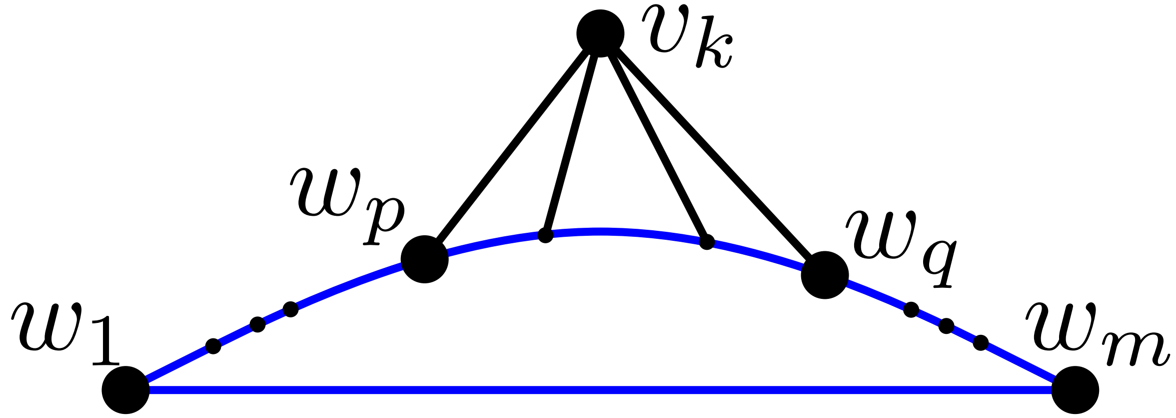
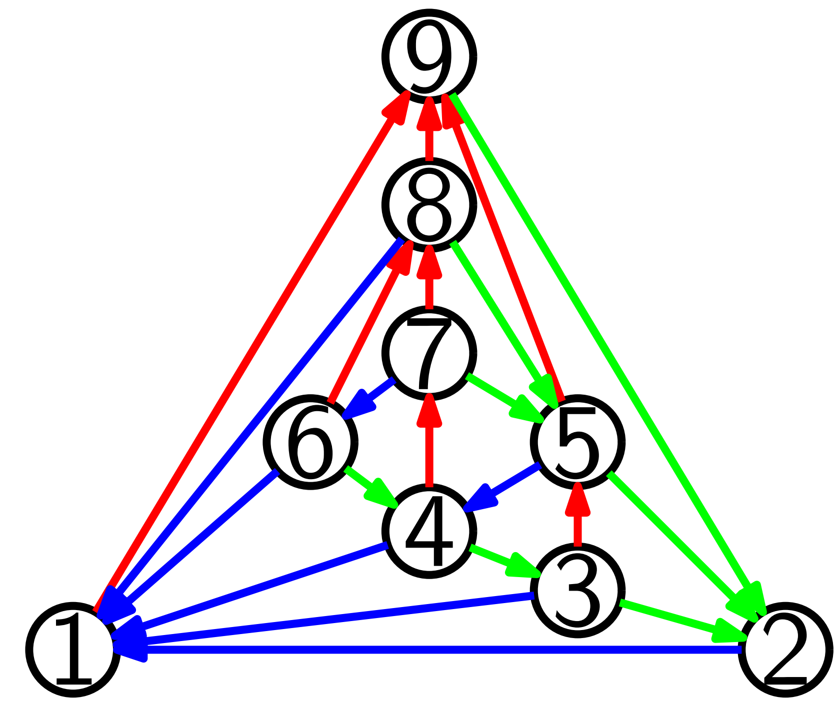
Canonical Ordering ⇒ Planar Straight-line Drawing

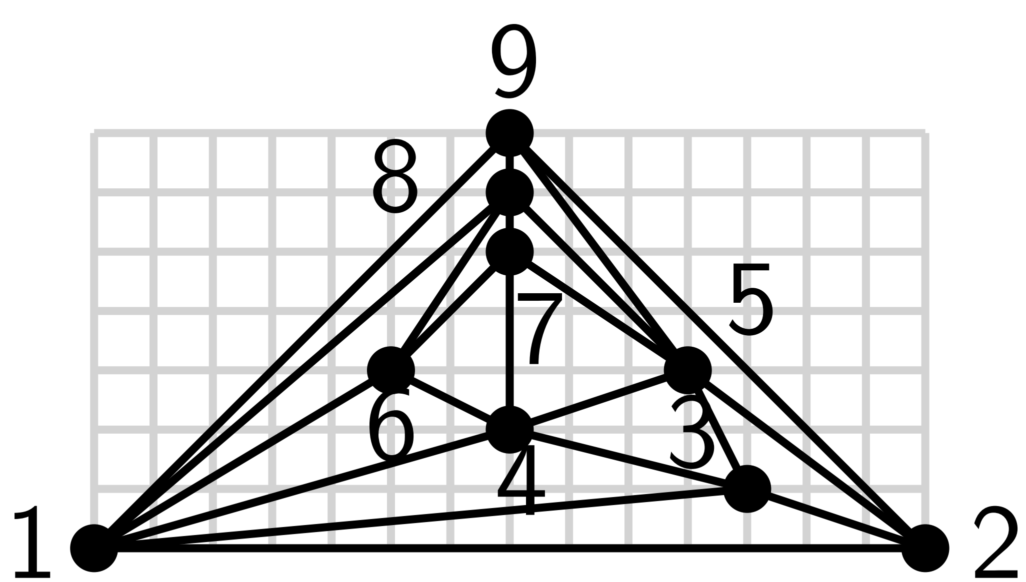




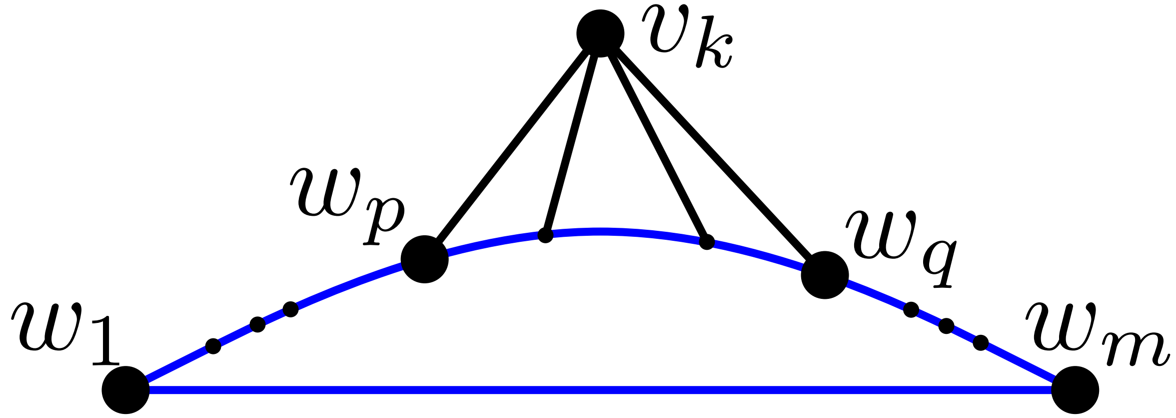
If $v \in \bigcup_{i=q}^{m} L(w_i)$ then $x(v) {+}{=} 2$.

If $v \in \bigcup_{i=p+1}^{q-1} L(w_i)$ then $x(v) {+}{=} 1$.

$L(v_k) = \{v_k\} \cup \bigcup_{i=p+1}^{q-1} L(w_i)$.



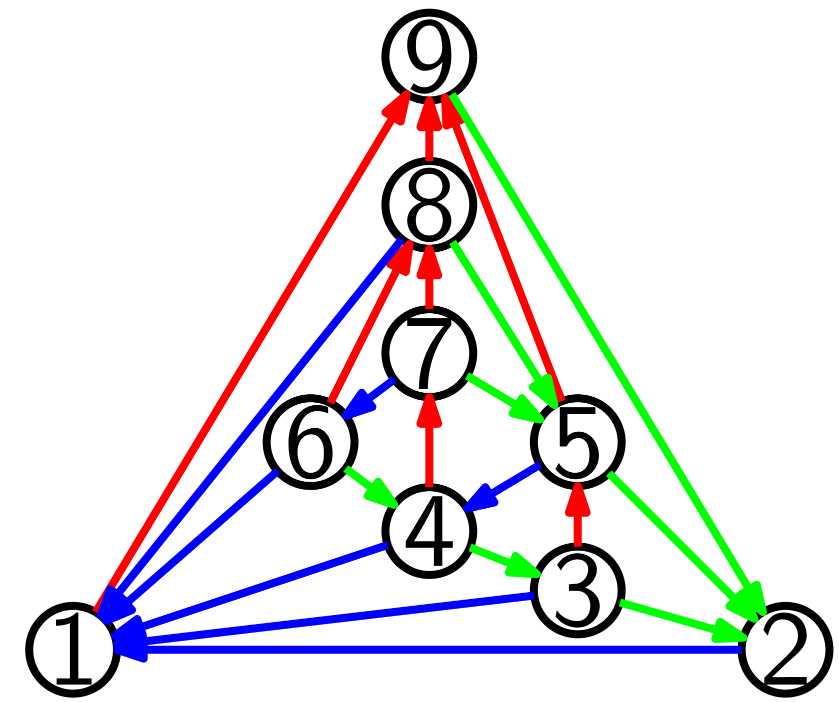
Canonical Ordering $\Rightarrow$ Planar Straight-line Drawing



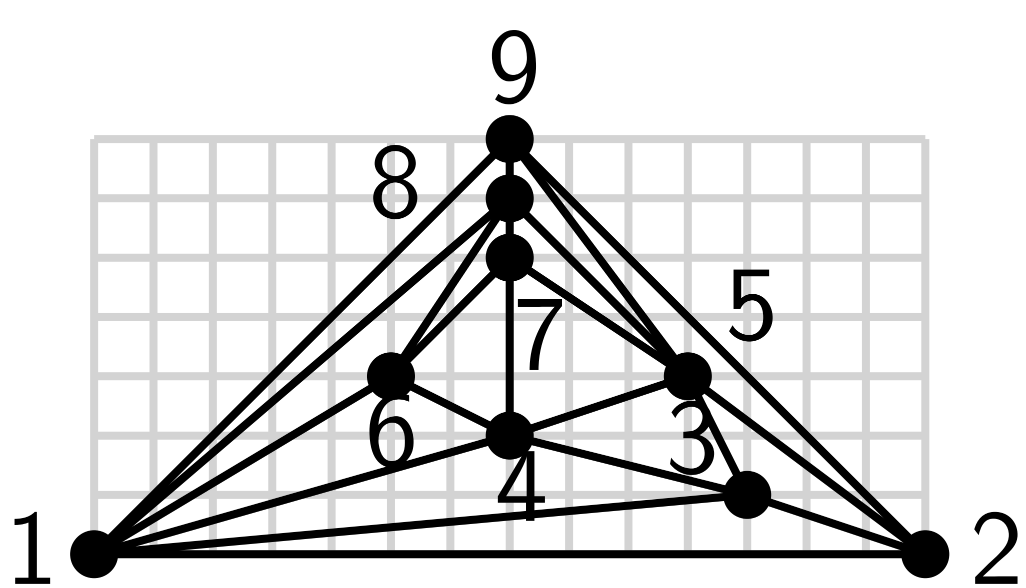
If $v \in \bigcup_{i=q}^{m} L(w_i)$ then $x(v) += 2$.

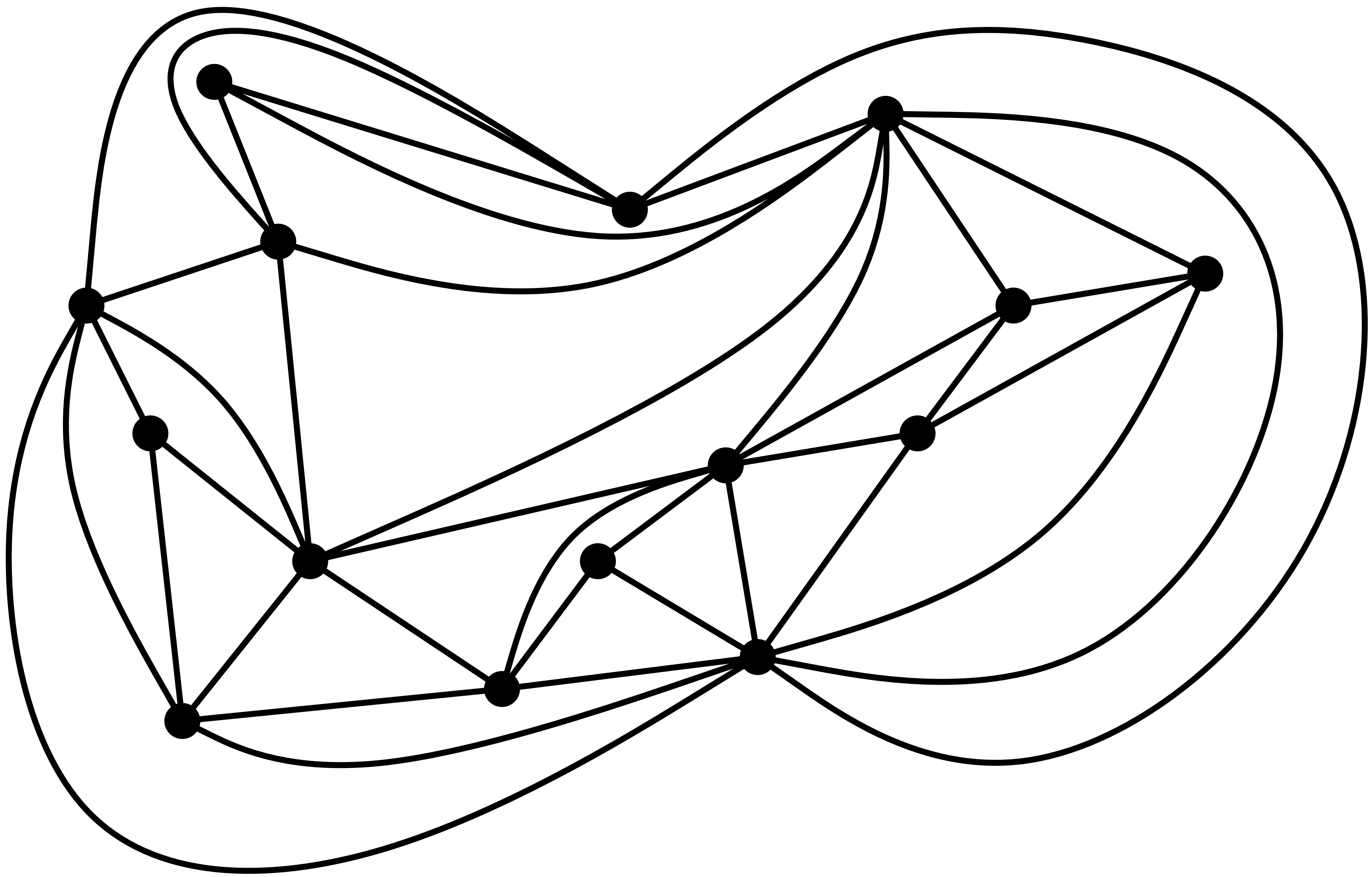
If $v \in \bigcup_{i=p+1}^{q-1} L(w_i)$ then $x(v) += 1$.

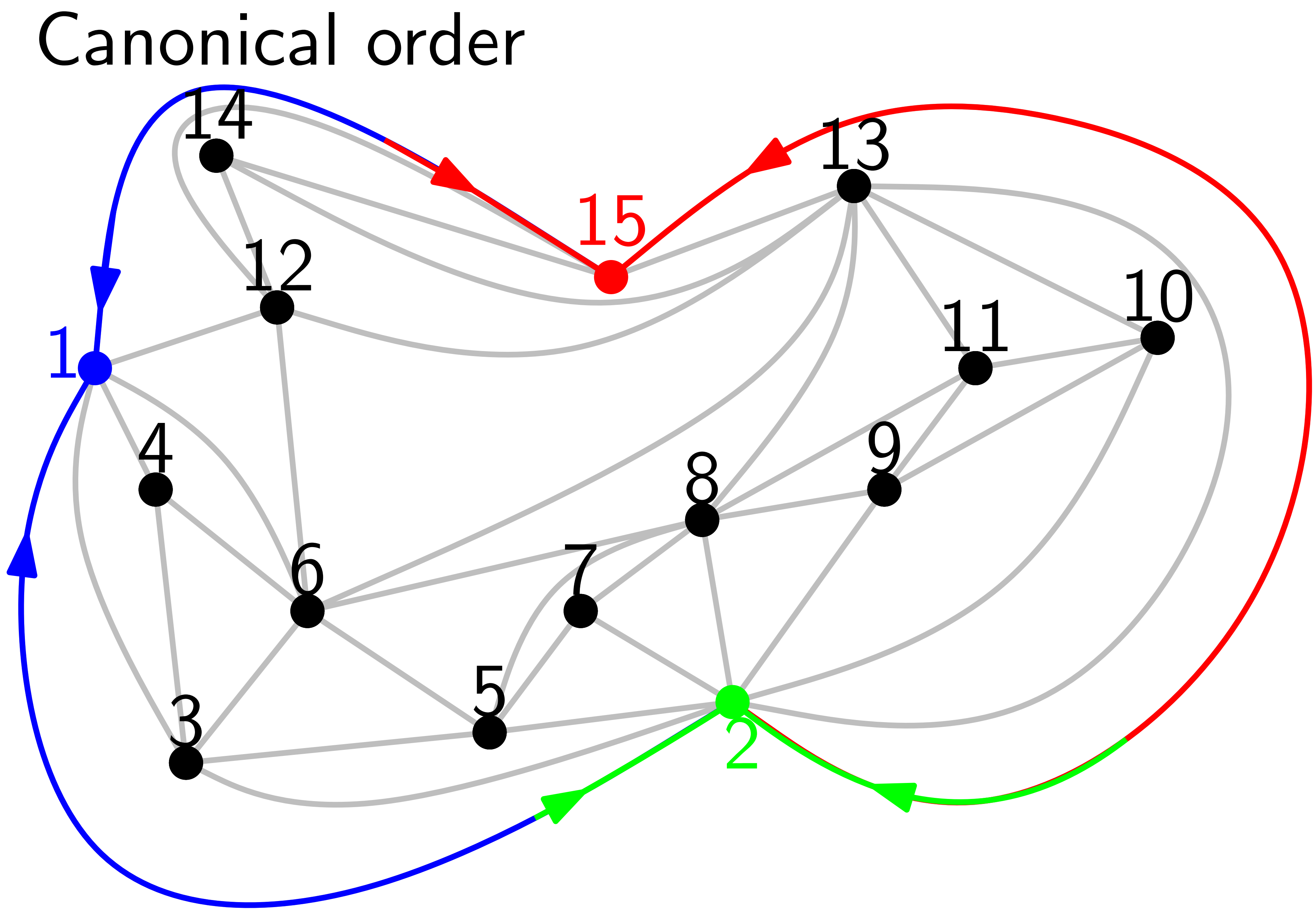
$L(v_k) = \{v_k\} \cup \bigcup_{i=p+1}^{q-1} L(w_i)$.



Schnyder Wood






Canonical order
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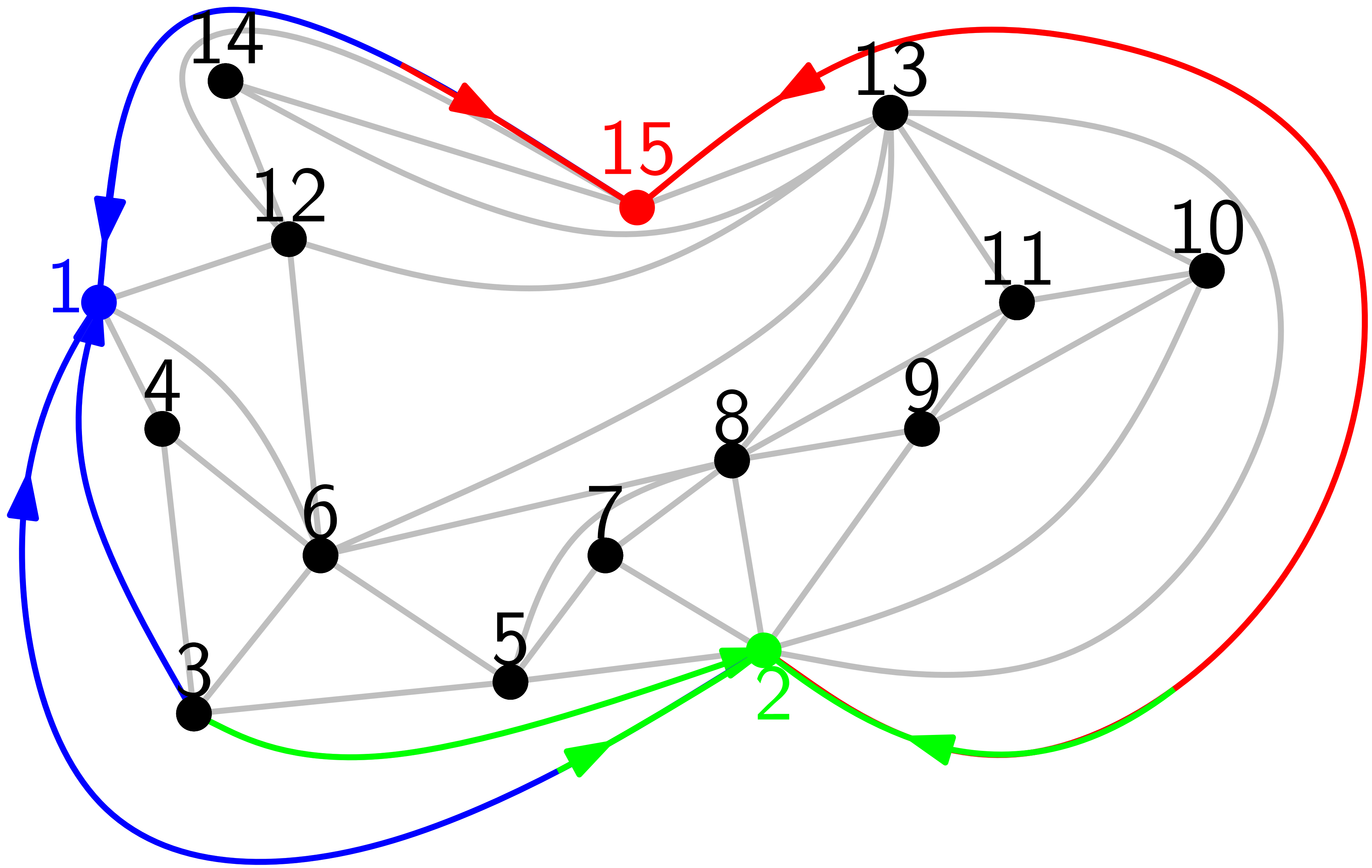
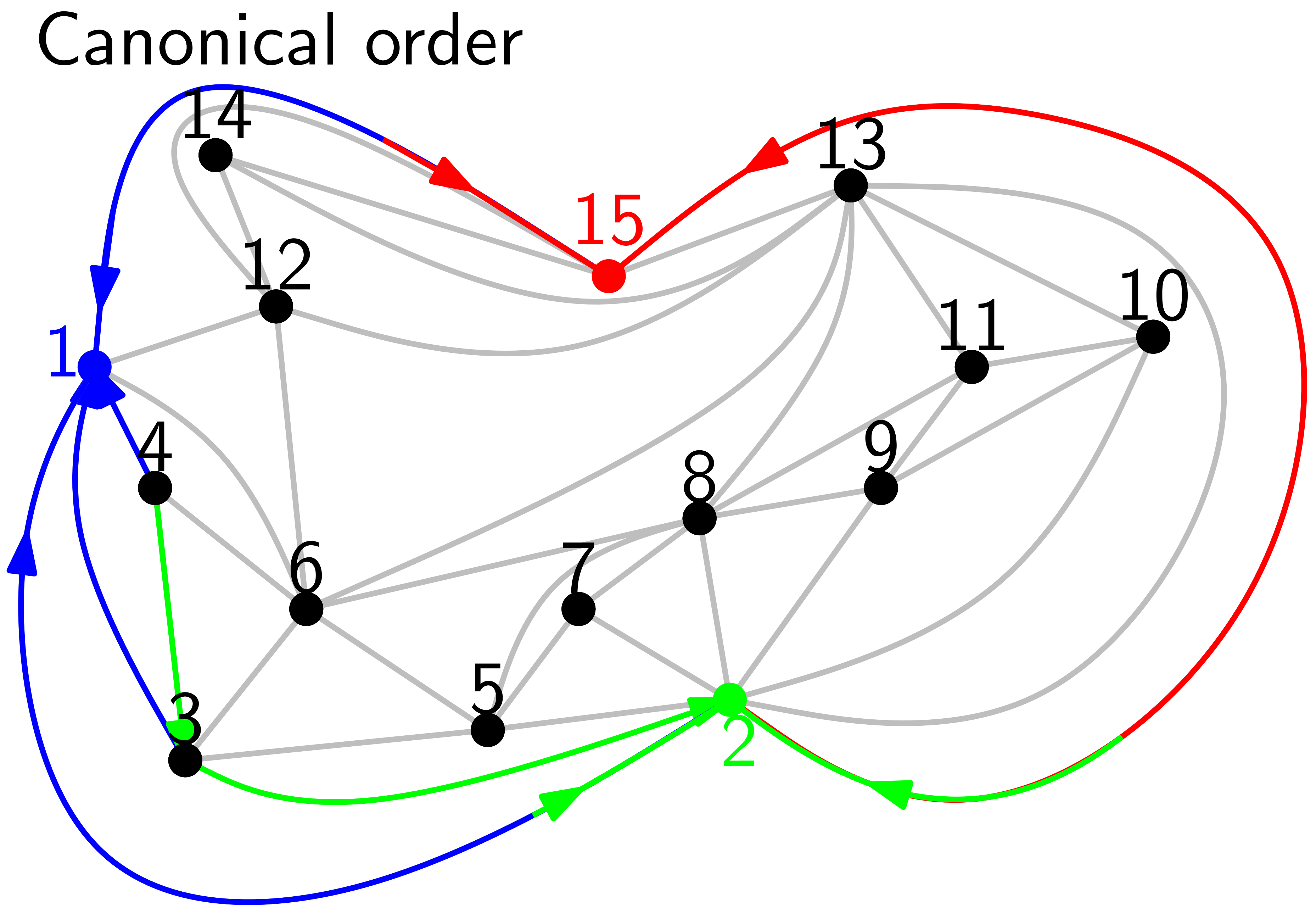
Canonical order





