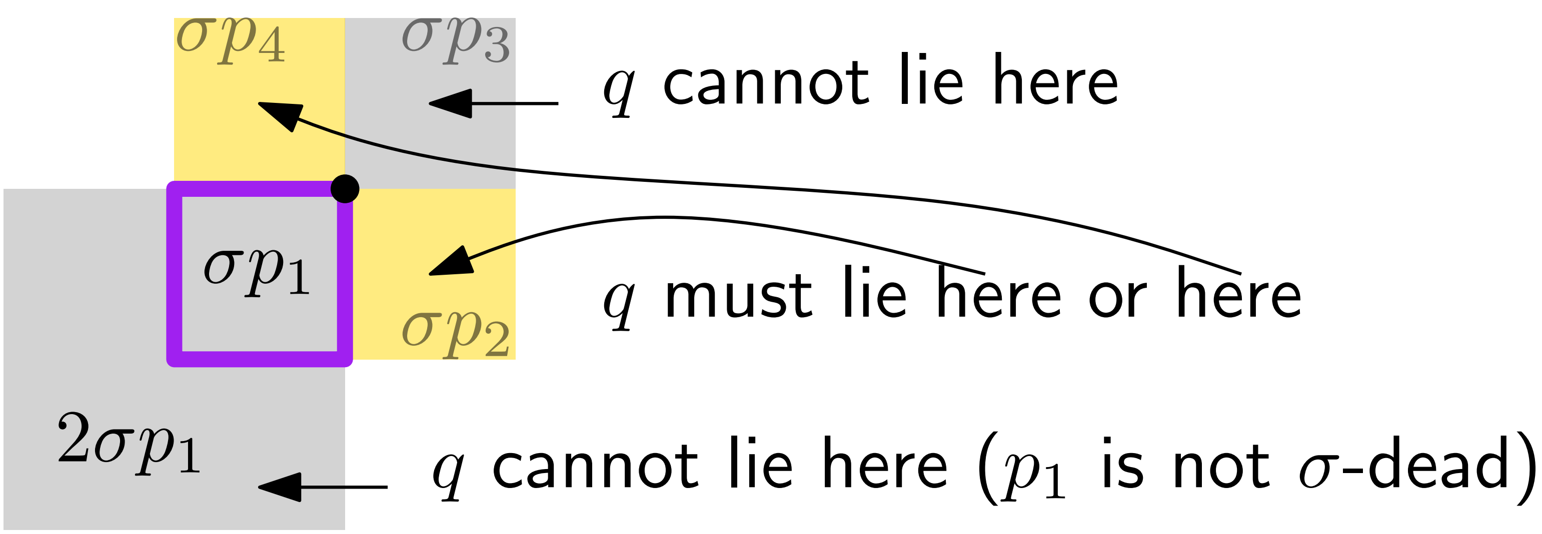
σp_i is p_i scaled by σ.

Call p_i σ-**dead** if $2\sigma p_i$ contains a point $q \neq p$,
else σ-**pending** if σp_i intersects σq_j and q_j is not σ-dead,
else σ-**alive**.



Lemma: A point may have at most two σ-pending squares.

Suppose p_1 is σ-pending and $\sigma p_1 \cap \sigma q_j \neq \emptyset$



Thus p_2 or p_4 is σ-dead.

If p has three σ-pending squares, at least two of $\{p_1, p_2, p_3, p_4\}$ are σ-dead. $\Rightarrow\Leftarrow$

Approximate largest label size

$\sigma = 0$

Repeat

Eliminate σ-dead squares

Assign a σ-alive square to every point with one.

Use 2SAT for the remaining set of points.

If some point has no square or 2SAT fails then return previous σ.

Increase σ to next interesting value.