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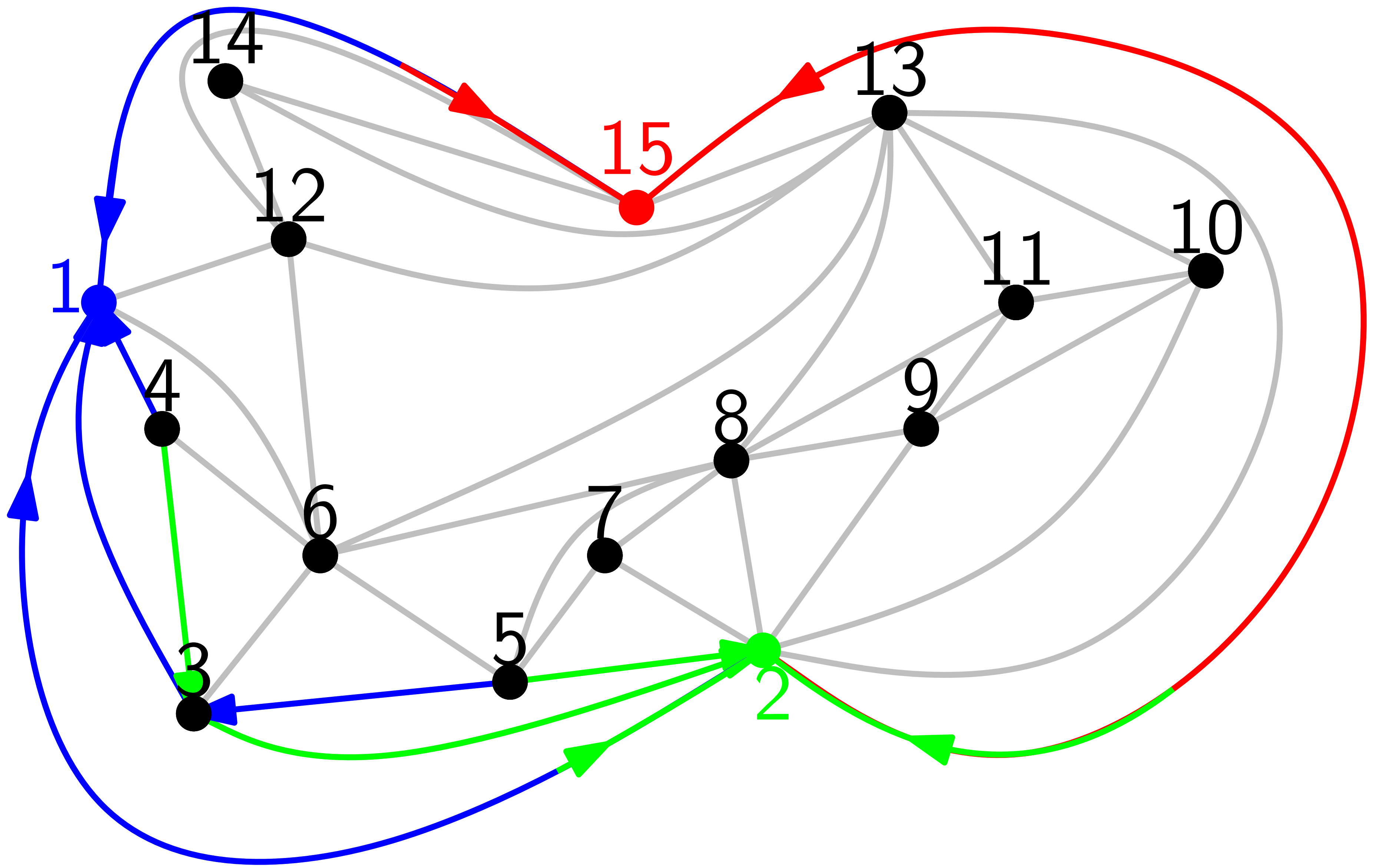

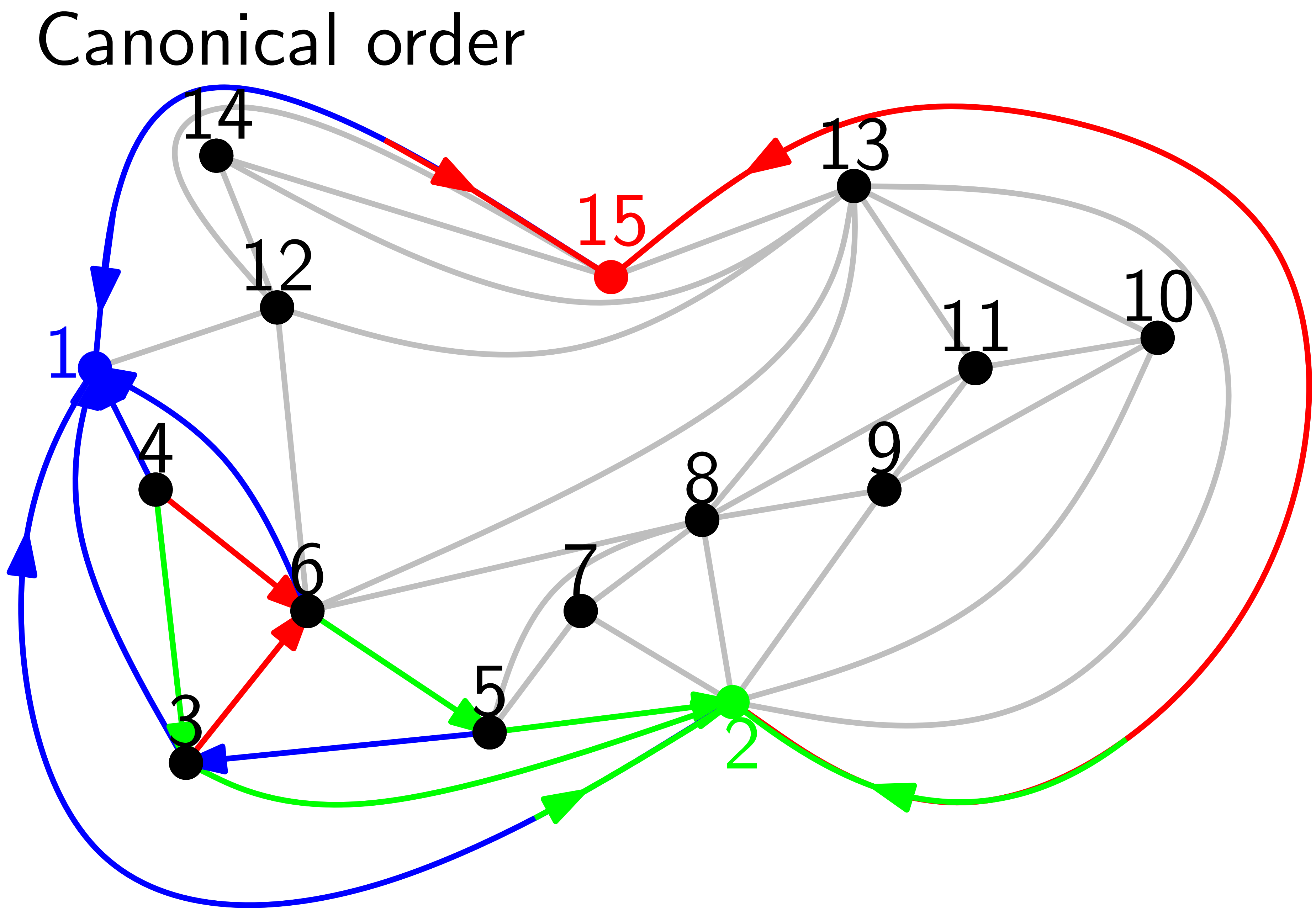
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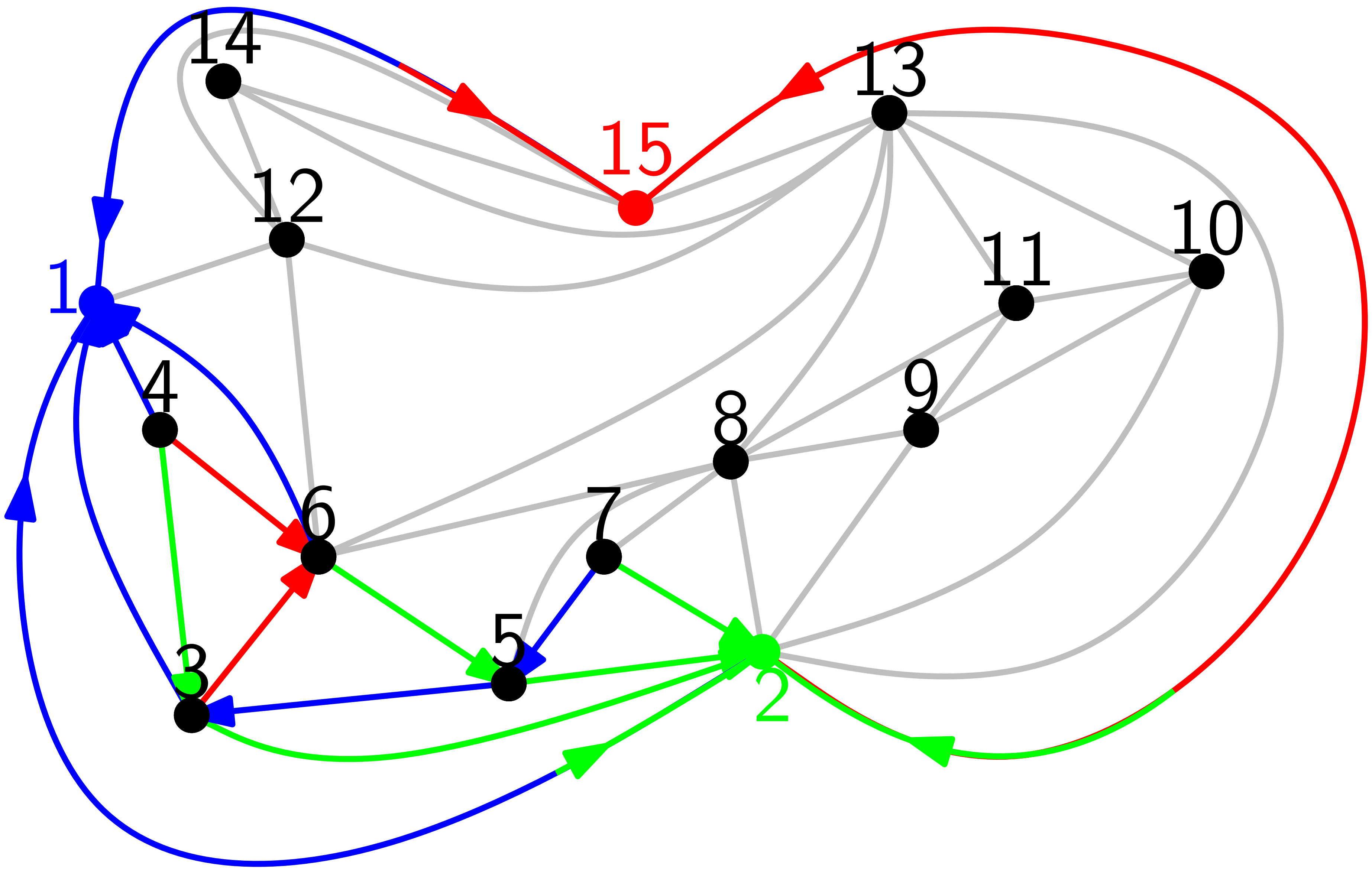
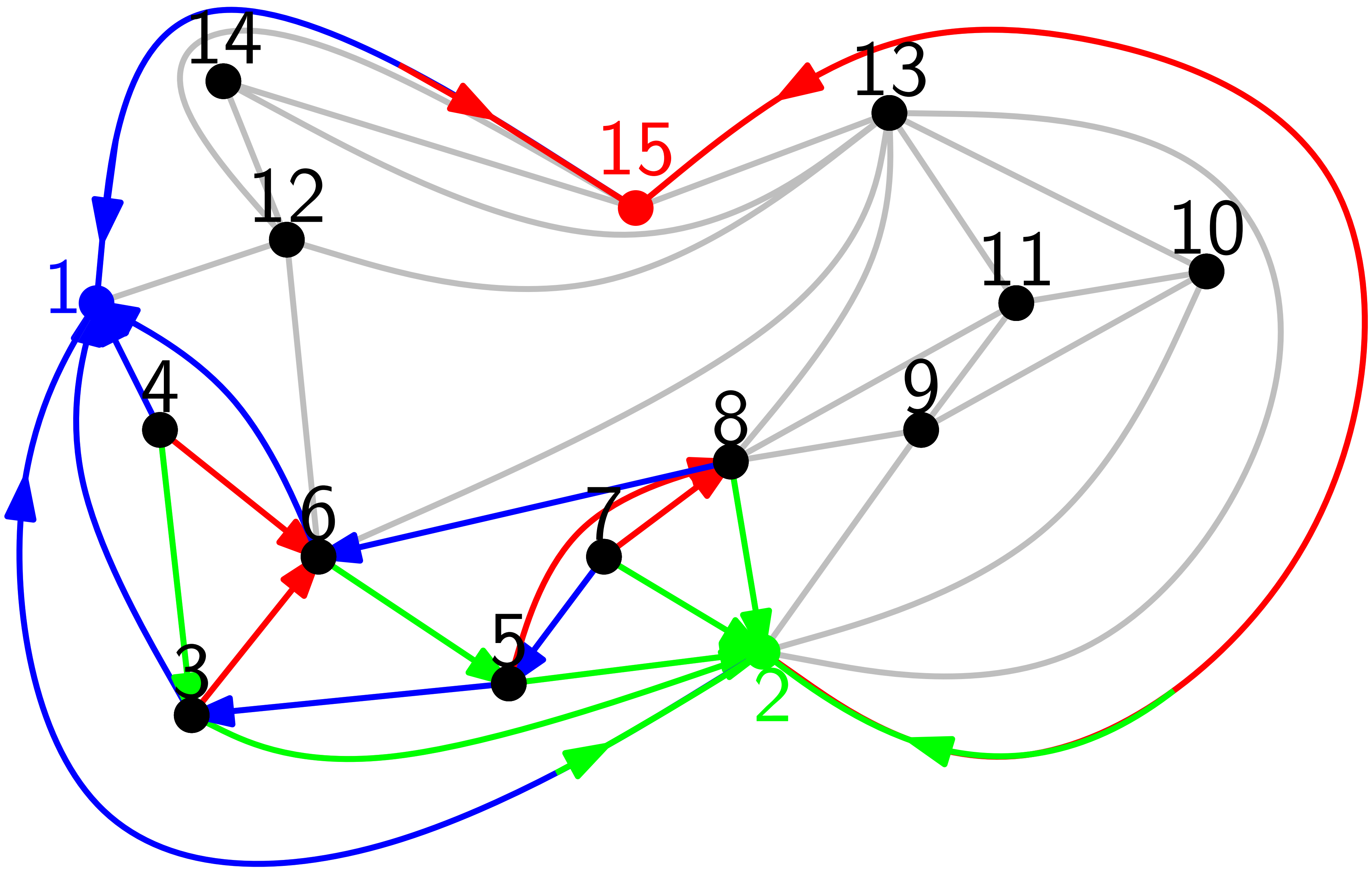
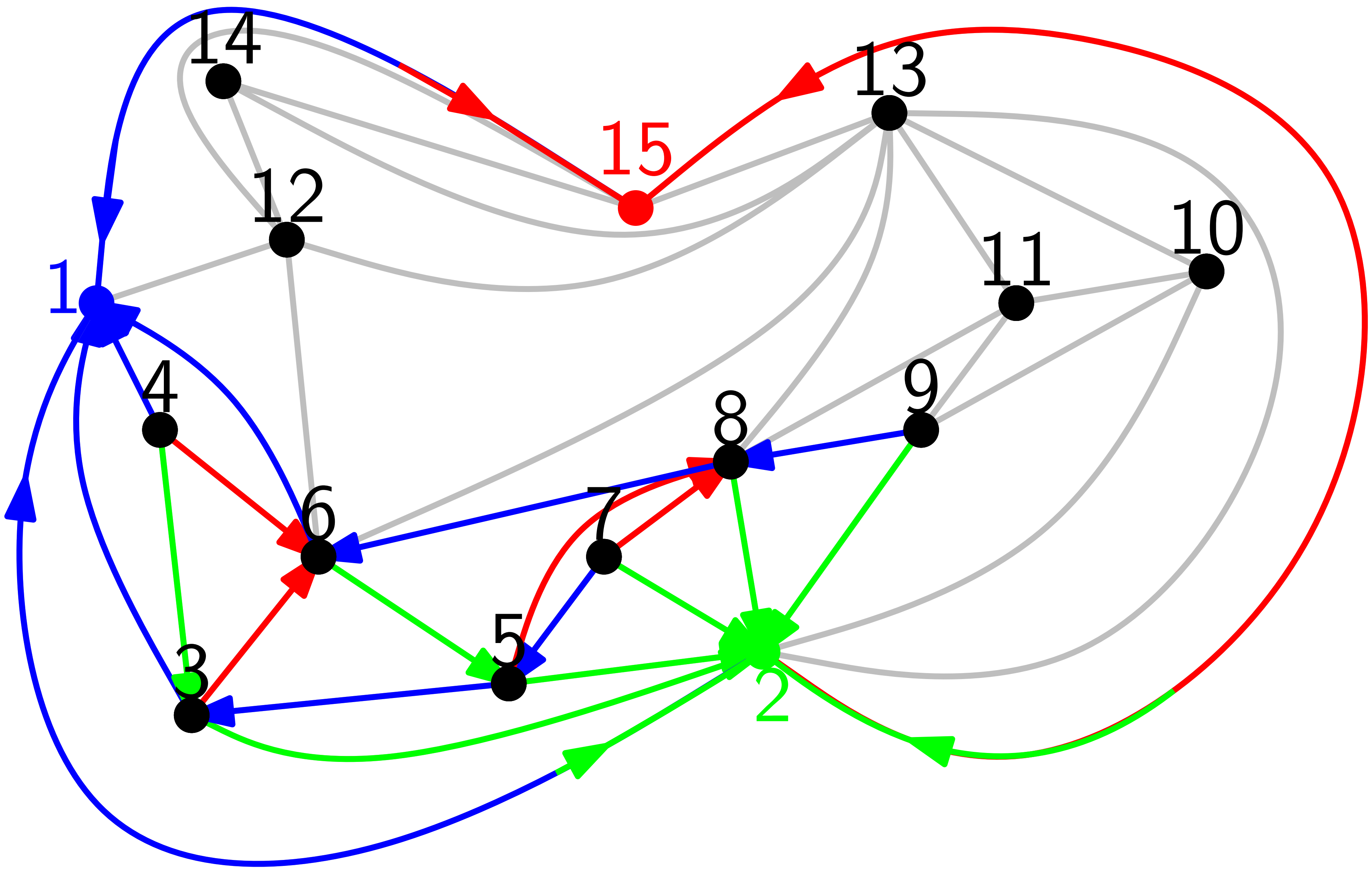


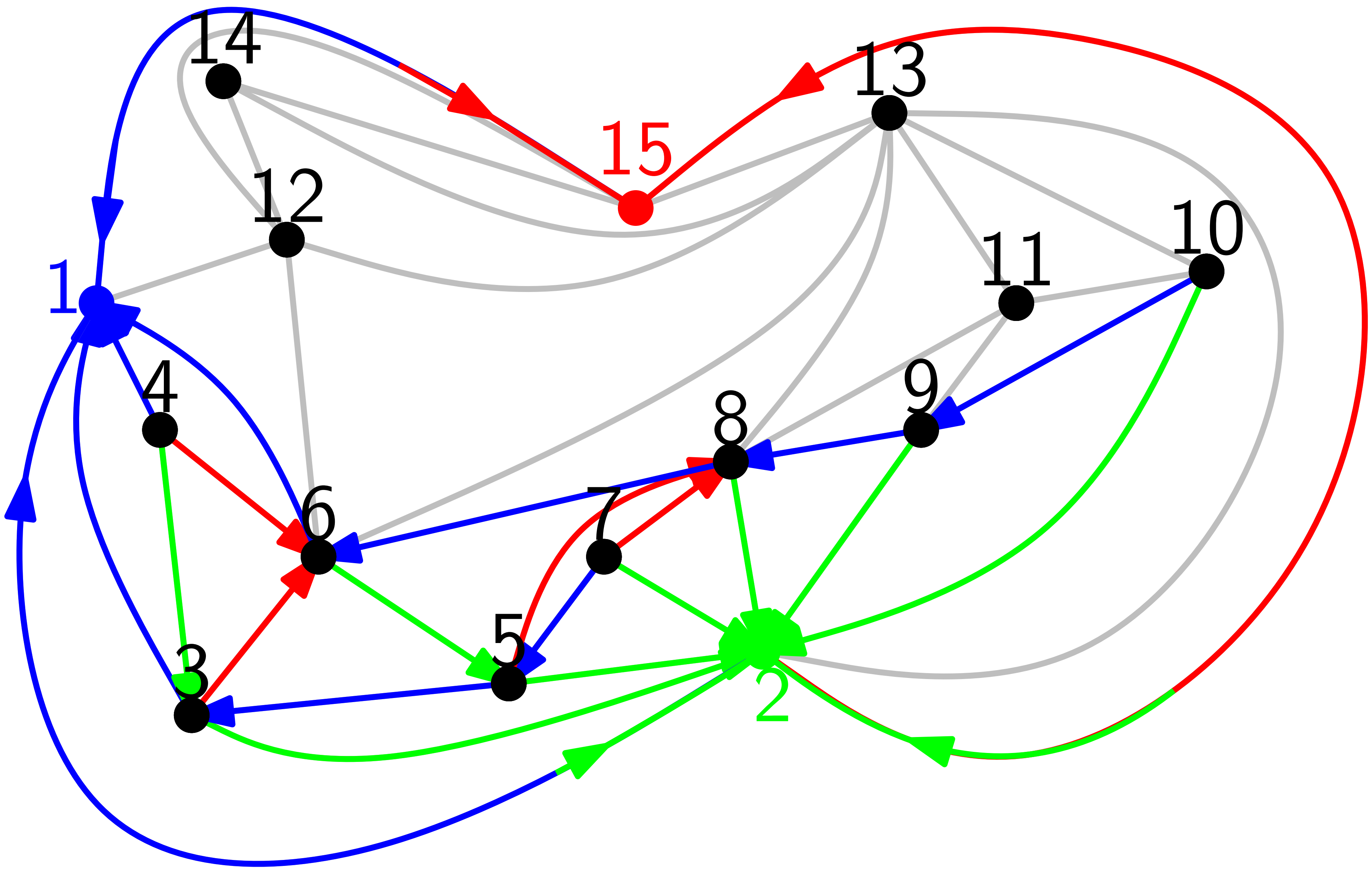
Canonical order

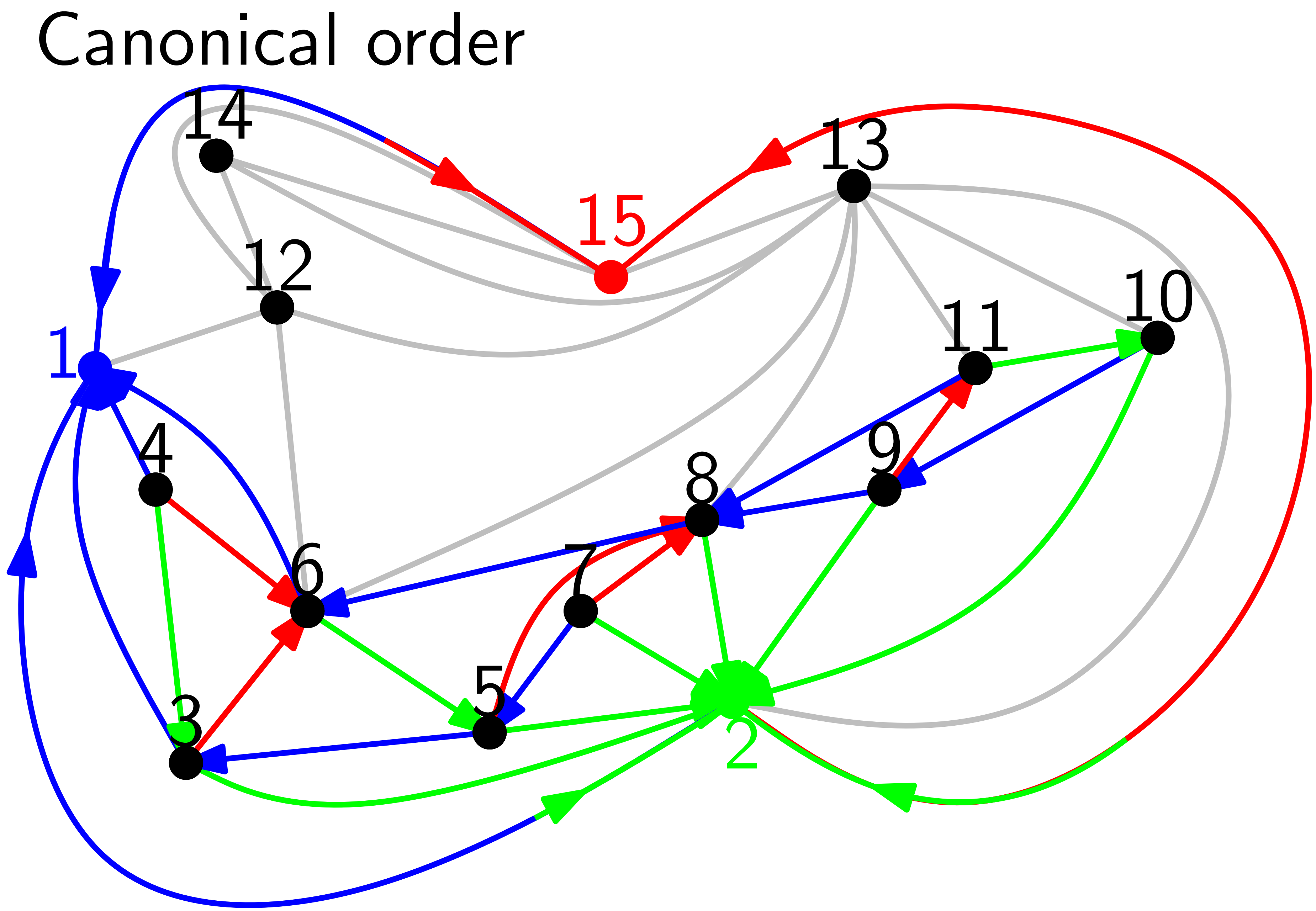


Canonical order



Canonical order





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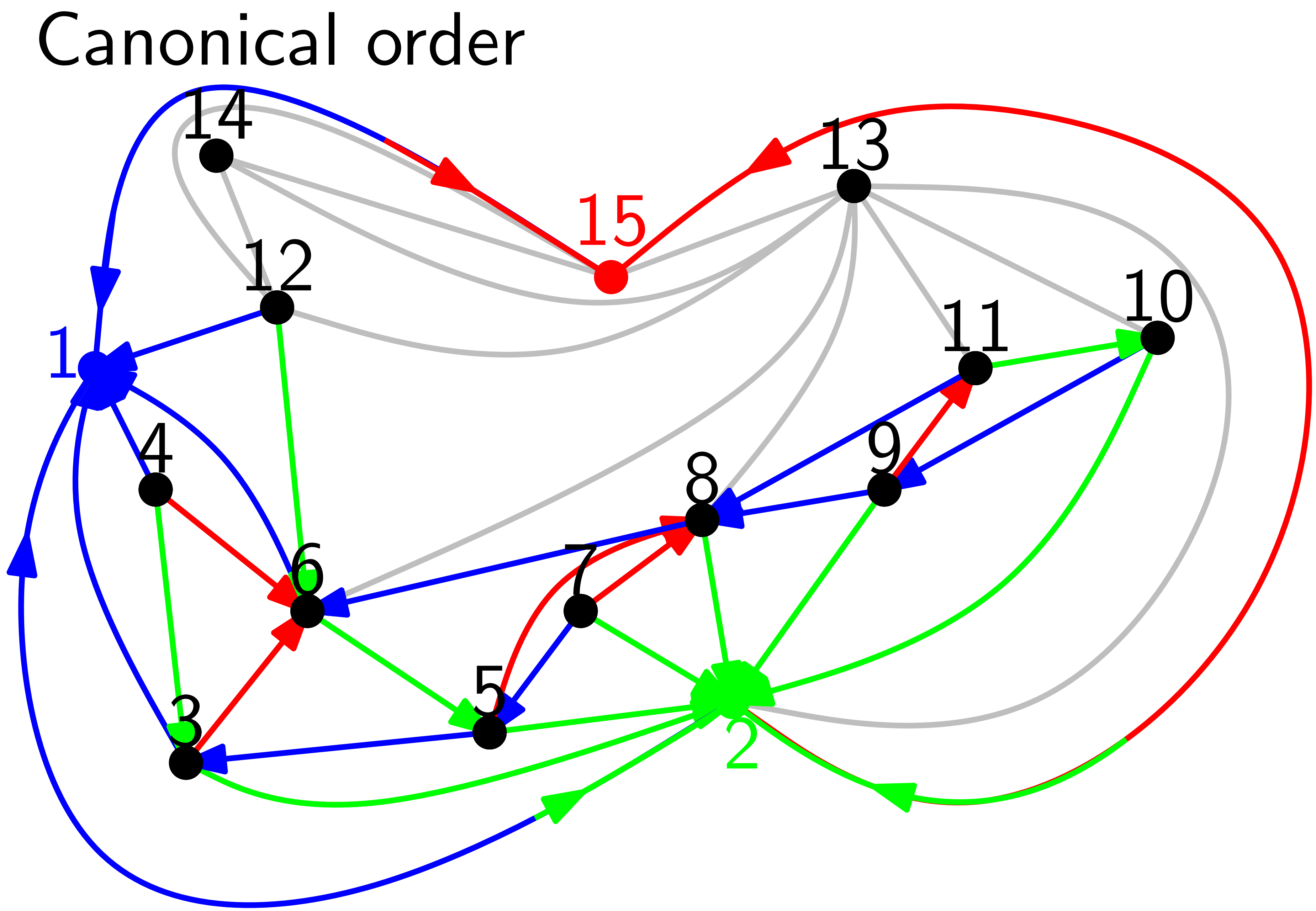

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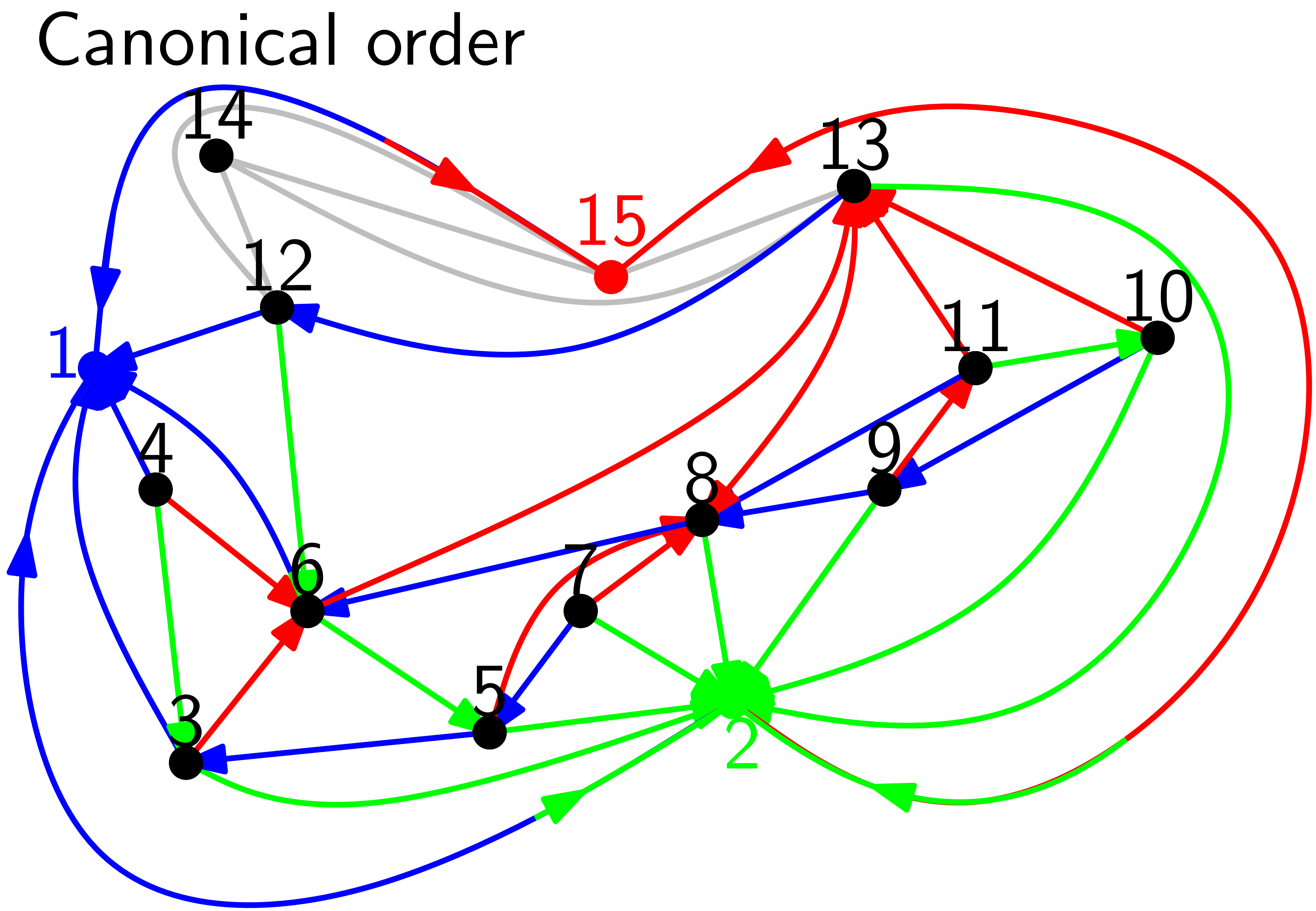

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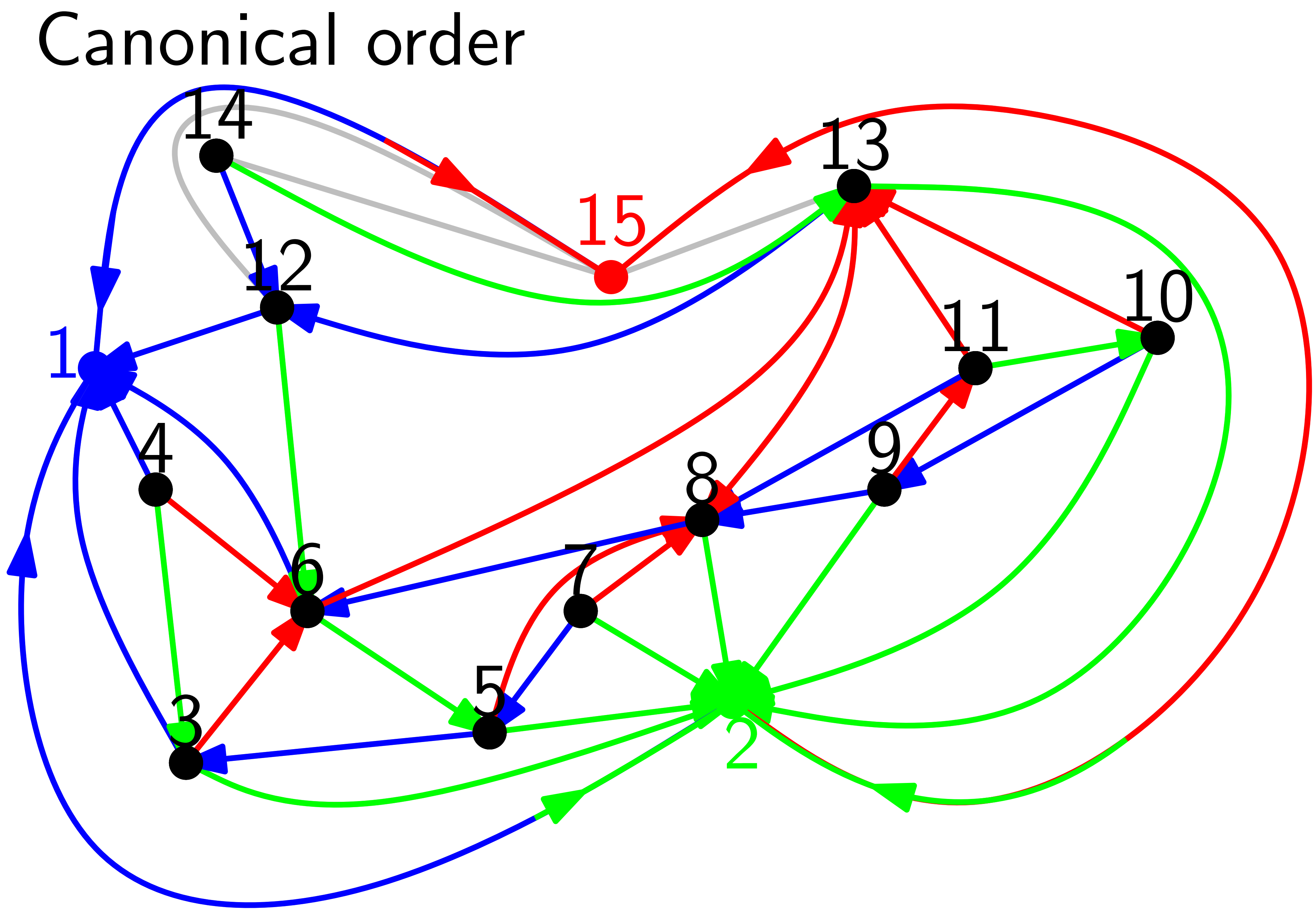

Canonical order
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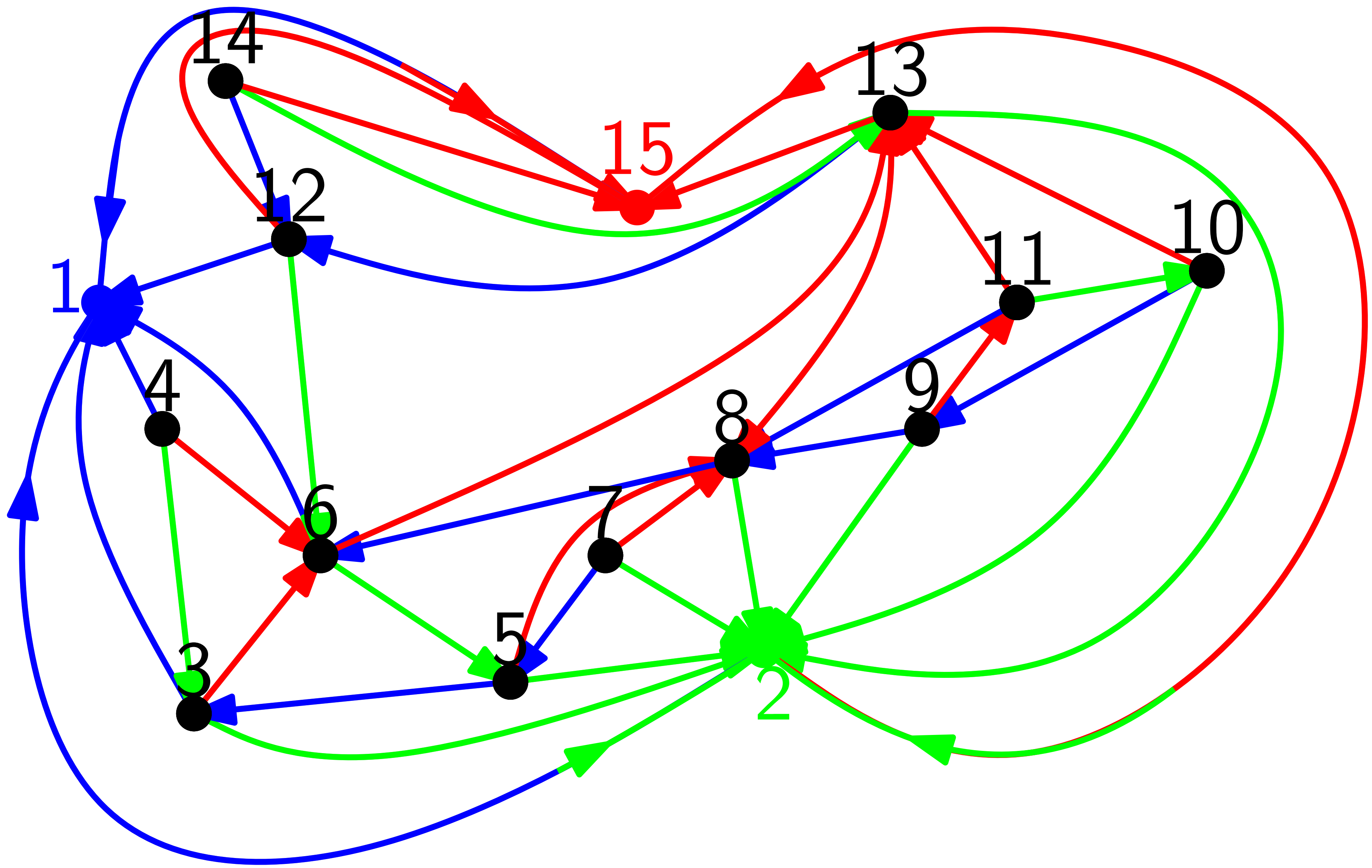
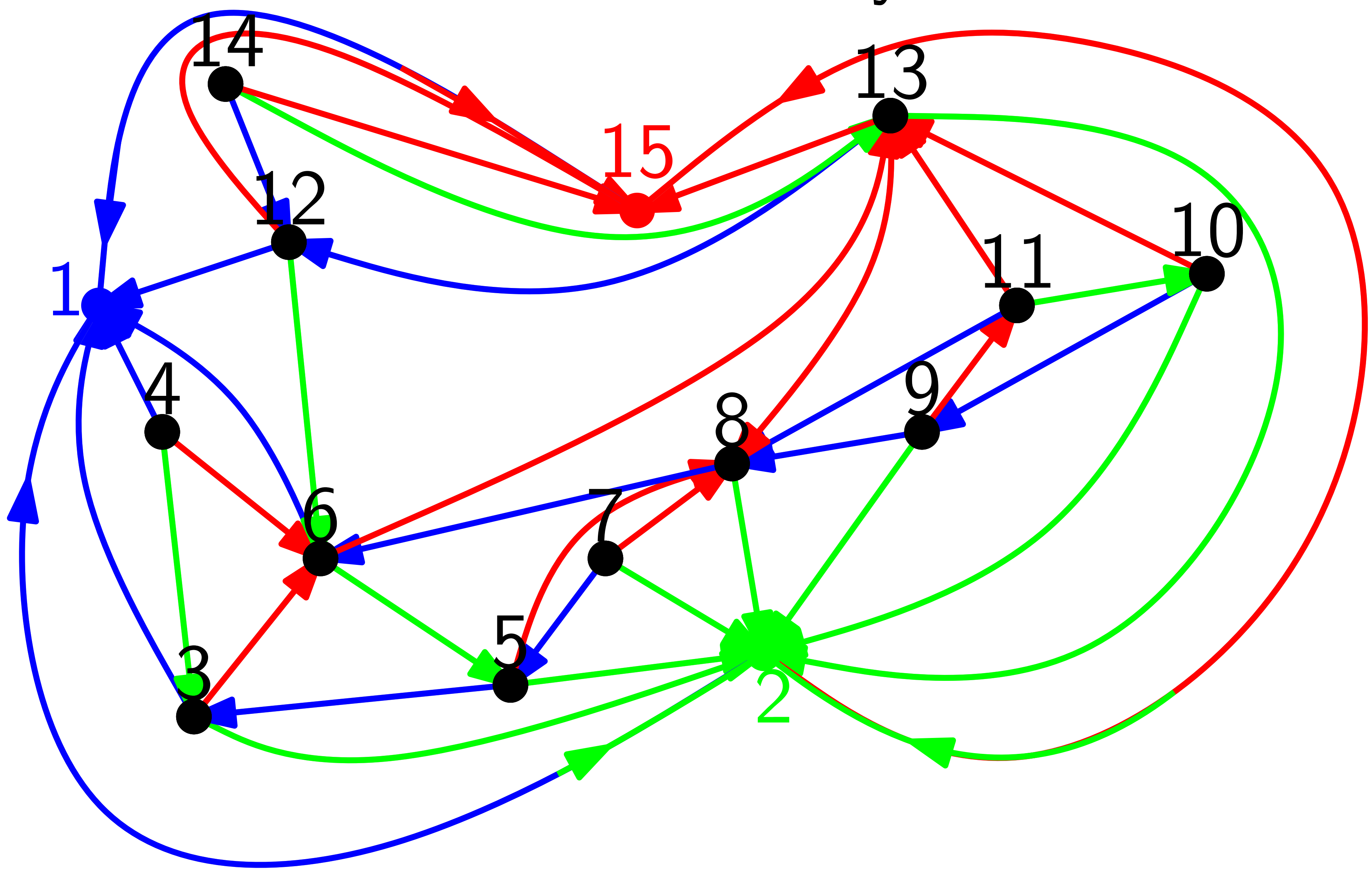
Canonical order



Canonical order $\Rightarrow$ Schnyder Wood



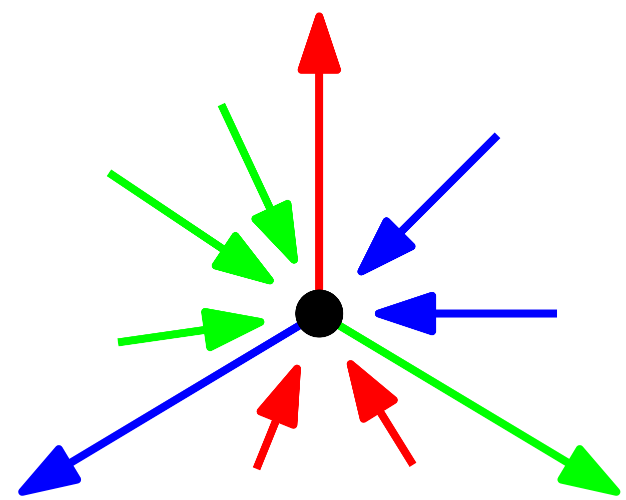
Schnyder Woods

Given a plane triangulation $G = (V, E)$

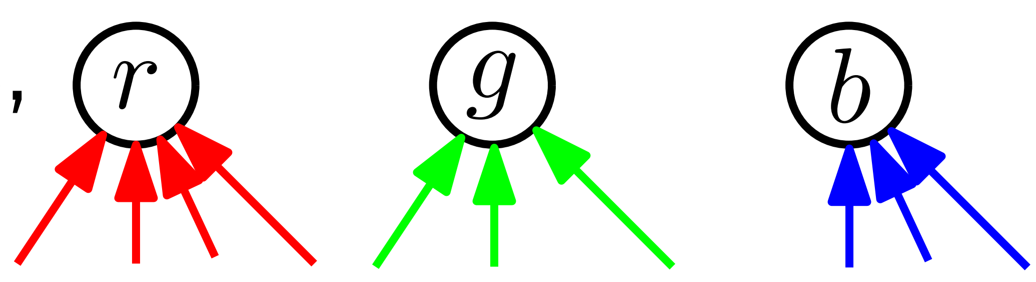
with vertices r, g, b on the outer face

a Schnyder wood is a coloring and orientation of the interior edges of G such that:

For every interior vertex,

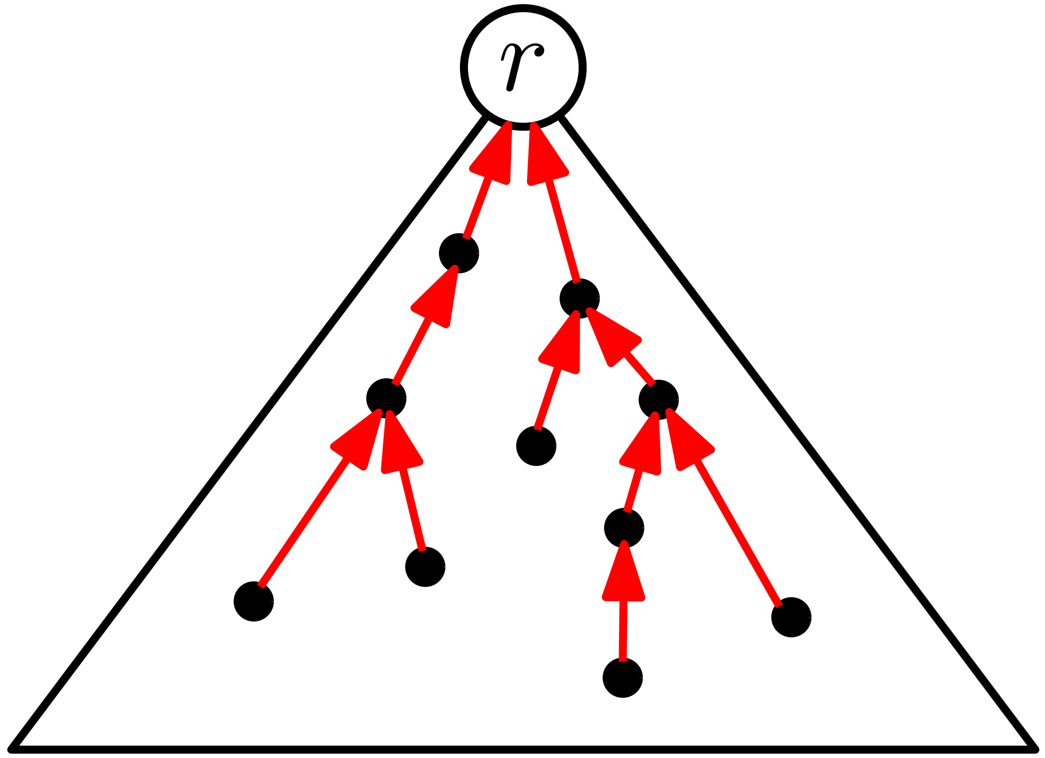


For exterior vertices,



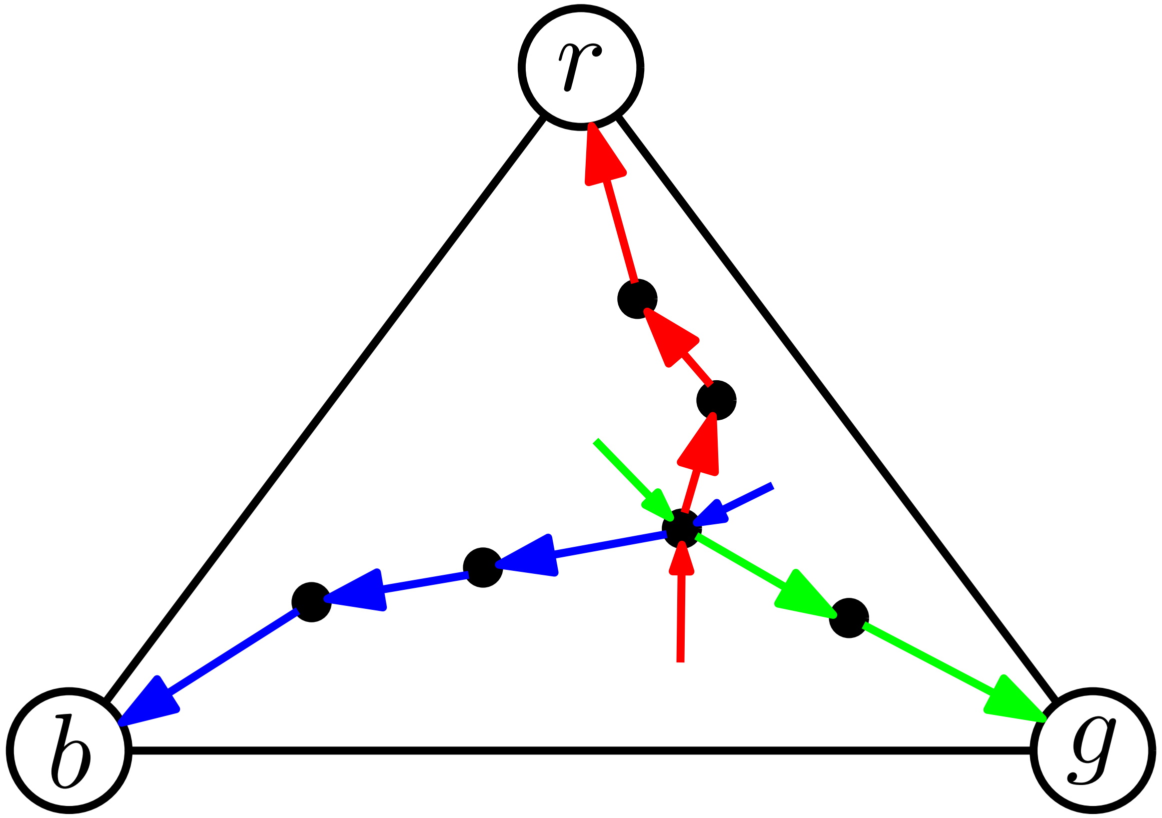
Schnyder Trees

The edges in one color class form a tree



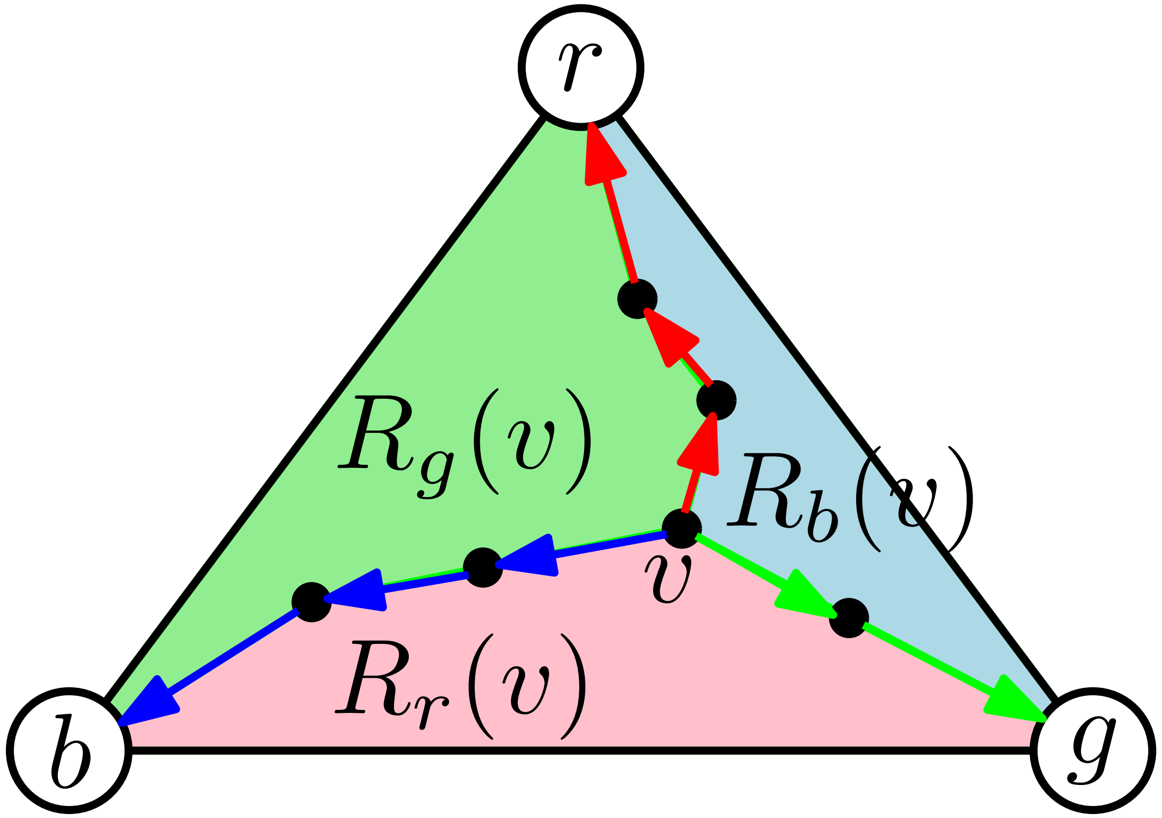
Schnyder Trees

Different colored ^{directed} paths share at most one vertex.



Schnyder Trees

Every vertex has three regions.



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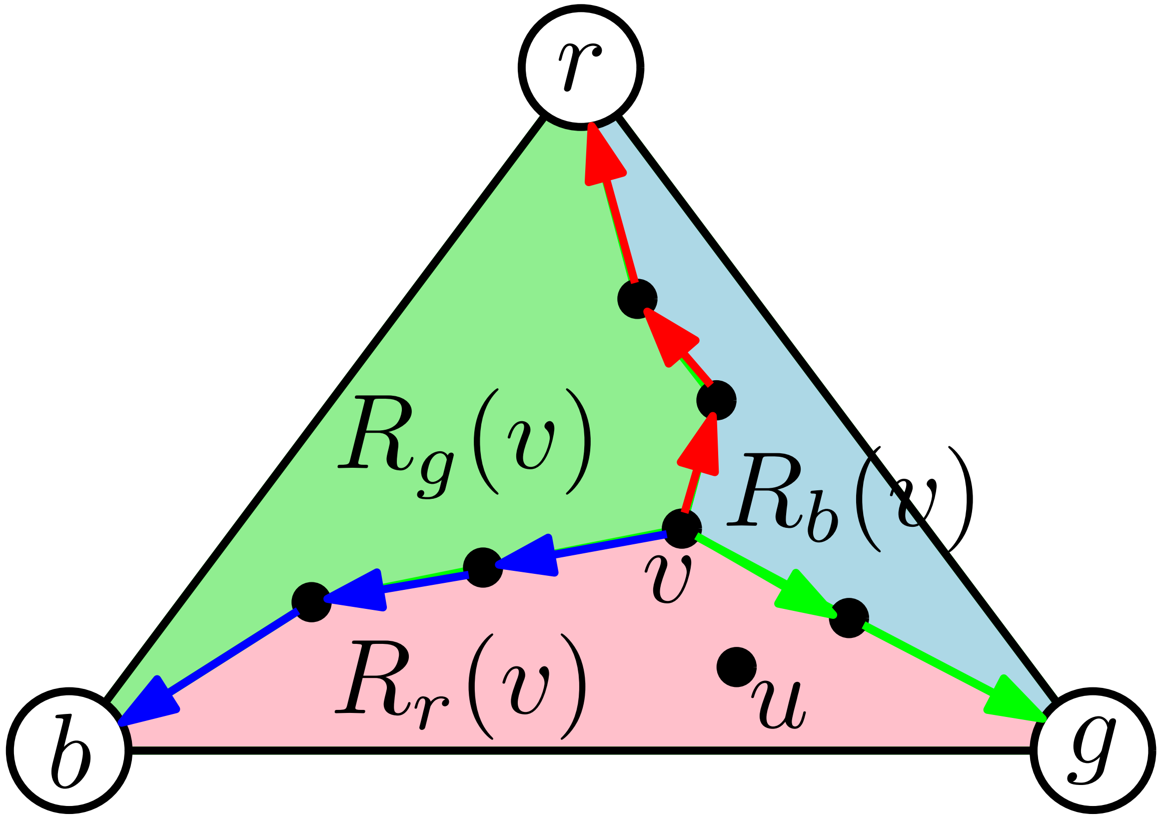
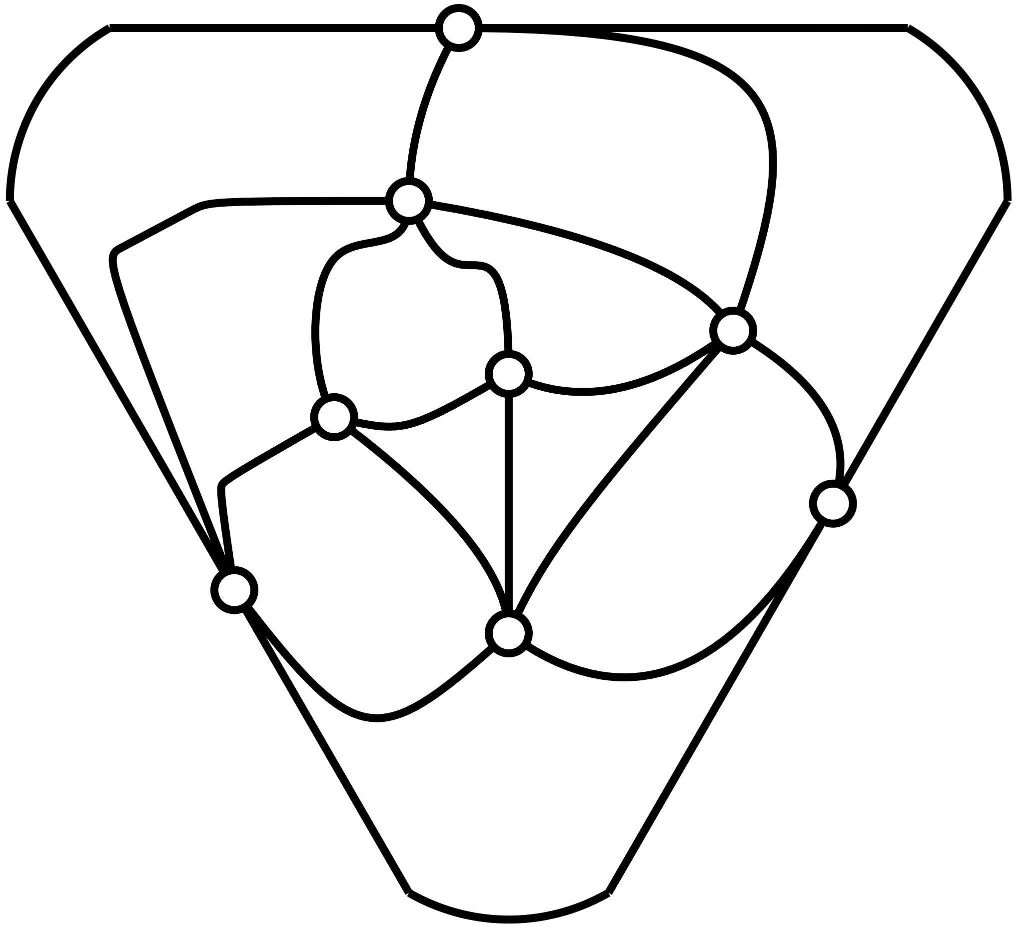


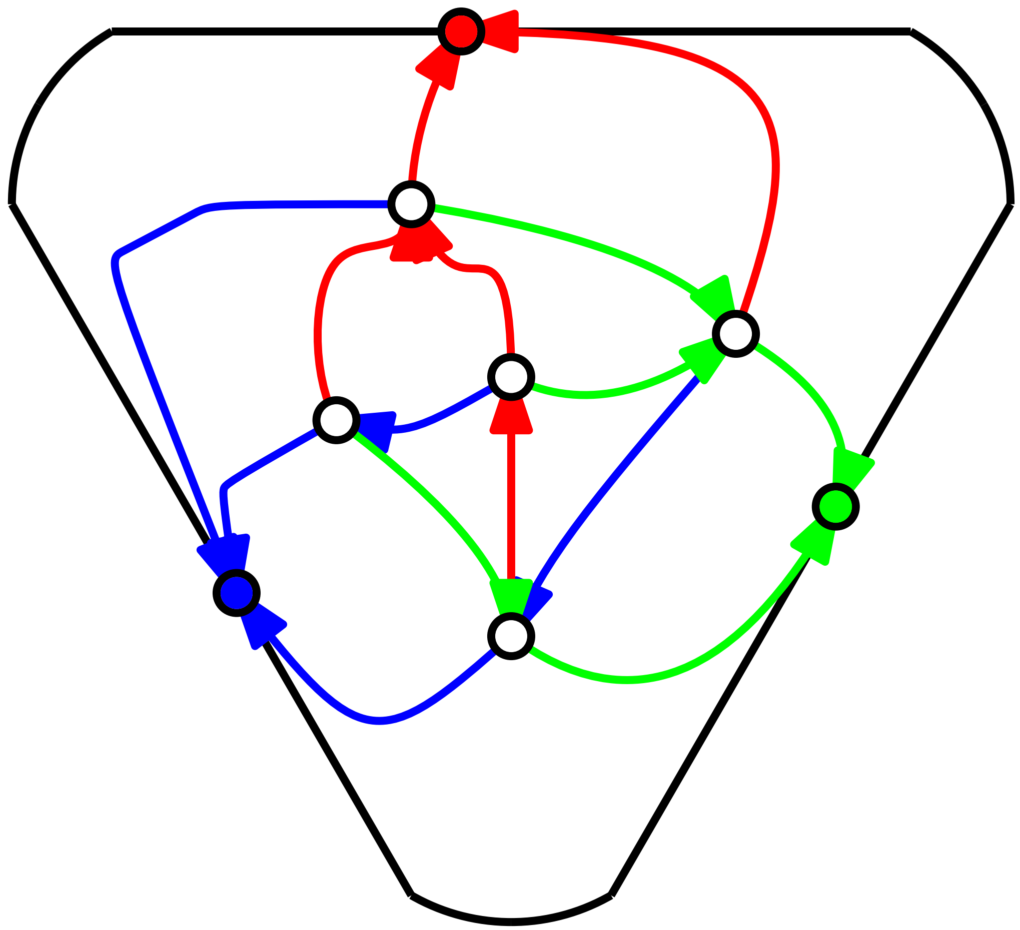
If $u \in R_c(v)$ then $R_c(u) \subset R_c(v)$.

To draw a plane triangulation...



To draw a plane triangulation...

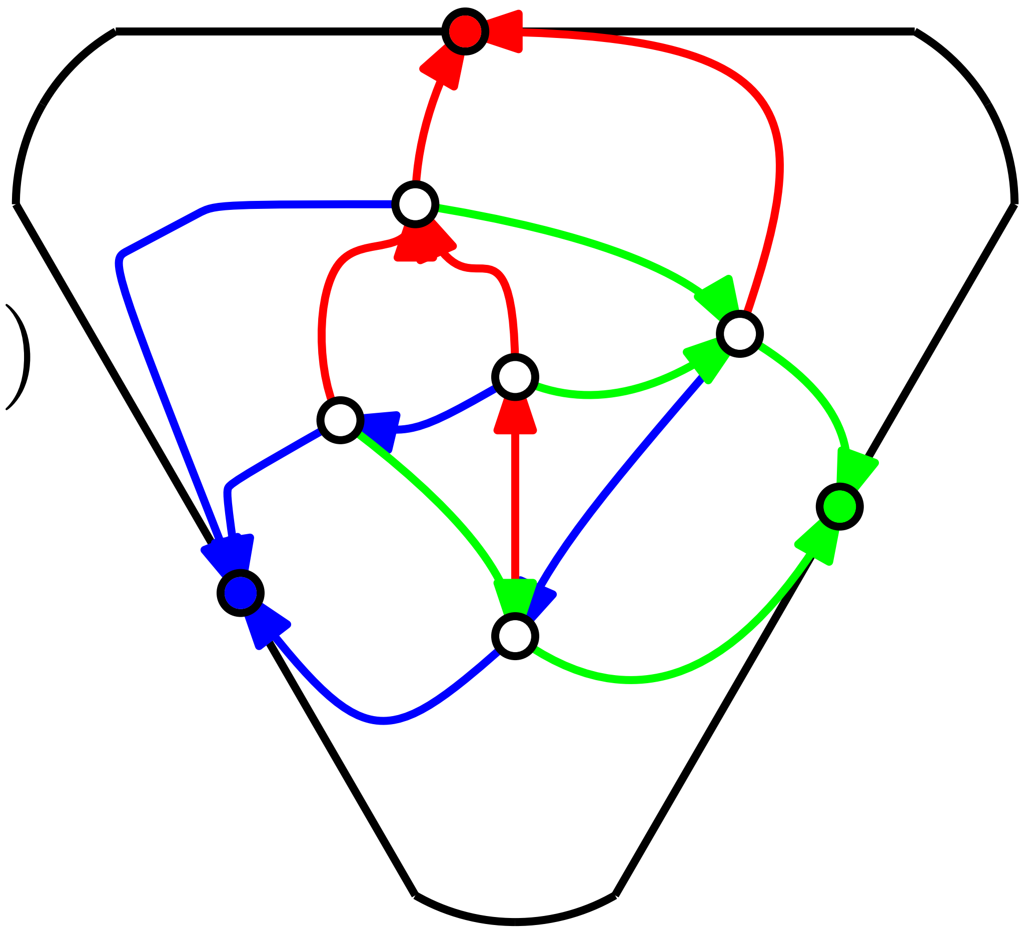
Make a Schnyder Wood



To draw a plane triangulation...

Make a Schnyder Wood

$\phi_c(v) = \#$ faces in $R_c(v)$

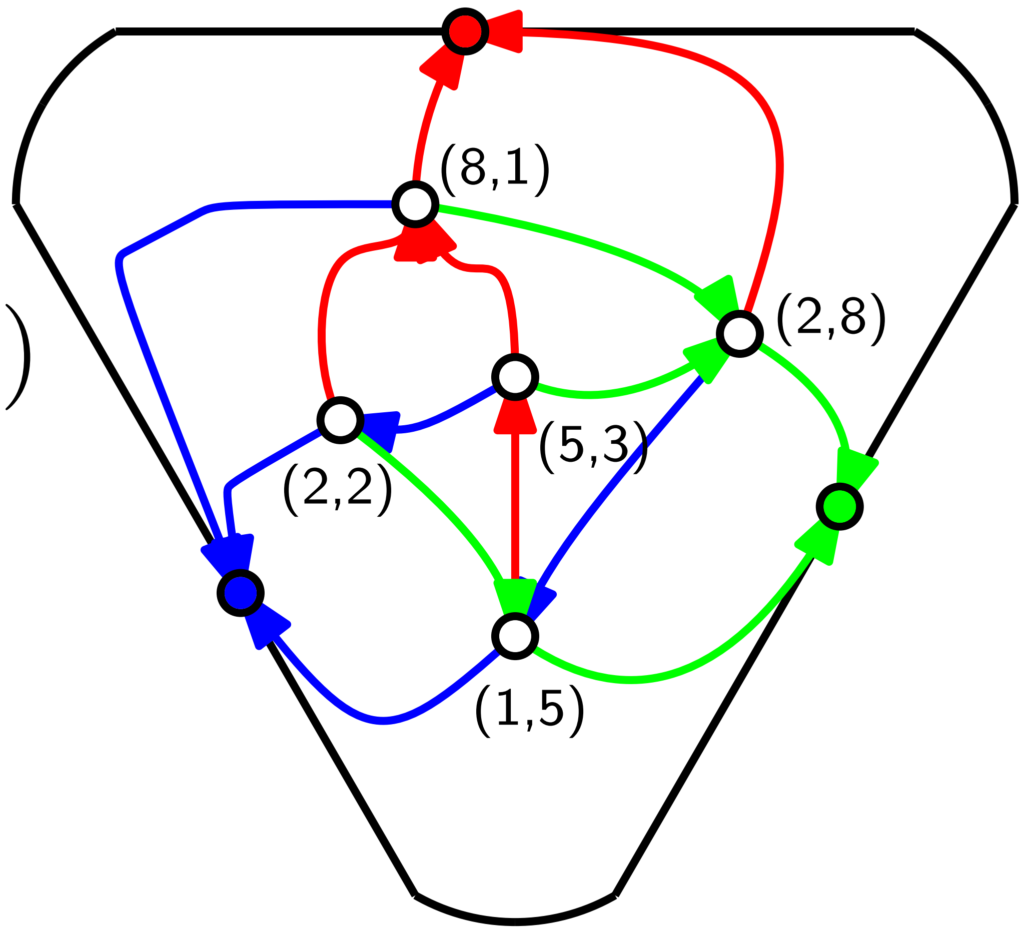


To draw a plane triangulation...

Make a Schnyder Wood

$\phi_c(v) = \#$ faces in $R_c(v)$

Draw v at $(\phi_r(v), \phi_g(v))$



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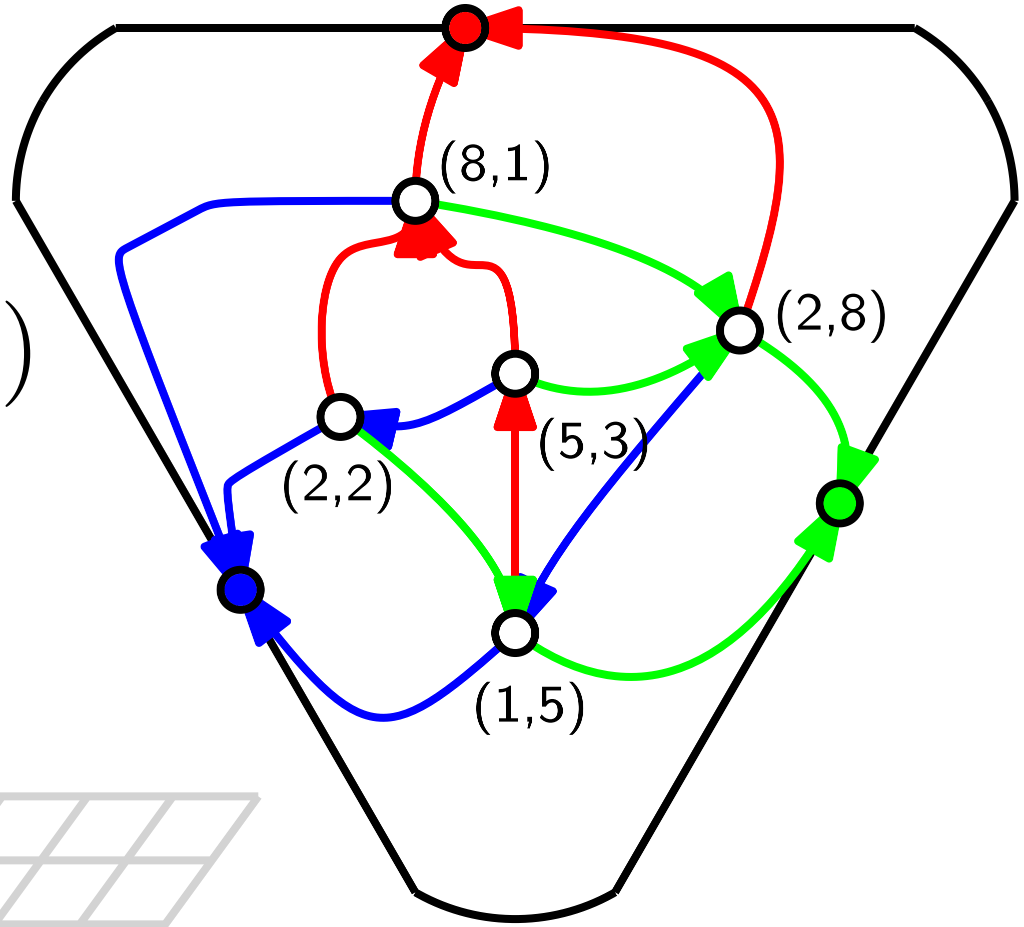
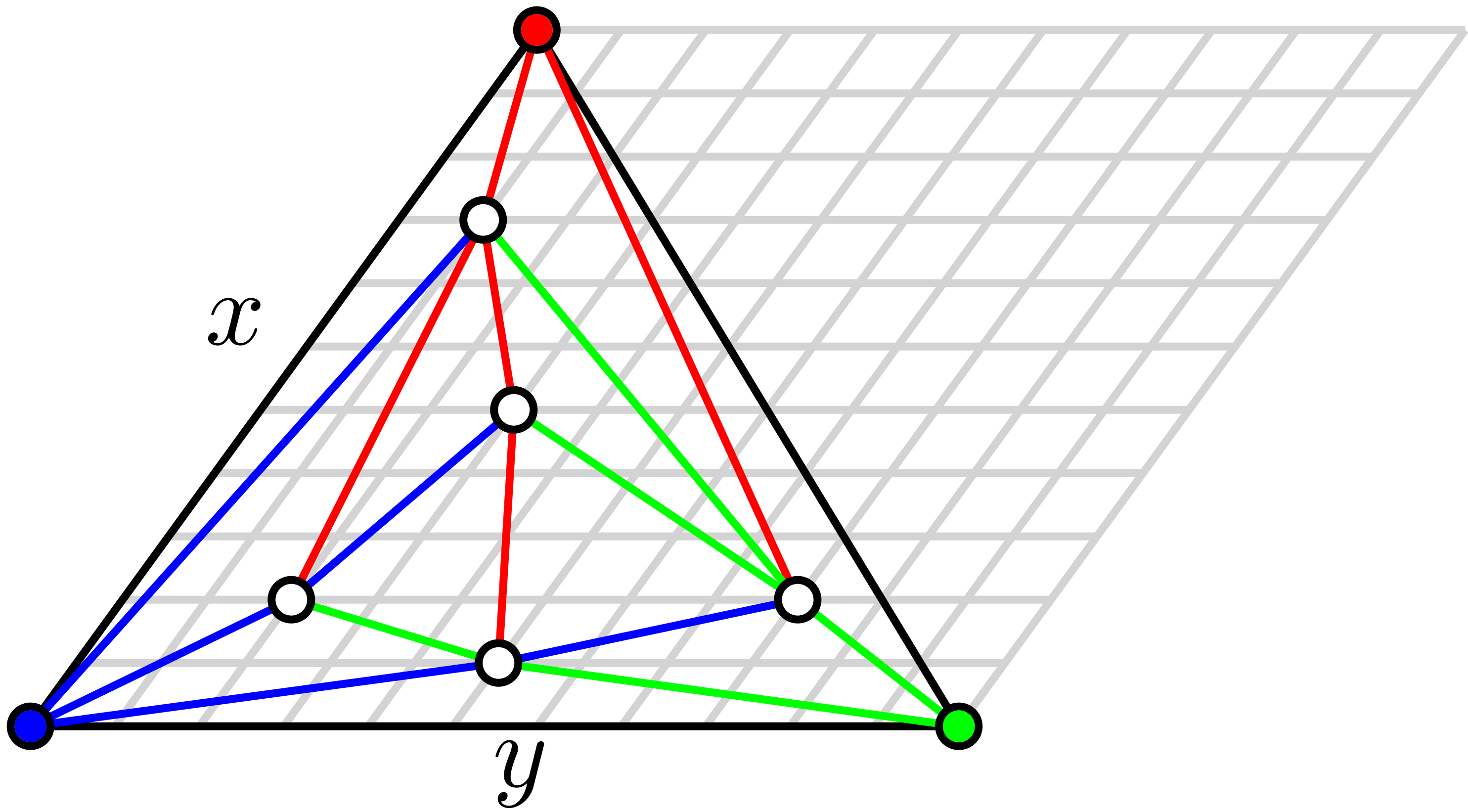
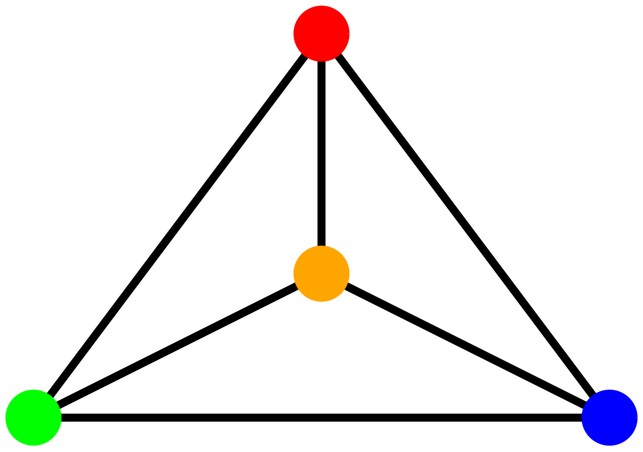




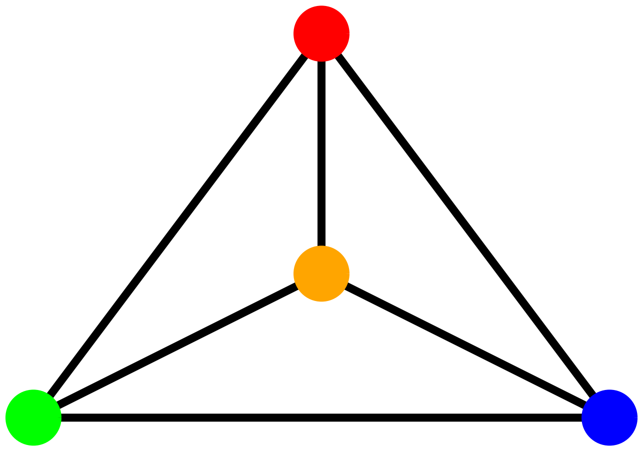
Proper contact representation

Farzad Fallahi's Ugrad Thesis 2017

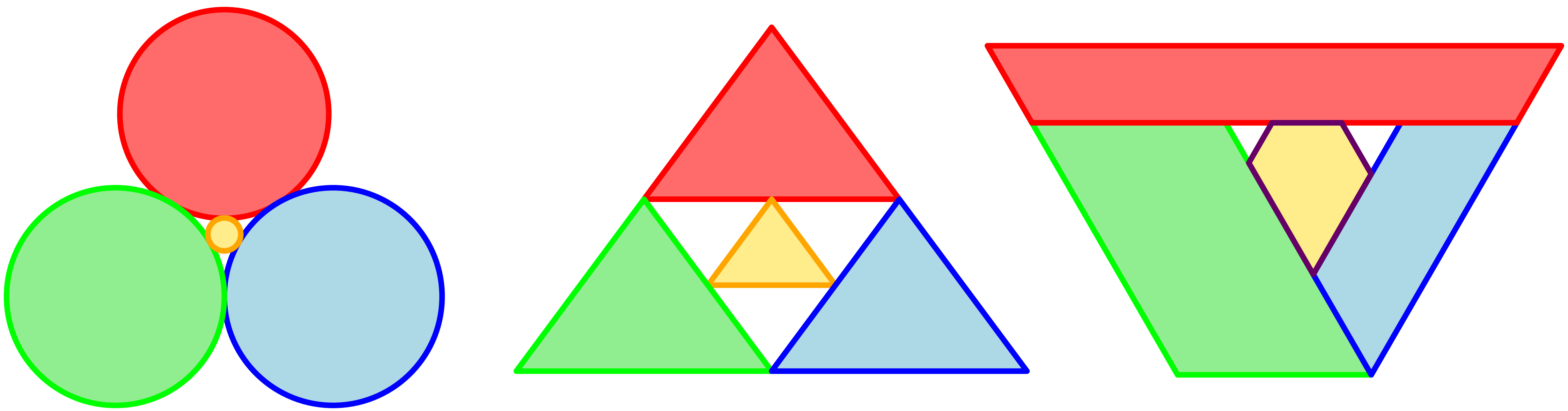


Proper contact representation

Farzad Fallahi's Ugrad Thesis 2017

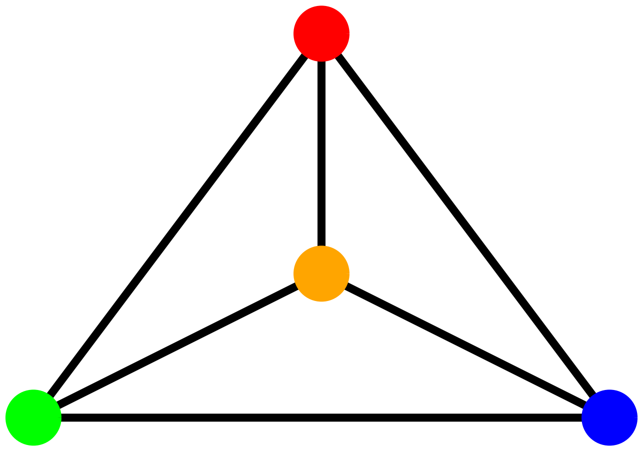


Contact Representations



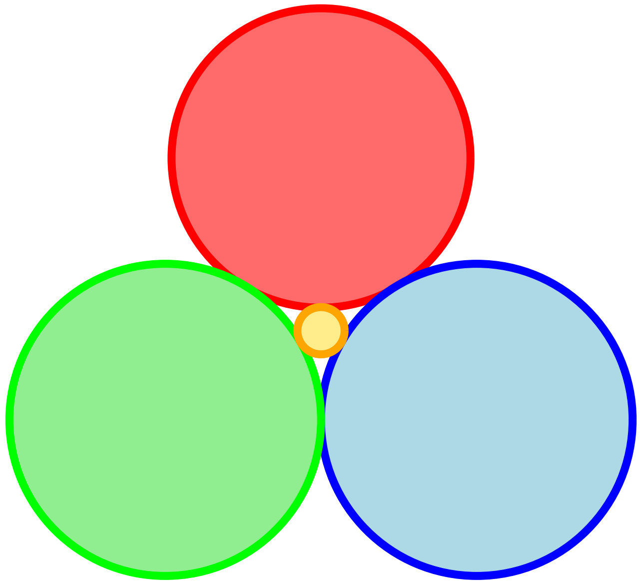
Proper contact representation

Farzad Fallahi's Ugrad Thesis 2017

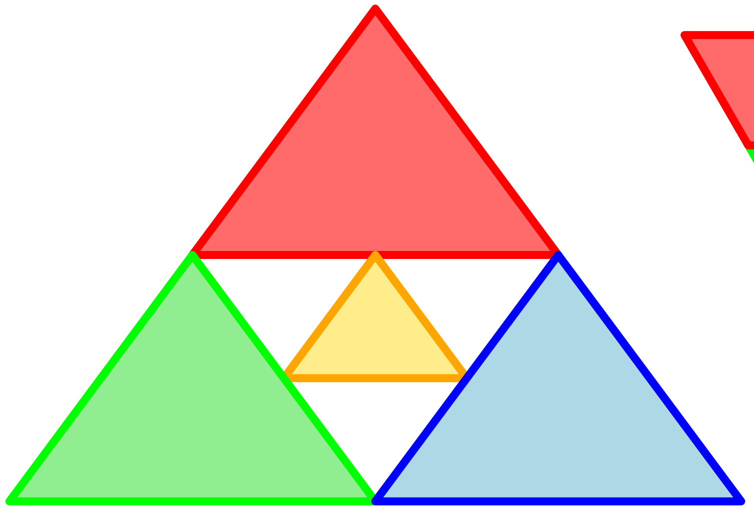


Contact Representations

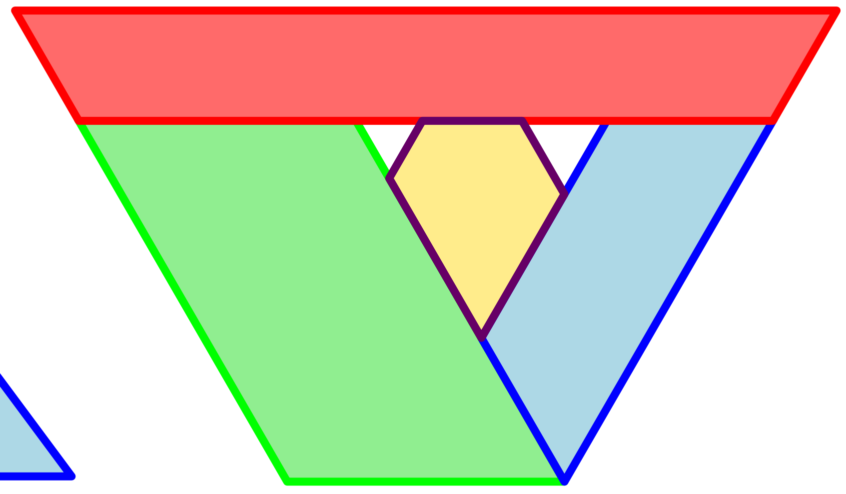
Symmetric



Simple



Face-to-face



Facts of Life

To represent all planar graphs with Face-to-Face contact, you need 6-sided polygons.

How Symmetric can they be?

Facts of Life

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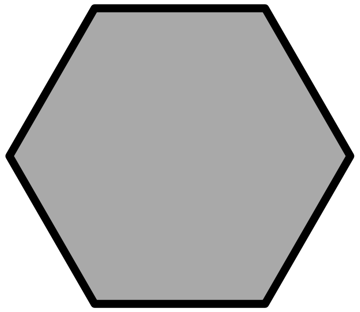
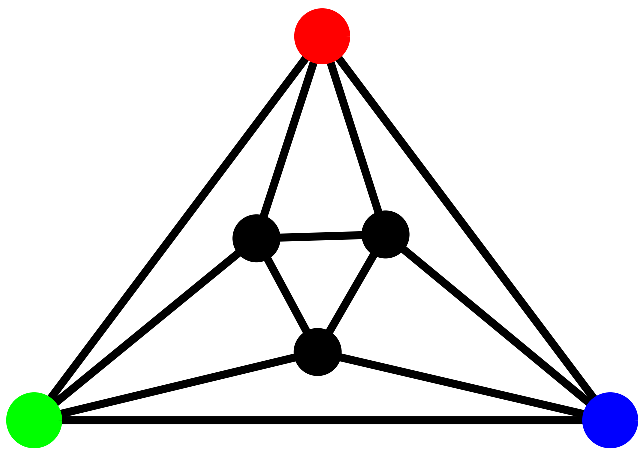
How Symmetric can they be?

Regular Hexagon



Too Symmetric



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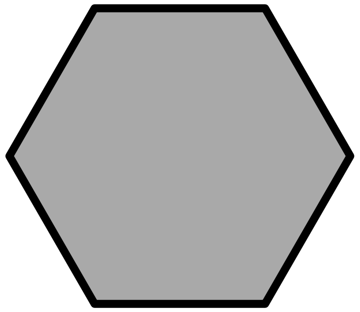

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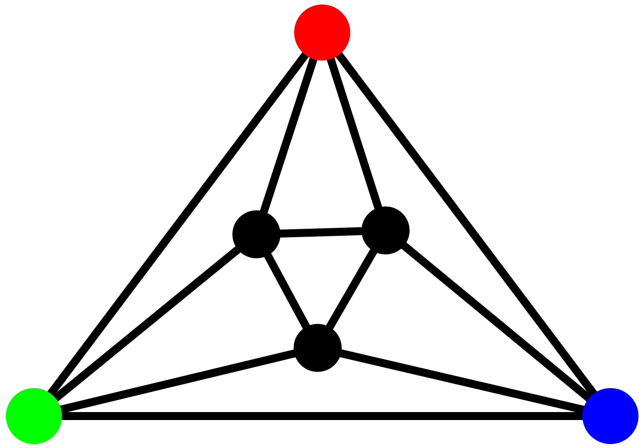
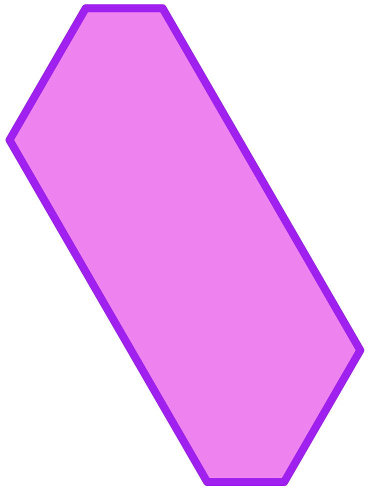
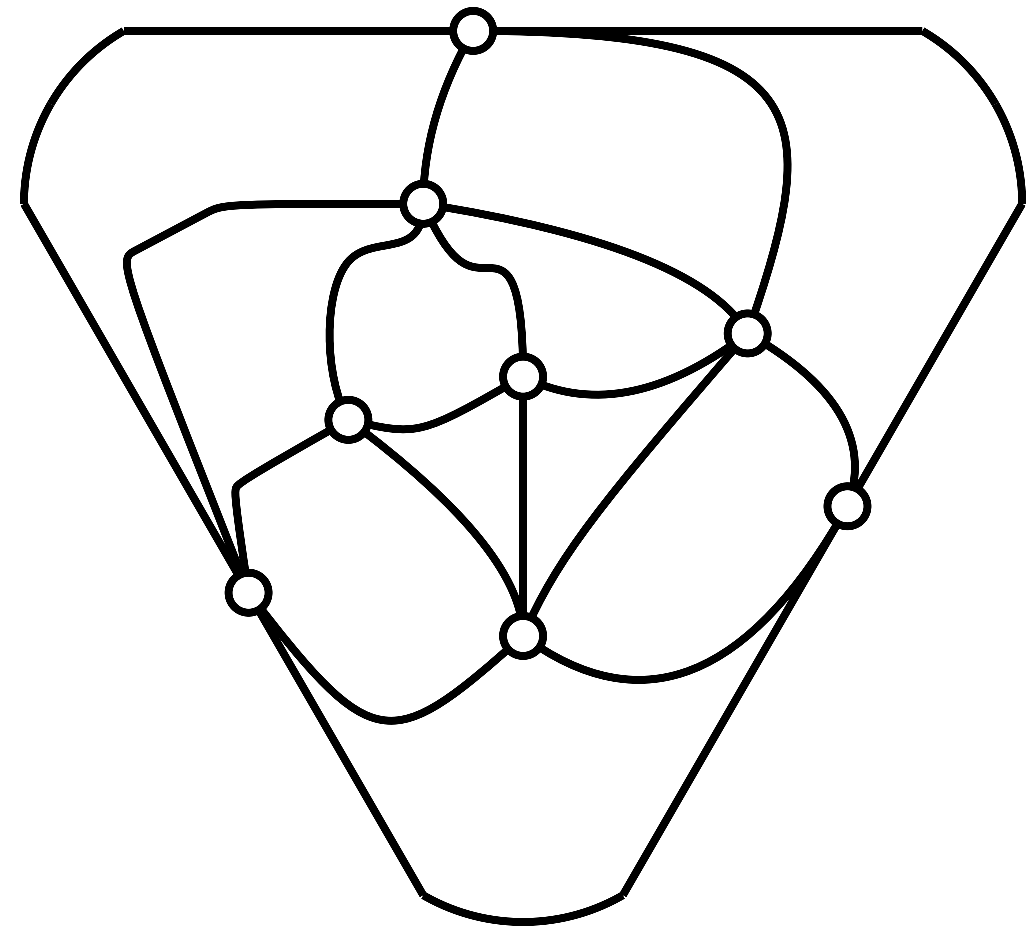


Equi-Parallel Hexagon



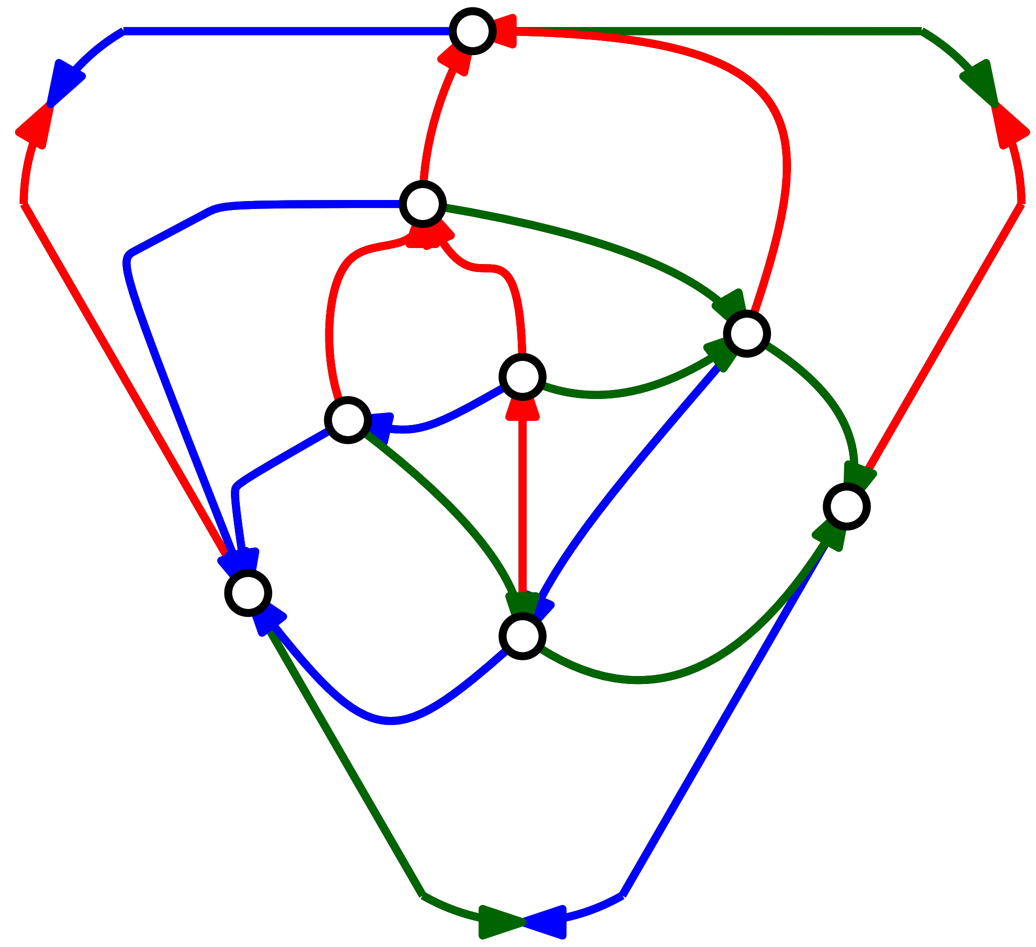
Just Symmetric enough.

Given a 3-connected, triangulated planar graph.



Given a 3-connected, triangulated planar graph.

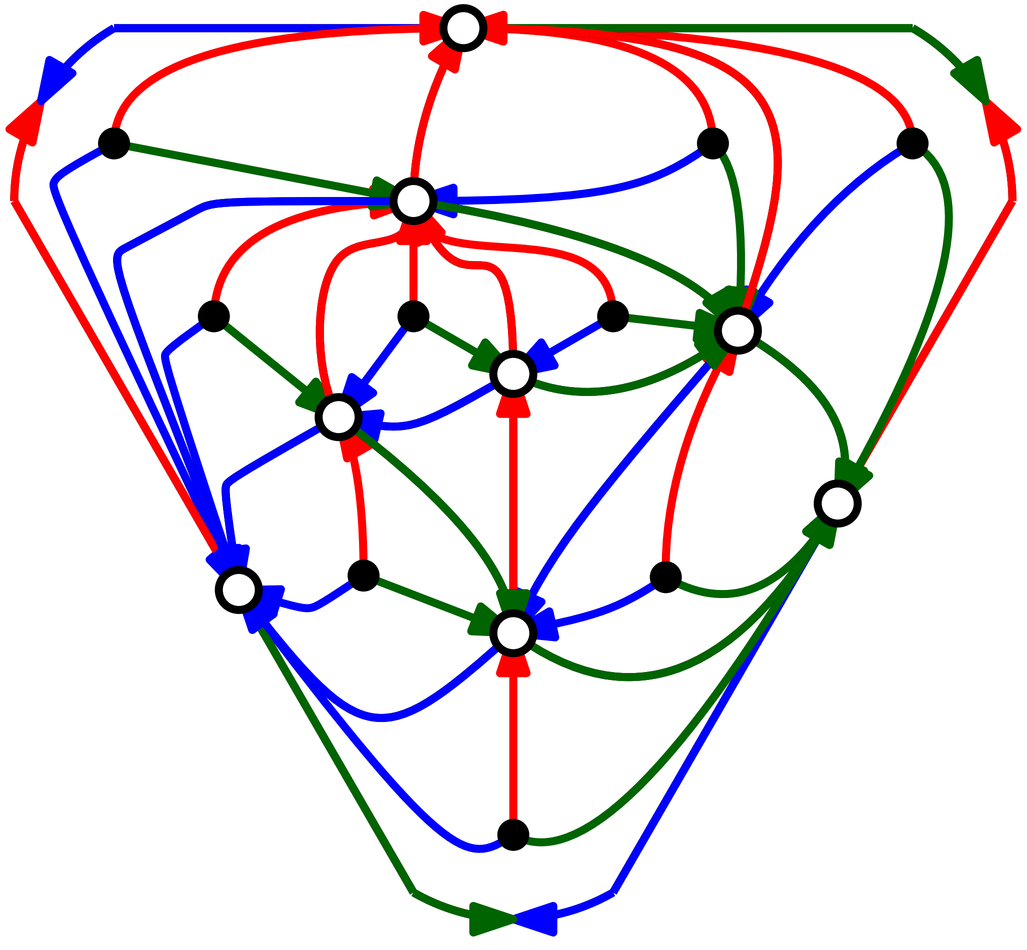
Color and direct edges to form a Schnyder wood.



Given a 3-connected, triangulated planar graph.

Color and direct edges to form a Schnyder wood.

Add dummy vertex in each acyclic face and connect into Schnyder wood.



Given a 3-connected, triangulated planar graph.

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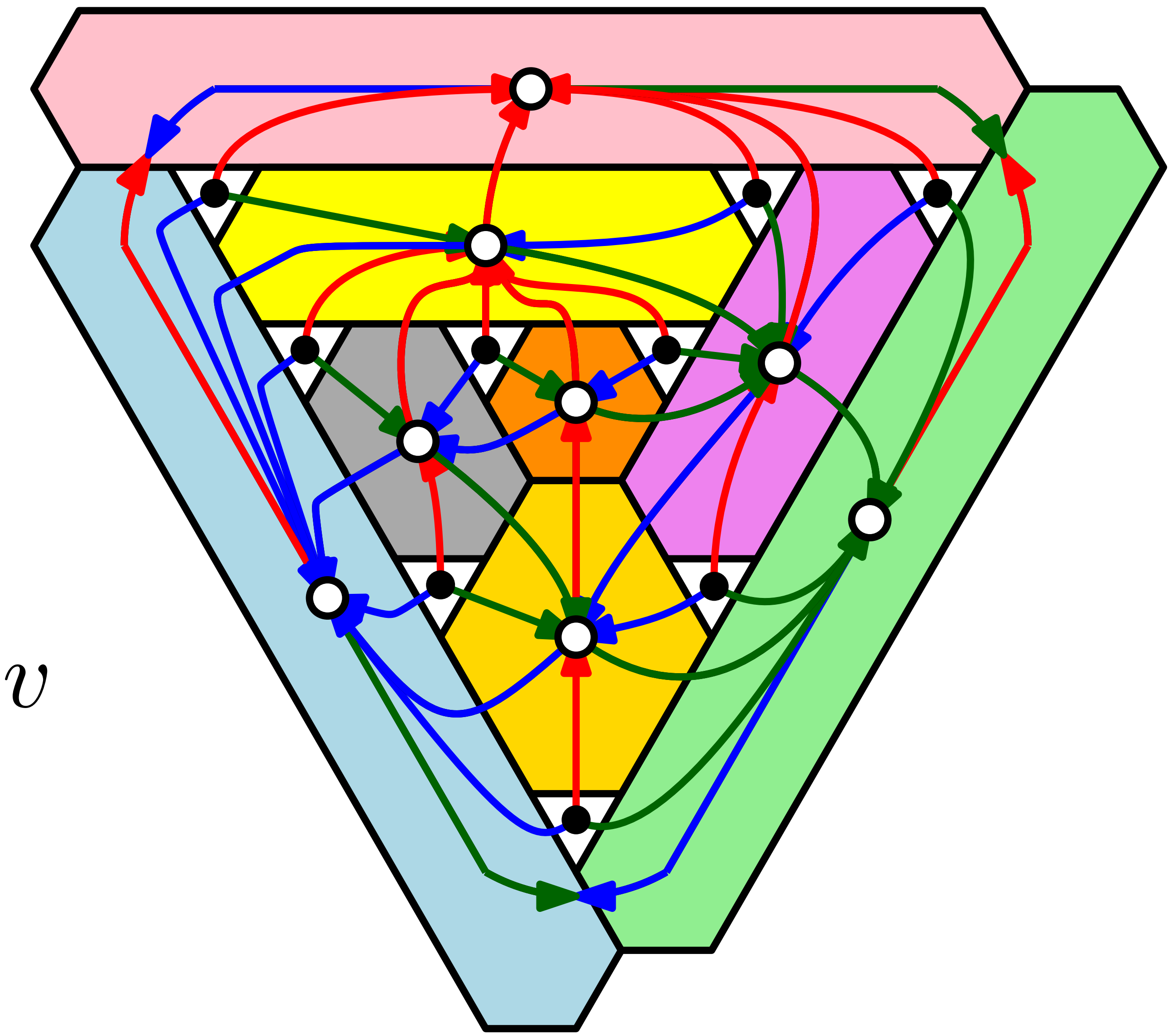
side lengths of hexagon v are numbers of leaves in T_0, T_1, T_2 below v.



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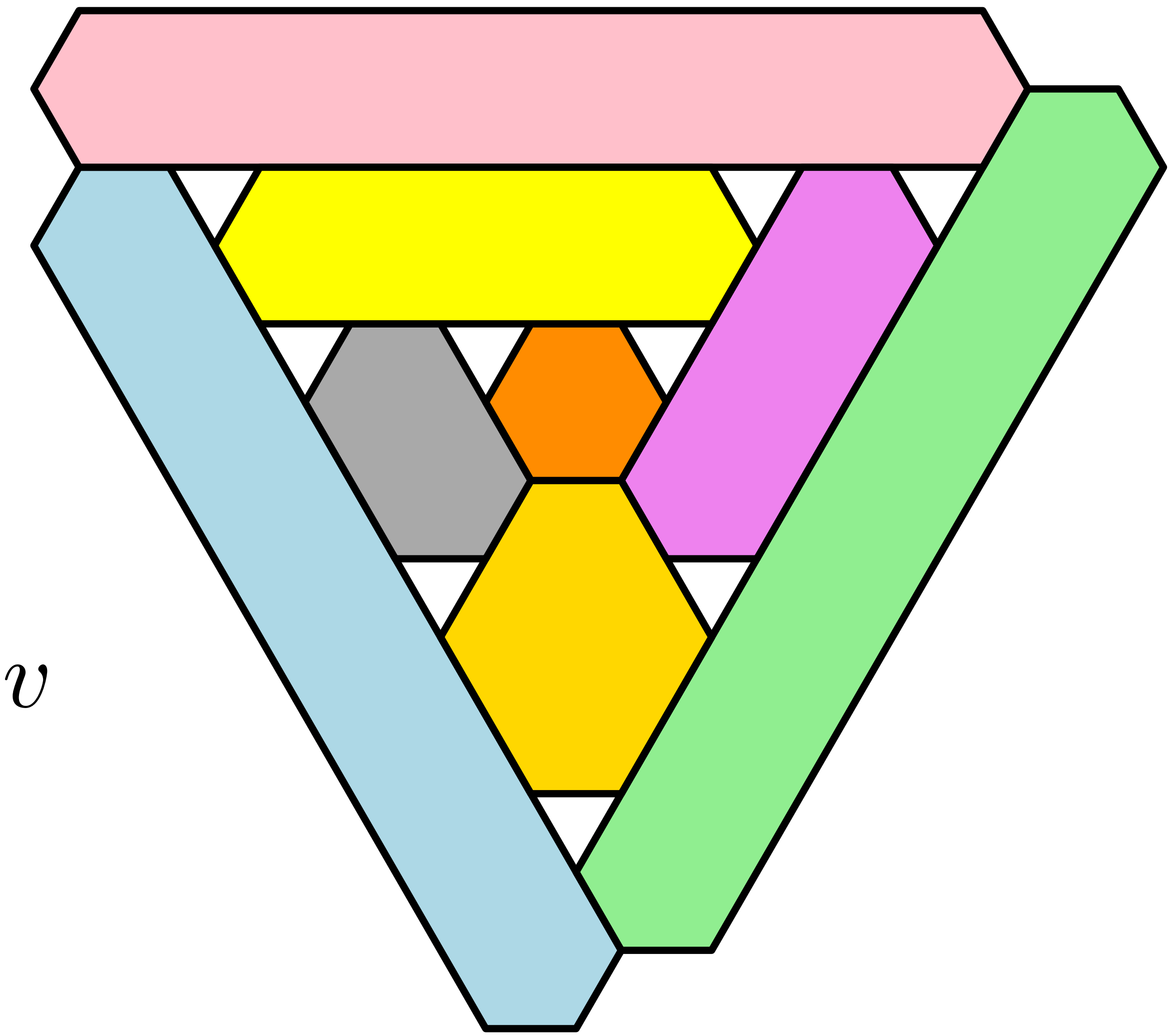

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side lengths of hexagon v are numbers of leaves in T_0, T_1, T_2 below v.

Ta da!

CONTACT REPRESENTATIONS OF NON-PLANAR GRAPHS IN 3D

with

Md. Jawaherul Alam

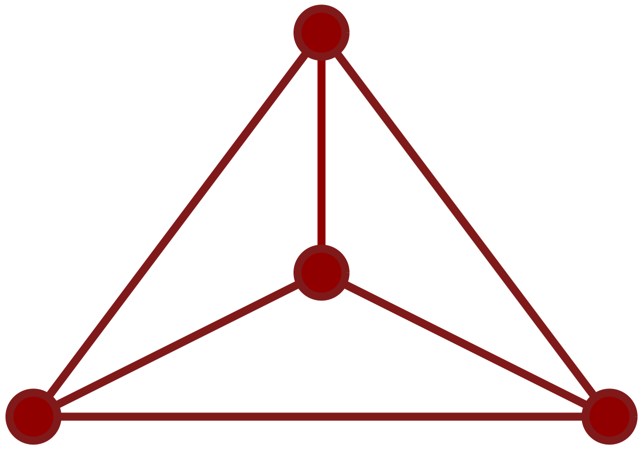
Stephen Kobourov

Sergey Pupyrev

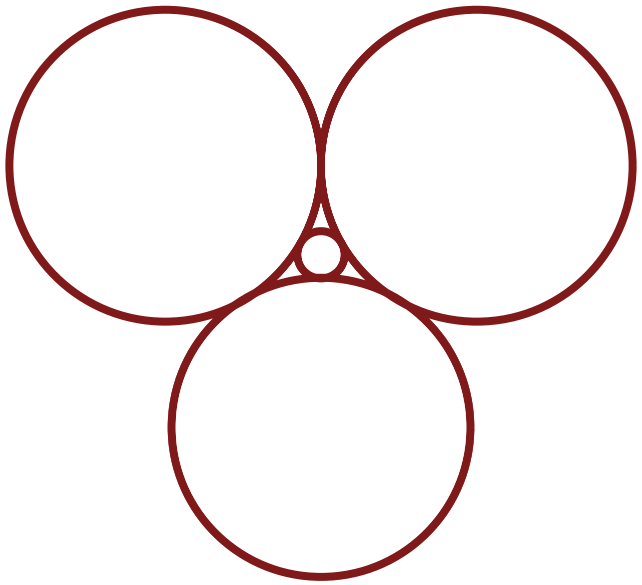
Jackson Toeniskoetter

Torsten Ueckerdt

Contact Representation

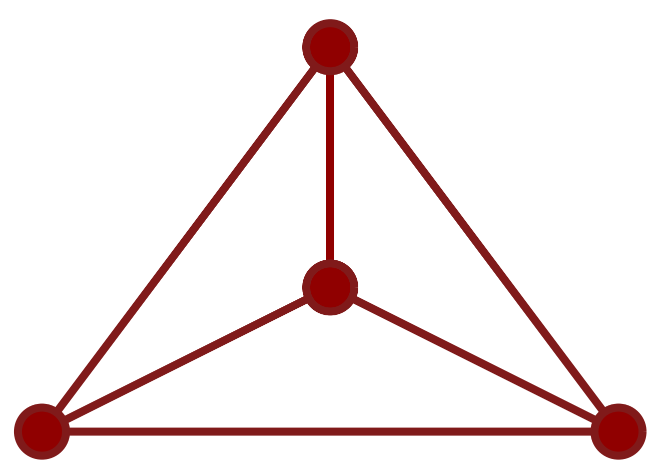


Contact Representation

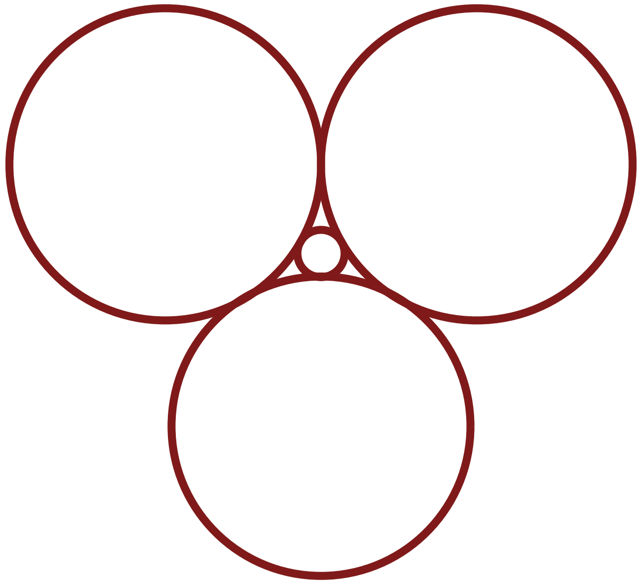


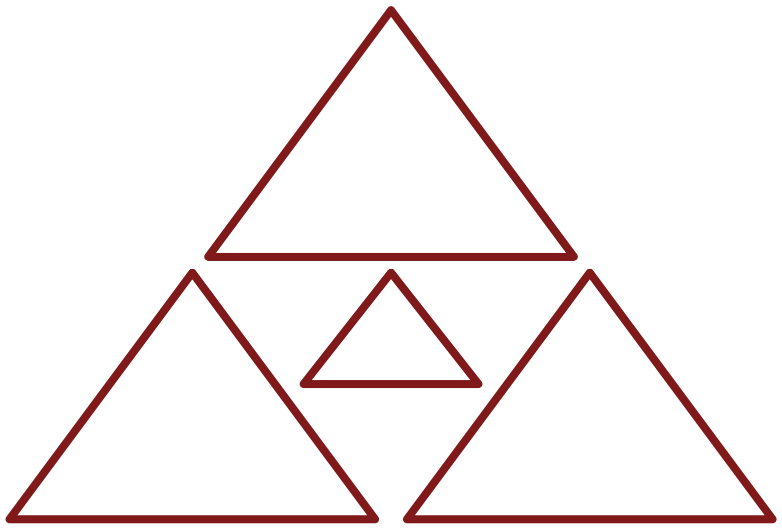
Vertices = Interior disjoint objects

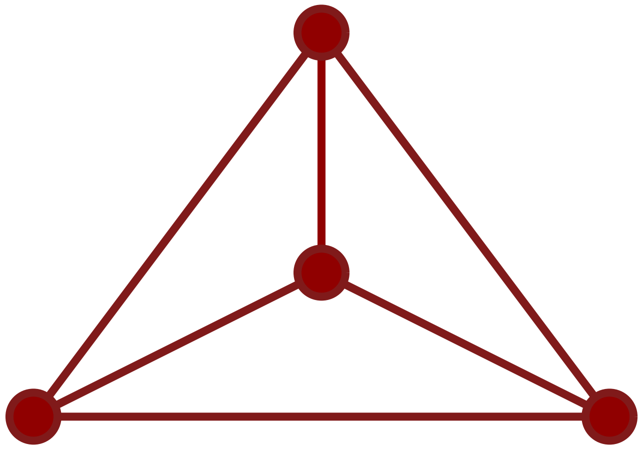
Edges = Contact



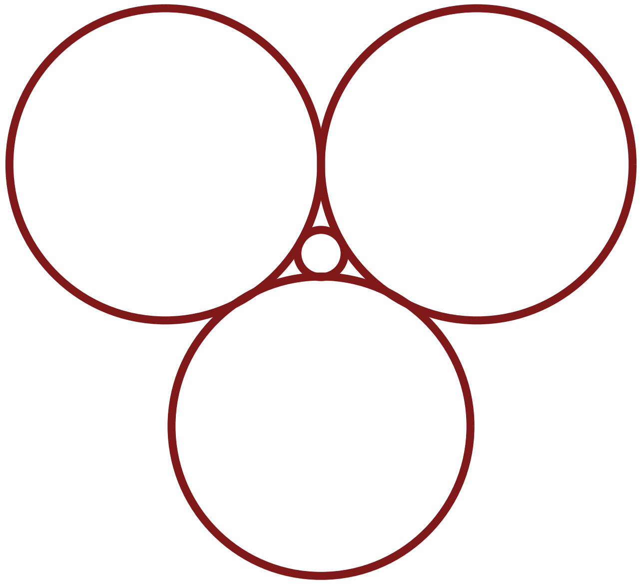
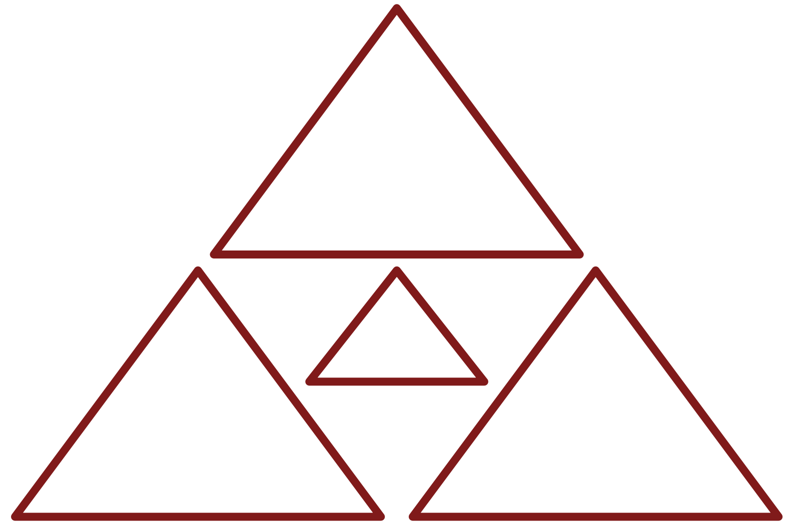
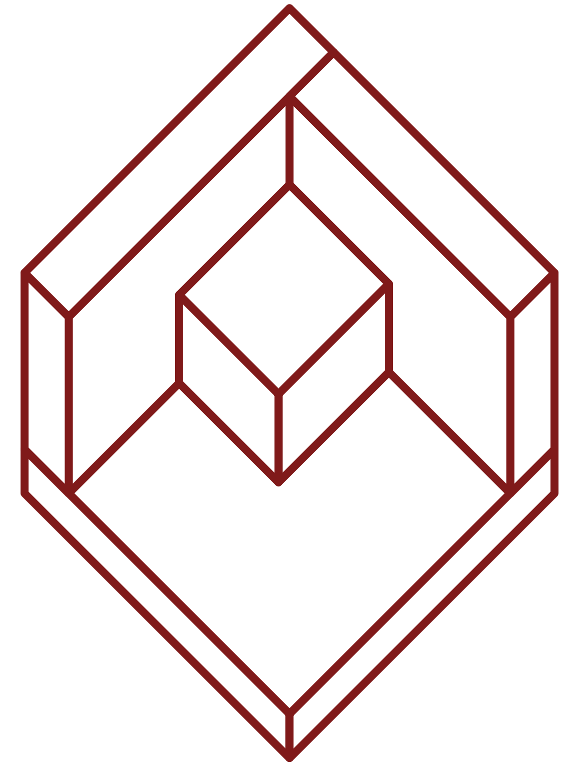
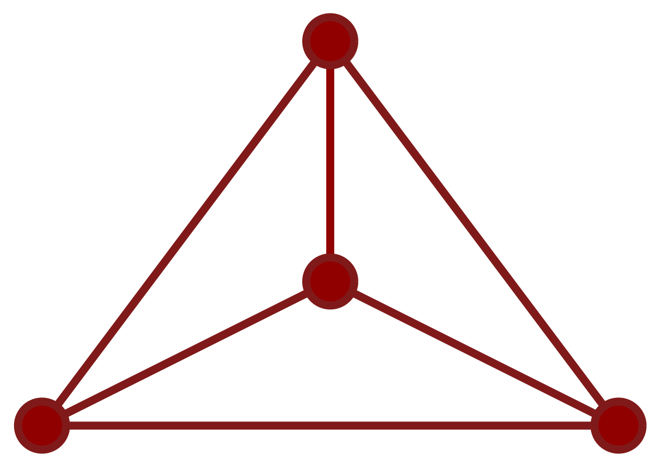
Contact Representation



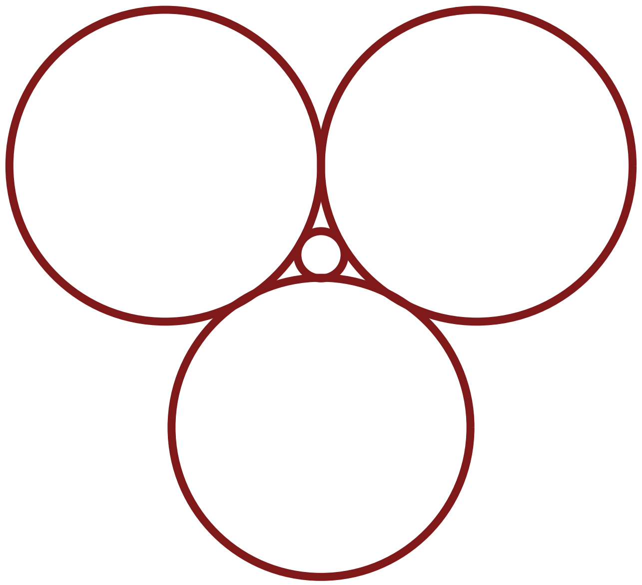
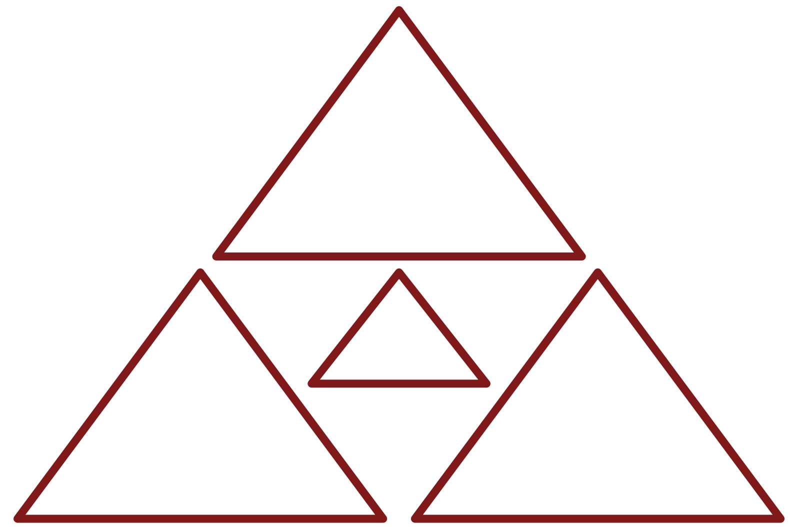
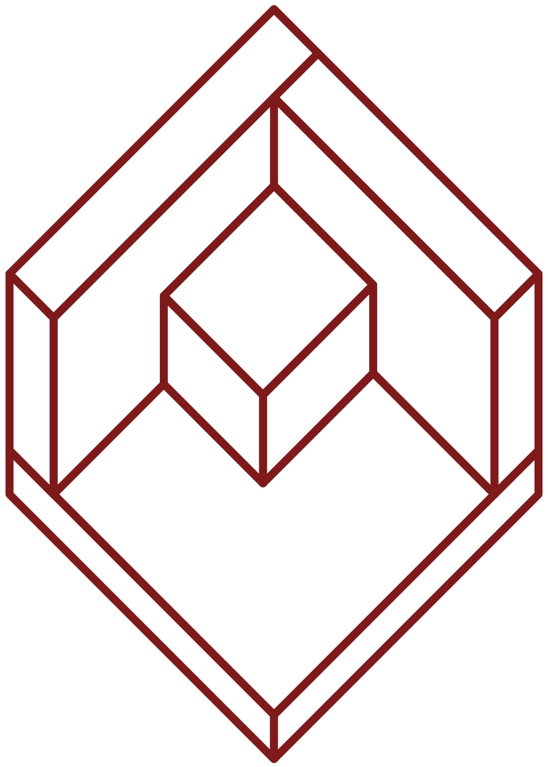




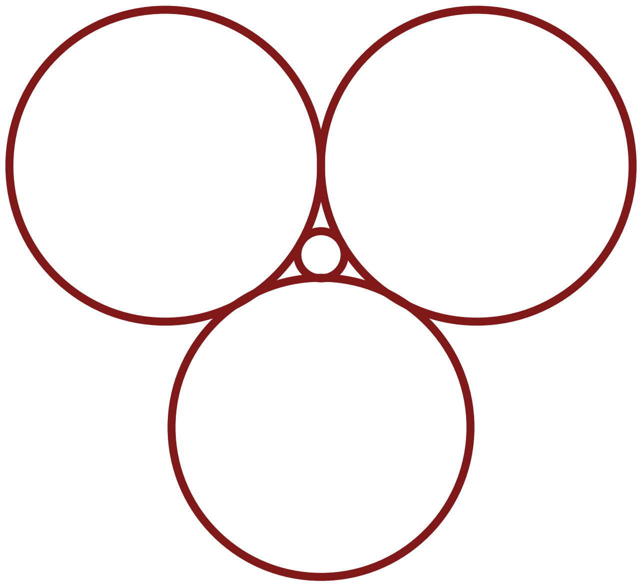
Contact Representation

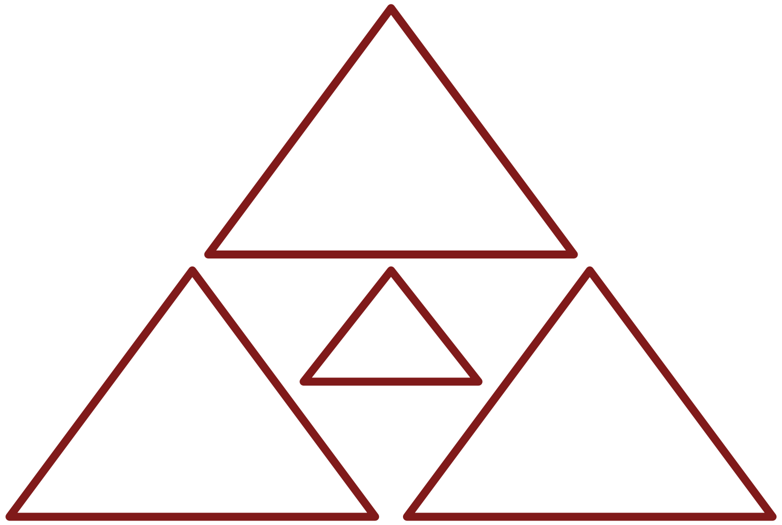





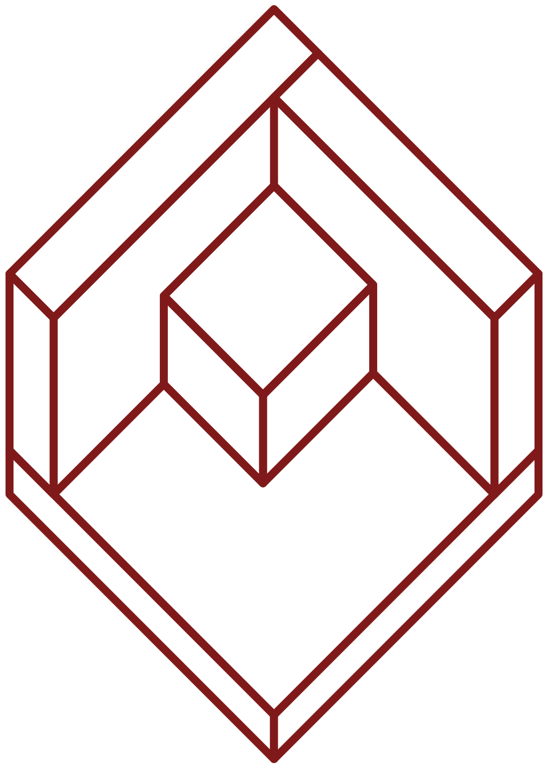
What graphs can be represented?

What graphs can be represented?







Planar Graphs

Kobe 36

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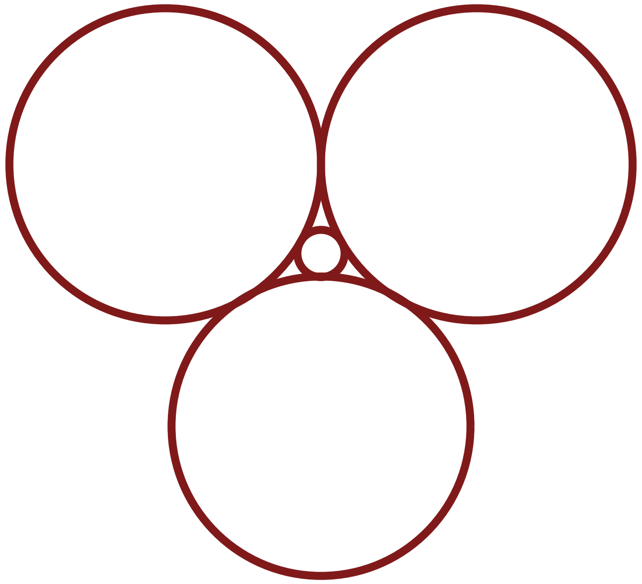
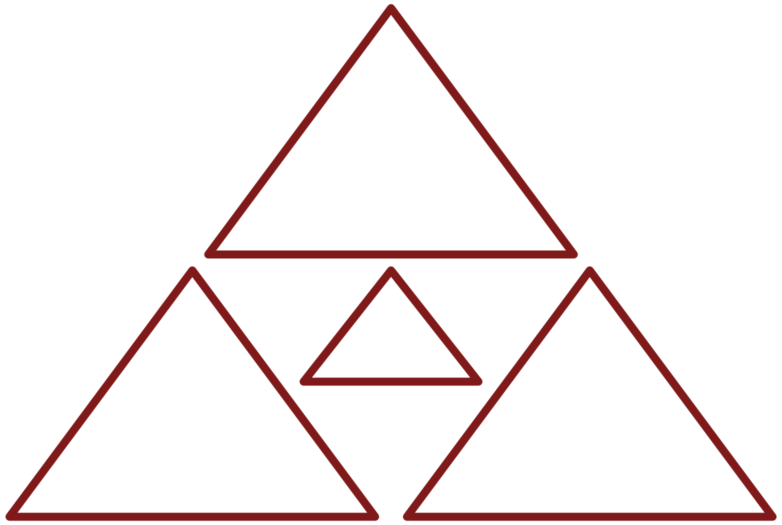


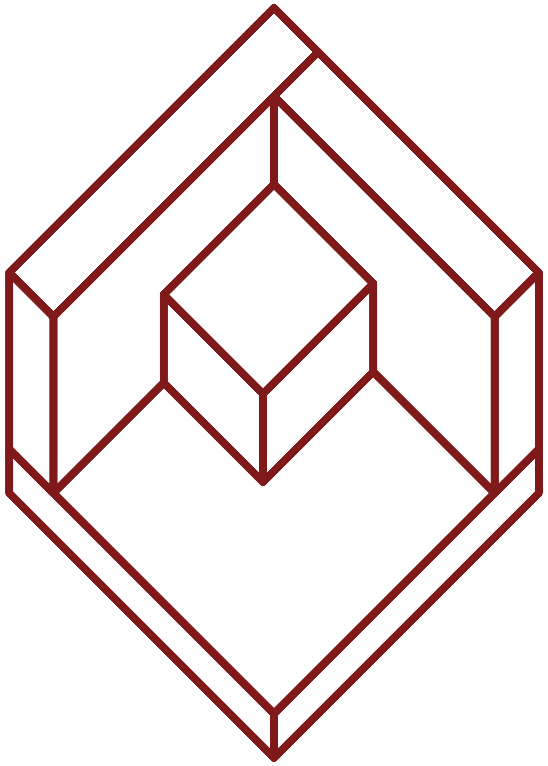
Planar Graphs

Kobe 36



Planar Graphs

De Fraysseix et al. 94



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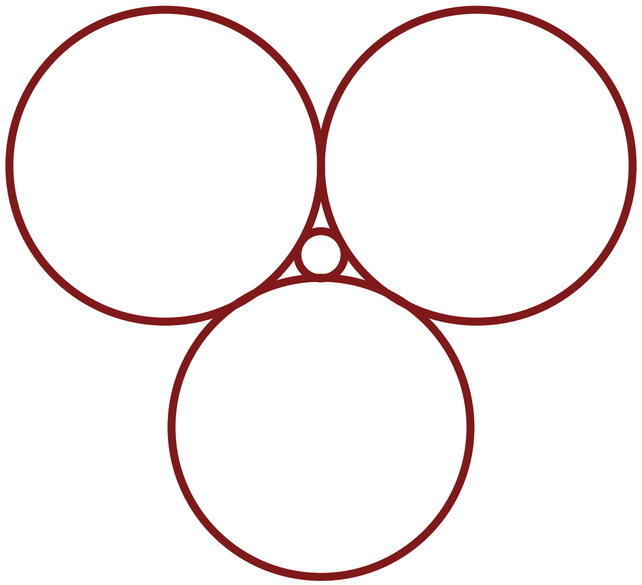
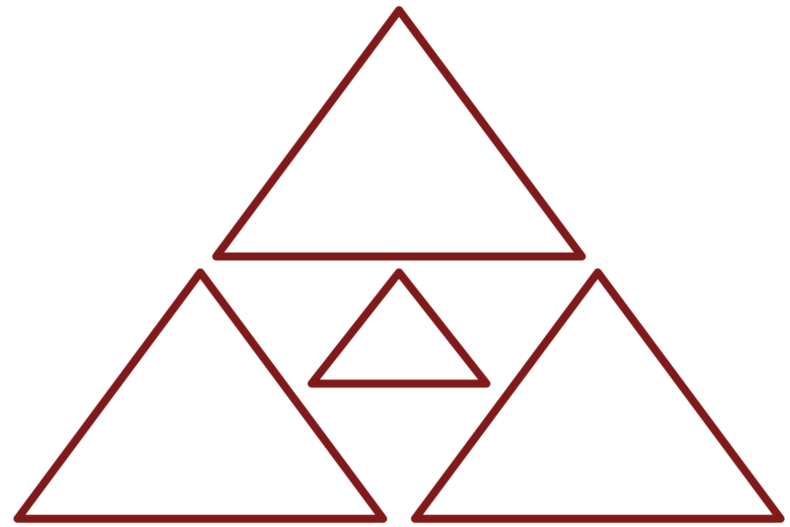
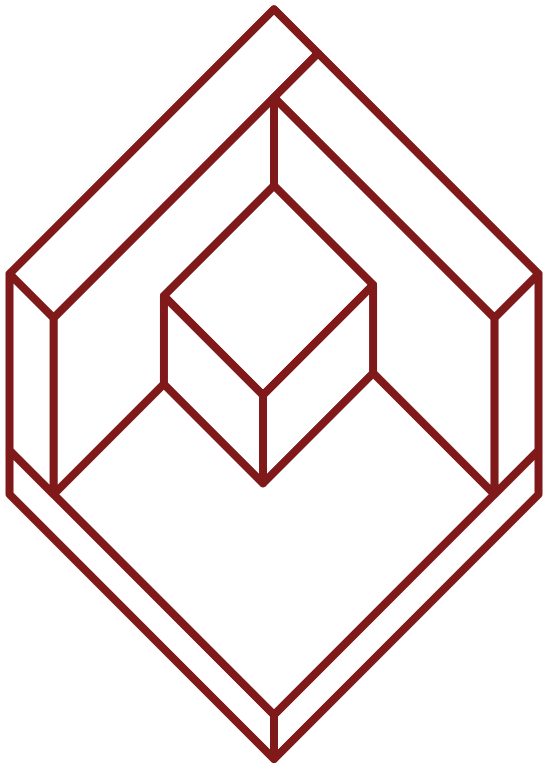


Planar Graphs
Kobe 36

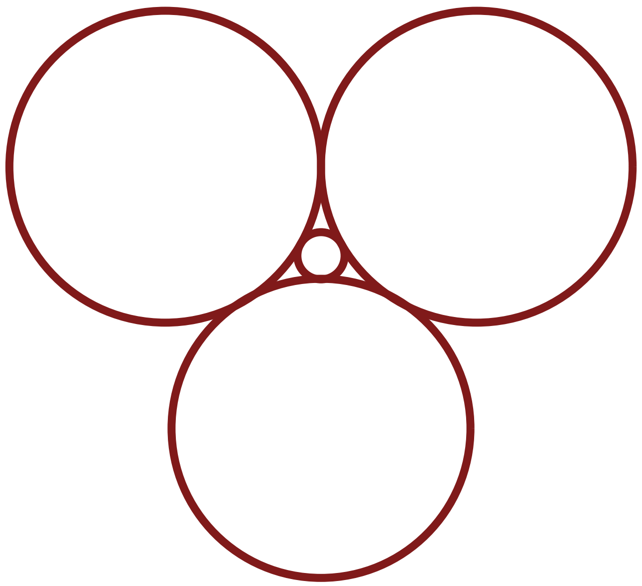


Planar Graphs
De Fraysseix et al. 94

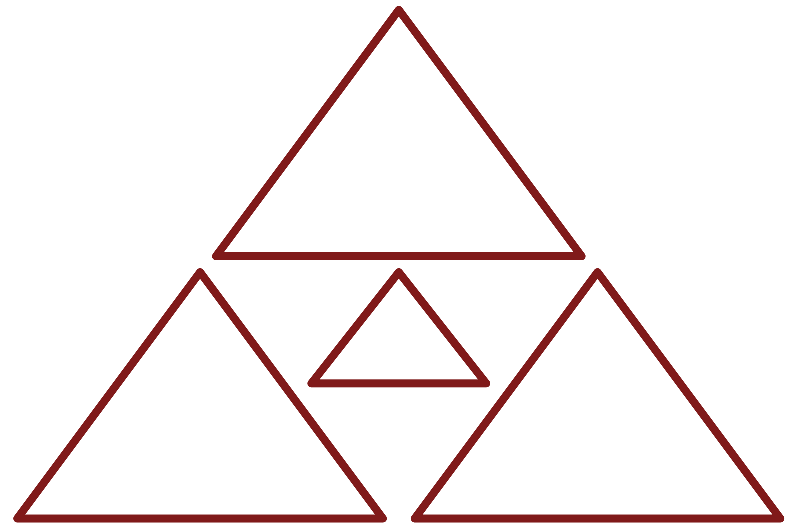


Planar Graphs
Thomassen 86

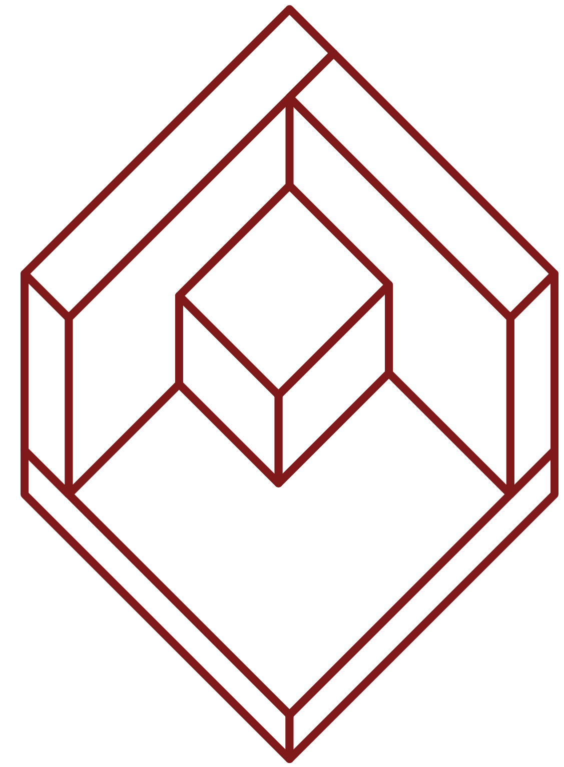
What graphs can be represented?



Planar Graphs
Kobe 36

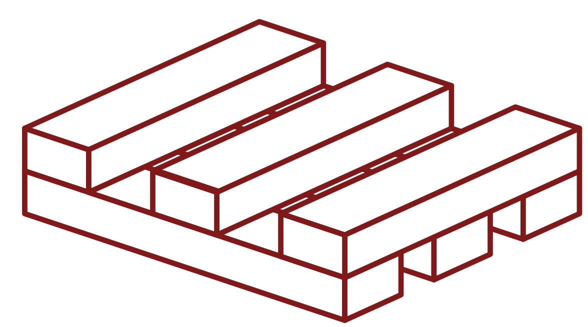


Planar Graphs
De Fraysseix et al. 94

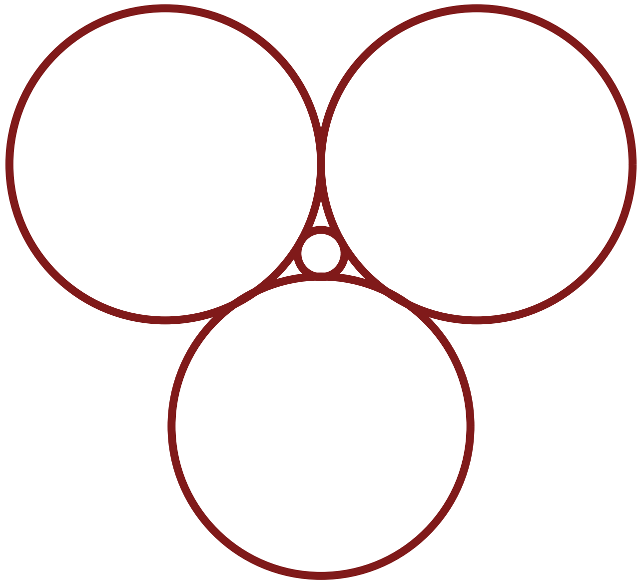


Planar Graphs
Thomassen 86

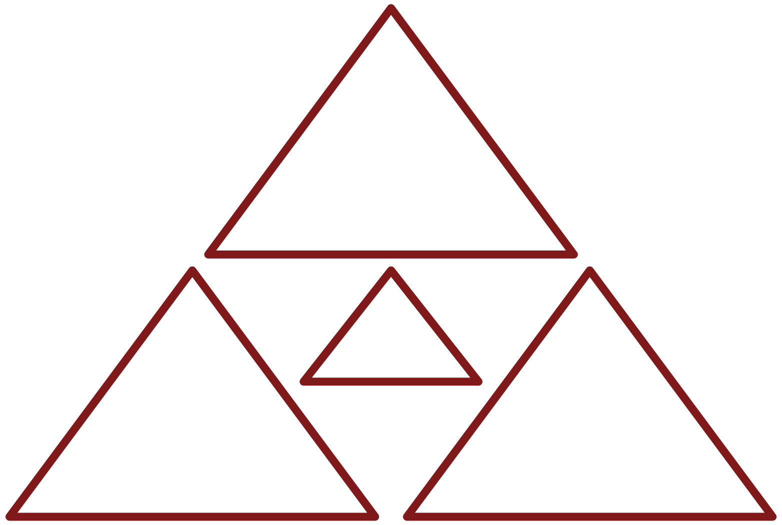
... and more.



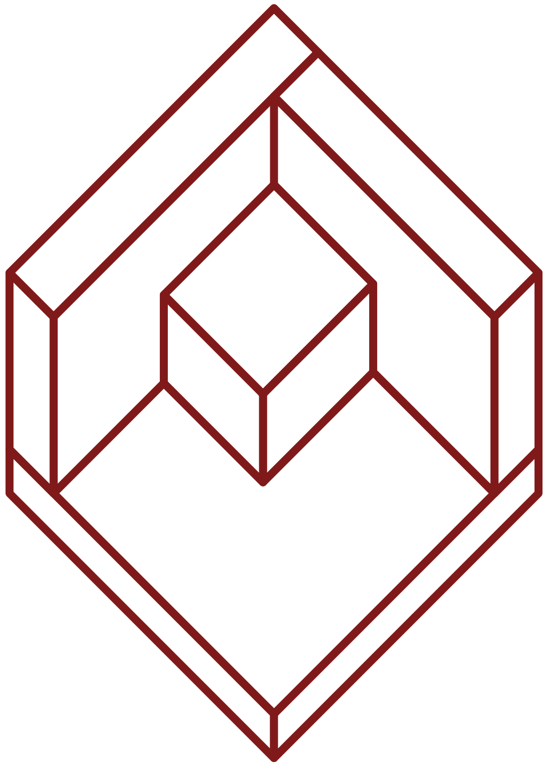
What graphs can be represented?



Planar Graphs
Kobe 36



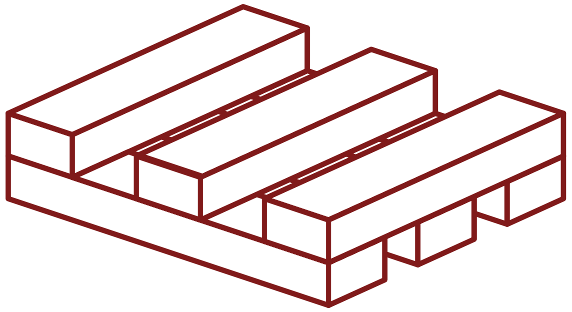
Planar Graphs
De Fraysseix et al. 94



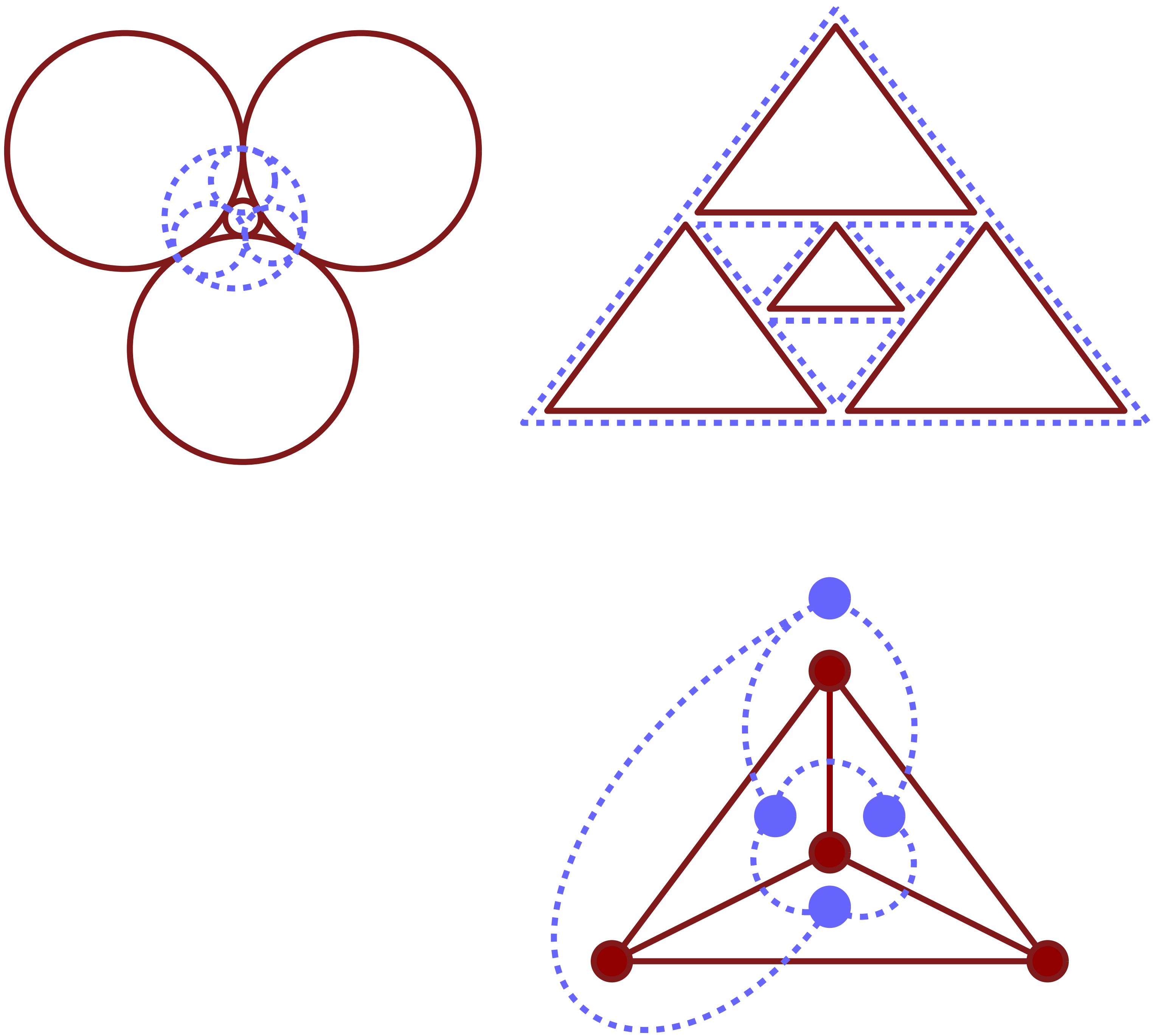
Planar Graphs
Thomassen 86

... and more.

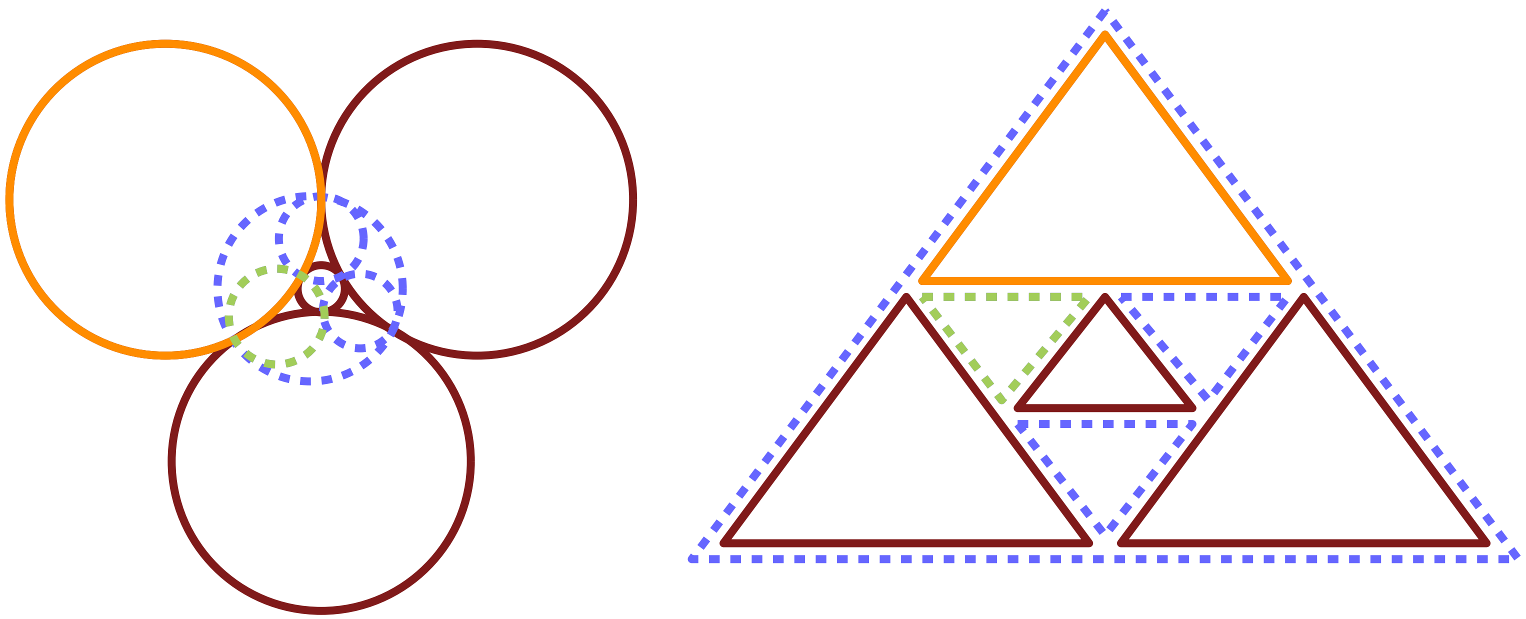
How much more?



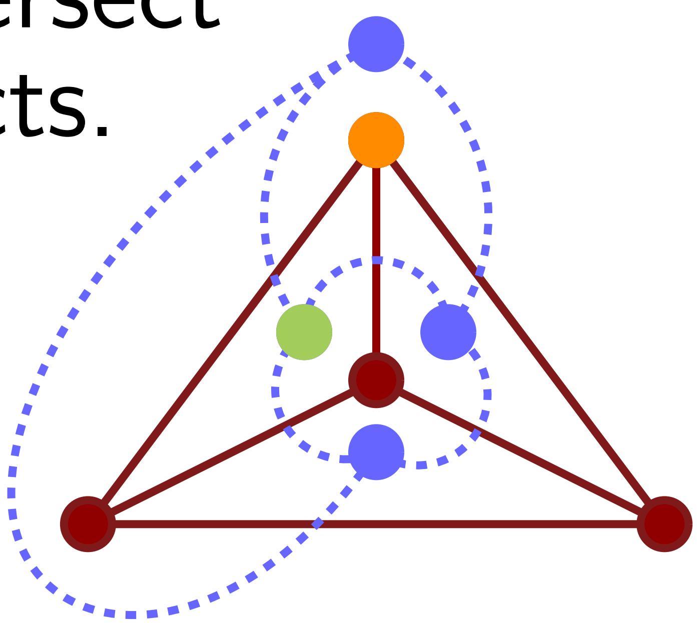
Simultaneous Primal-Dual Contact Representation



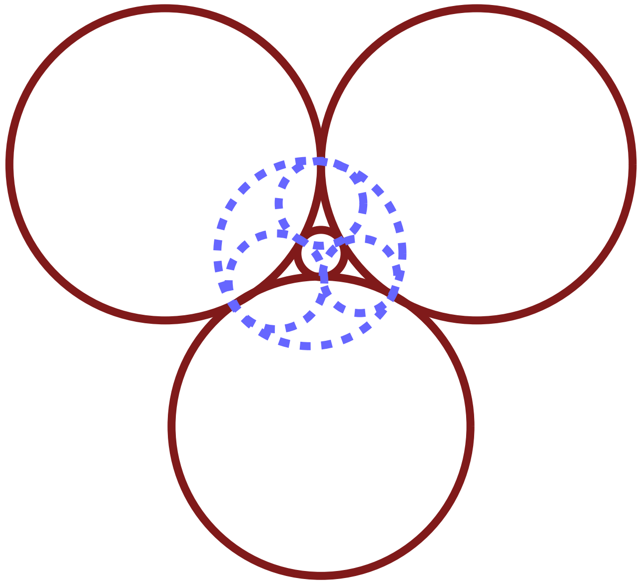
Simultaneous Primal-Dual Contact Representation



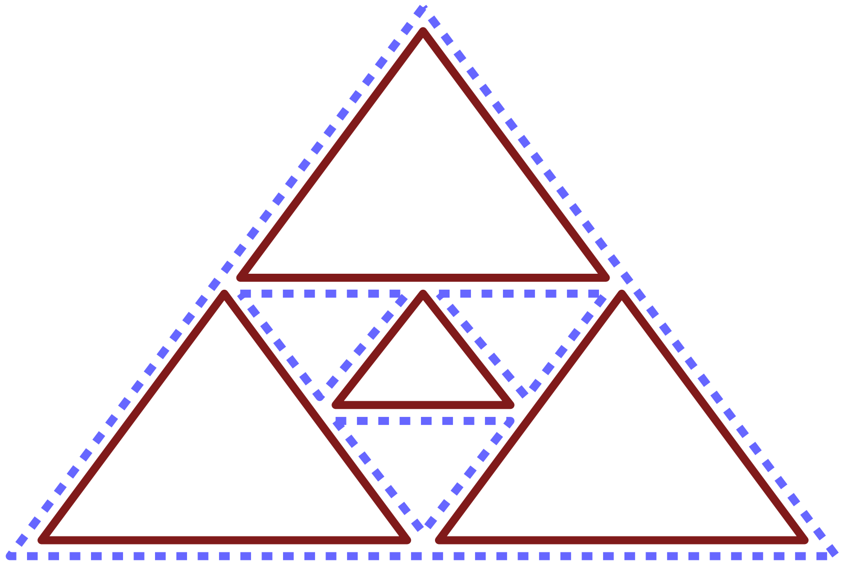
Vertex objects intersect
incident face objects.



Simultaneous Primal-Dual Contact Representation



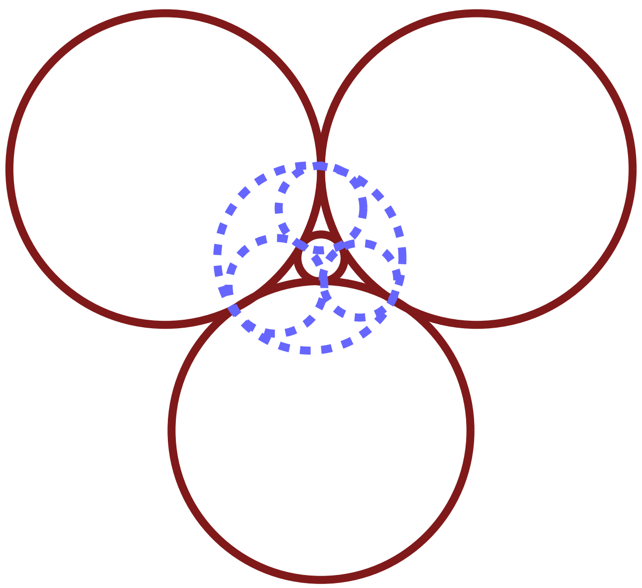
Andreev 70



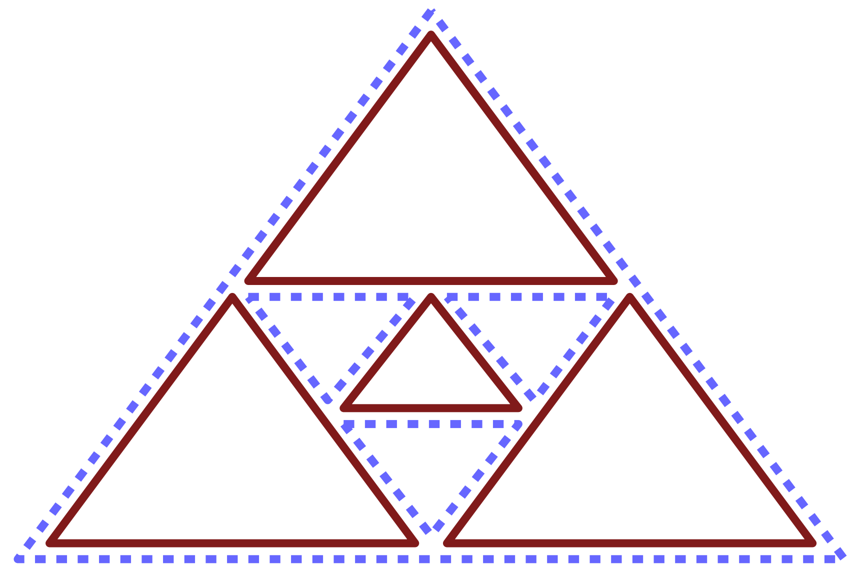
Gonçalves 12

3-connected planar graph & dual

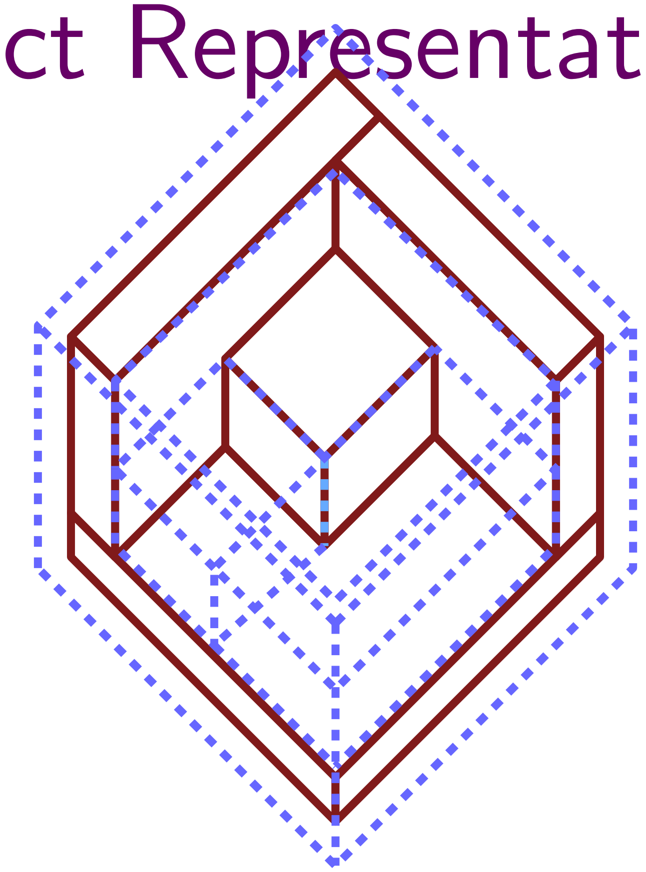
Simultaneous Primal-Dual Contact Representation



Andreev 70



Gonçalves 12

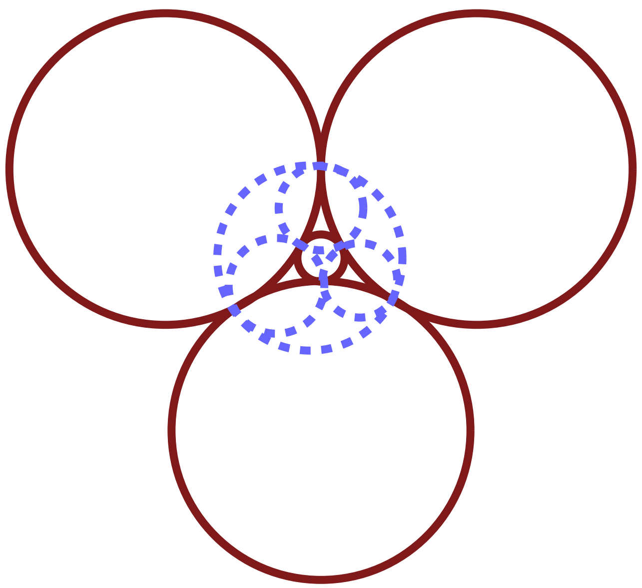


This paper

3-connected planar graph & dual

Thm 1 Every 3-connected planar graph admits a proper primal-dual 3D box-contact representation.

Simultaneous Primal-Dual Contact Representation



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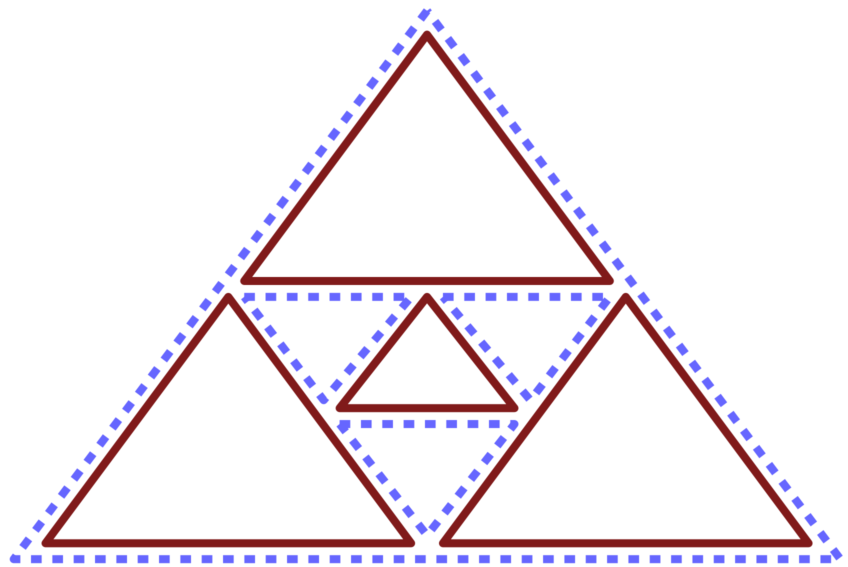
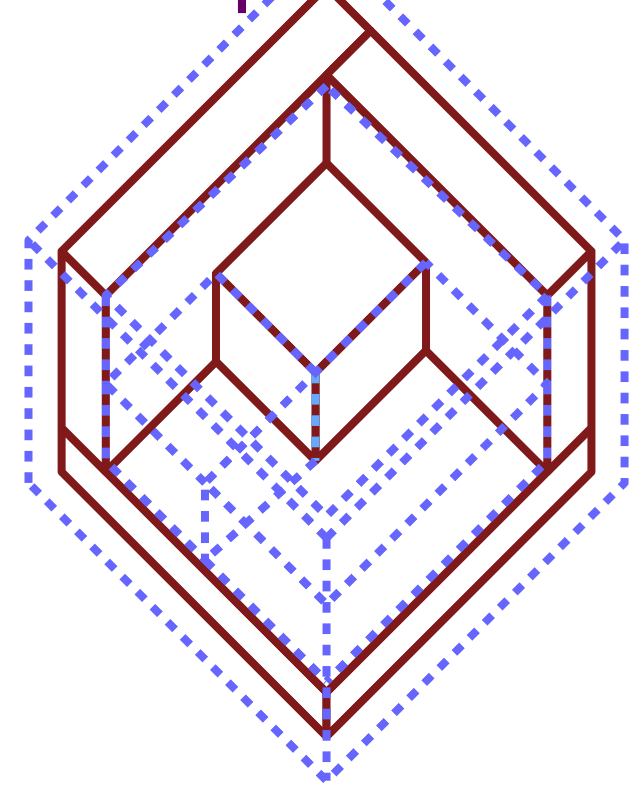


Gonçalves 12



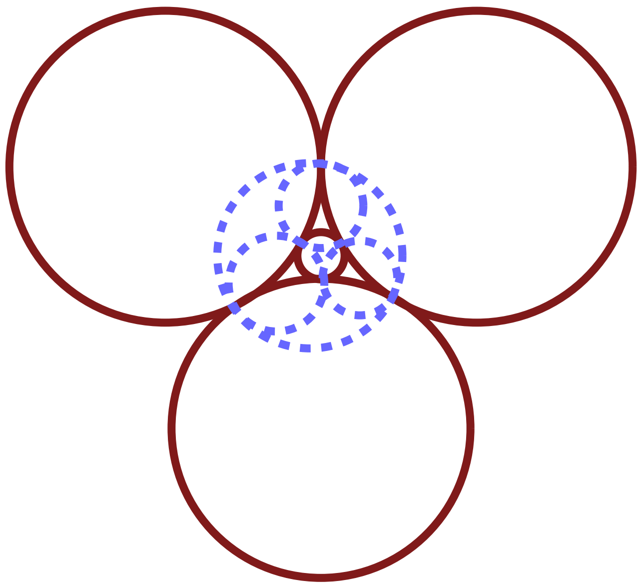
This paper

3-connected planar graph & dual

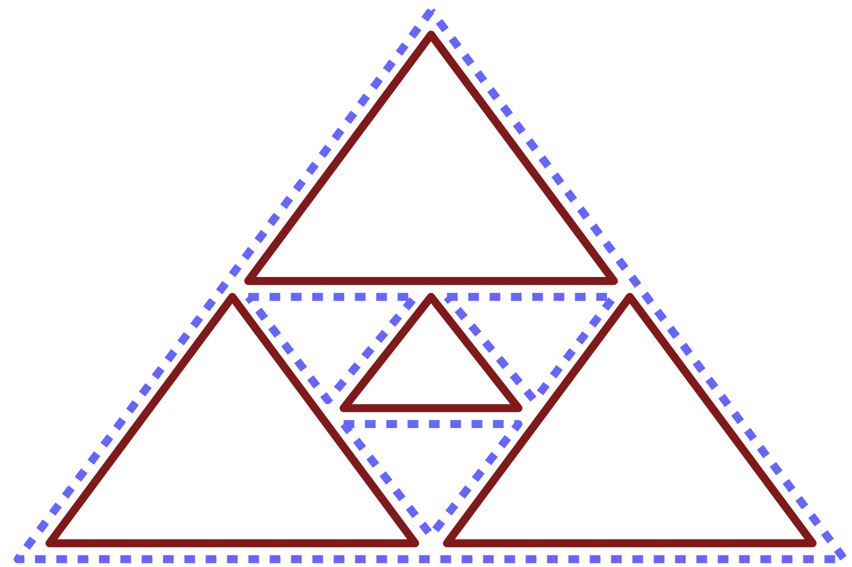
Thm 1 Every 3-connected planar graph admits a proper primal-dual 3D box-contact representation.

proper: face-to-face contact

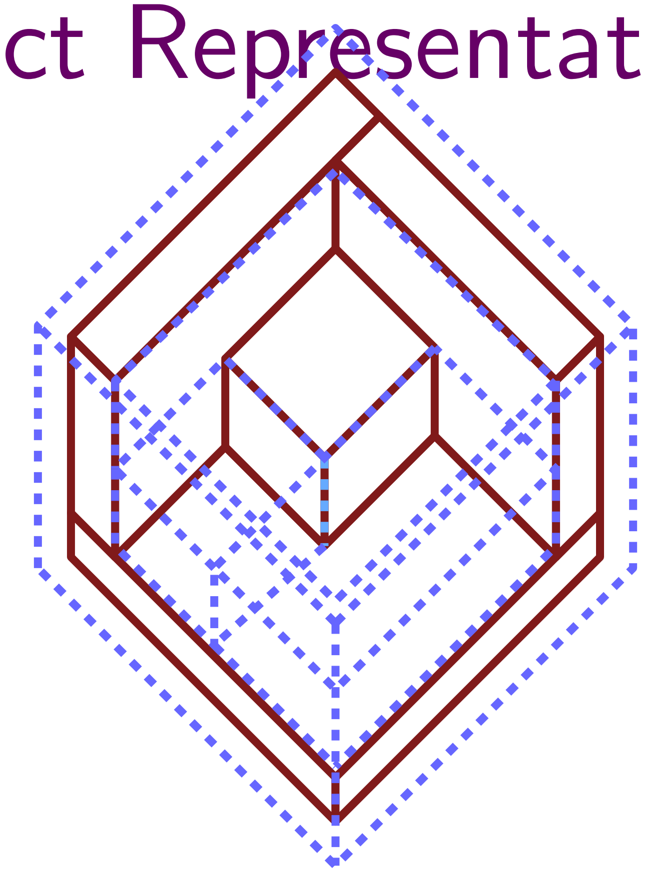
Simultaneous Primal-Dual Contact Representation



Andreev 70



Gonçalves 12



This paper

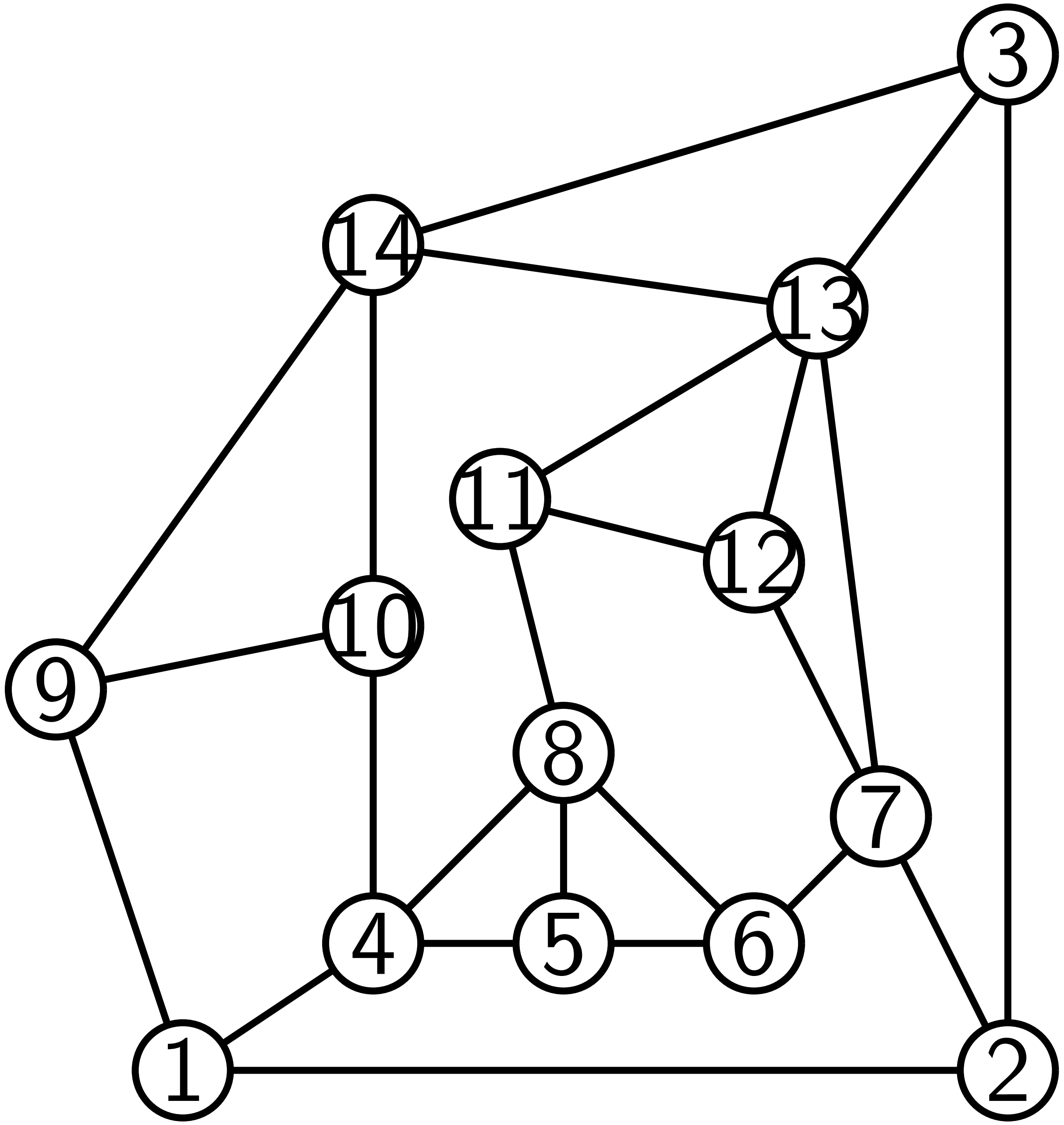
3-connected planar graph & dual

Thm 1 Every 3-connected planar graph admits a proper primal-dual 3D box-contact representation.

And it can be computed in linear time.

Schnyder Wood

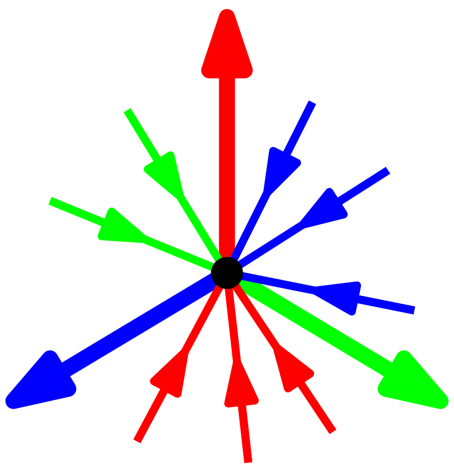
Edge orientation and coloring of 3-connected planar graph using 3 colors so that



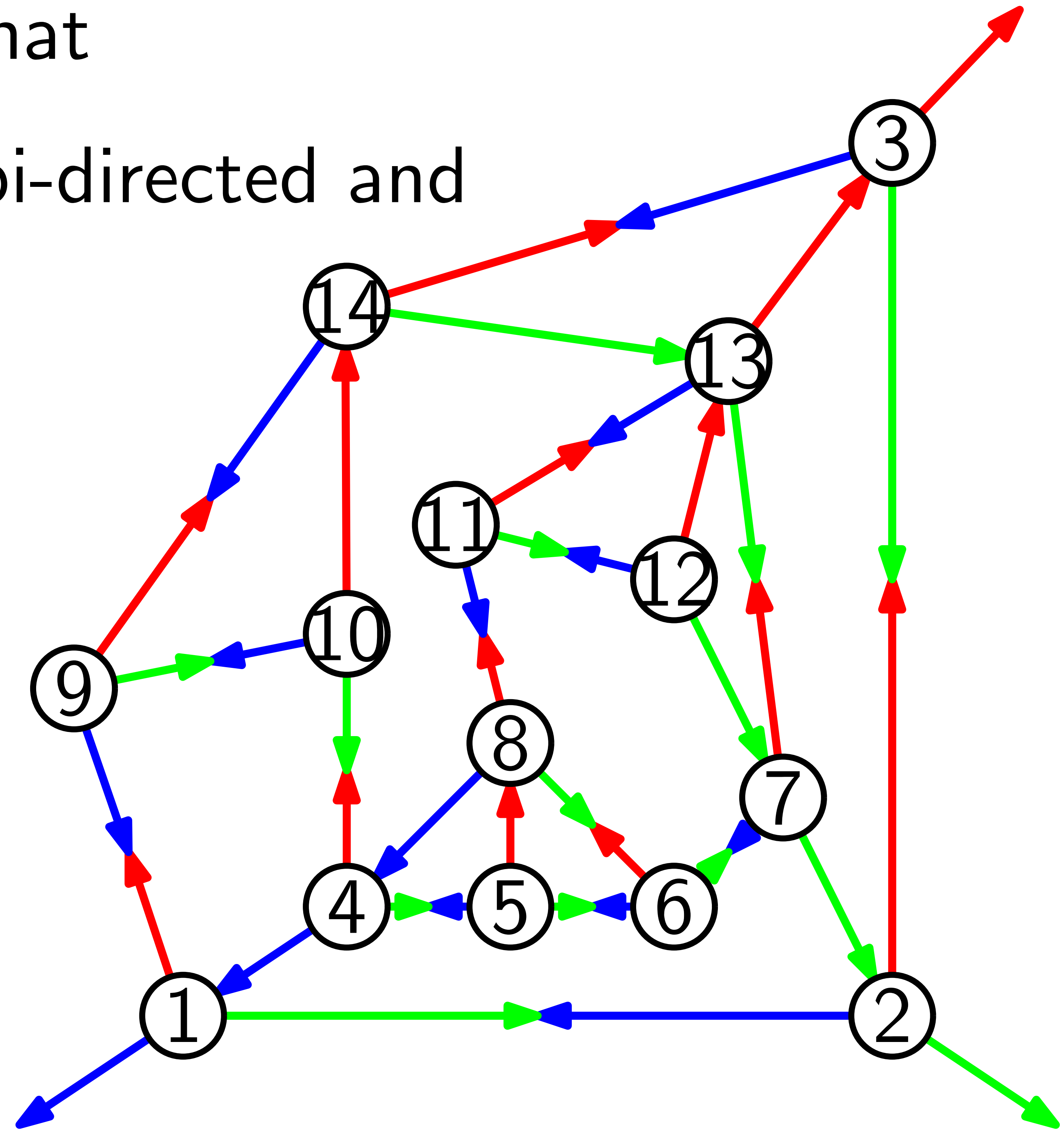
Schnyder Wood

Edge orientation and coloring of 3-connected planar graph using 3 colors so that

1. Every edge is uni- or bi-directed and each direction colored.

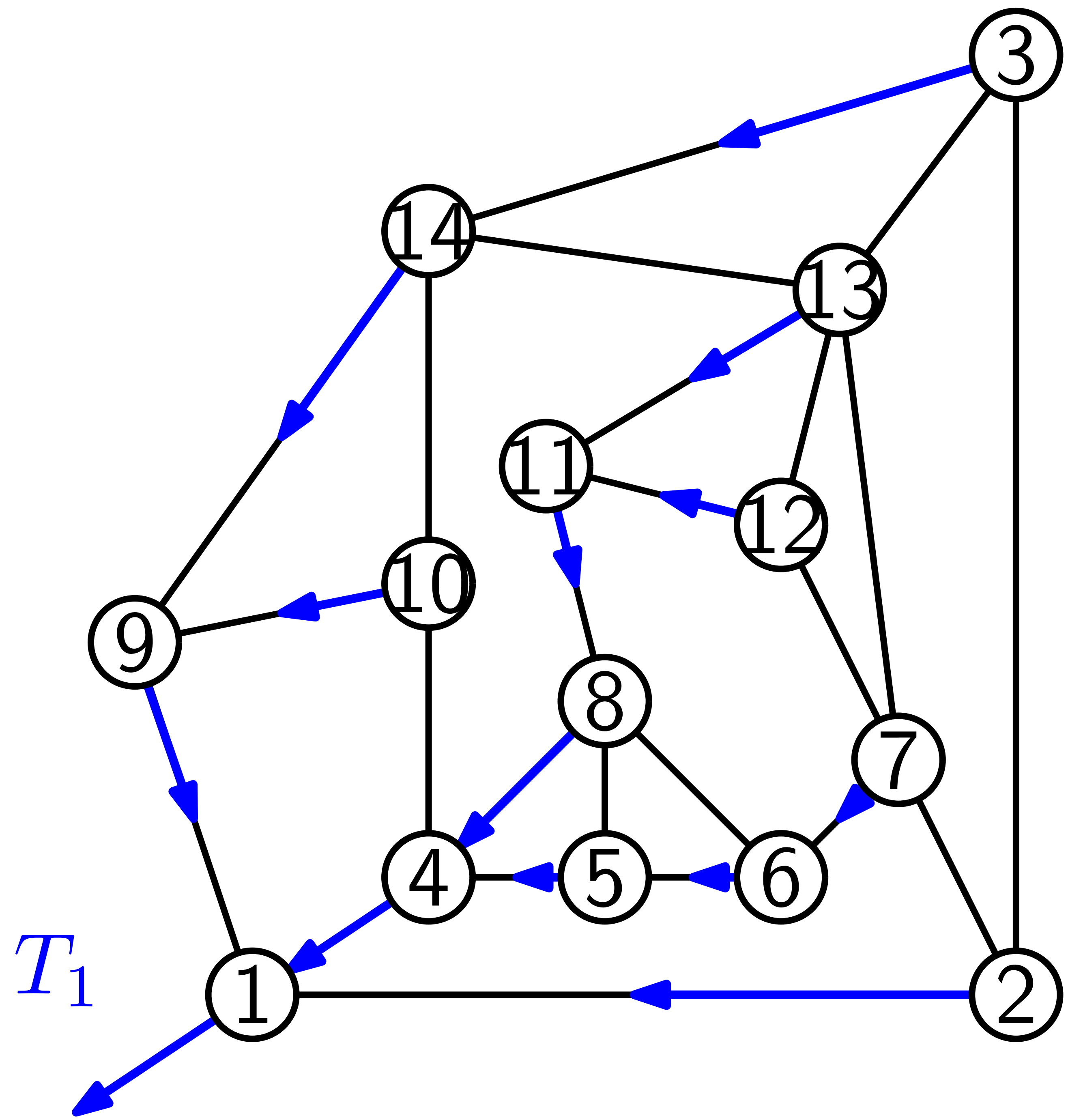
2. 

3. No cycle in one color.



Schnyder Wood

Each color class forms a tree.



Schnyder Wood

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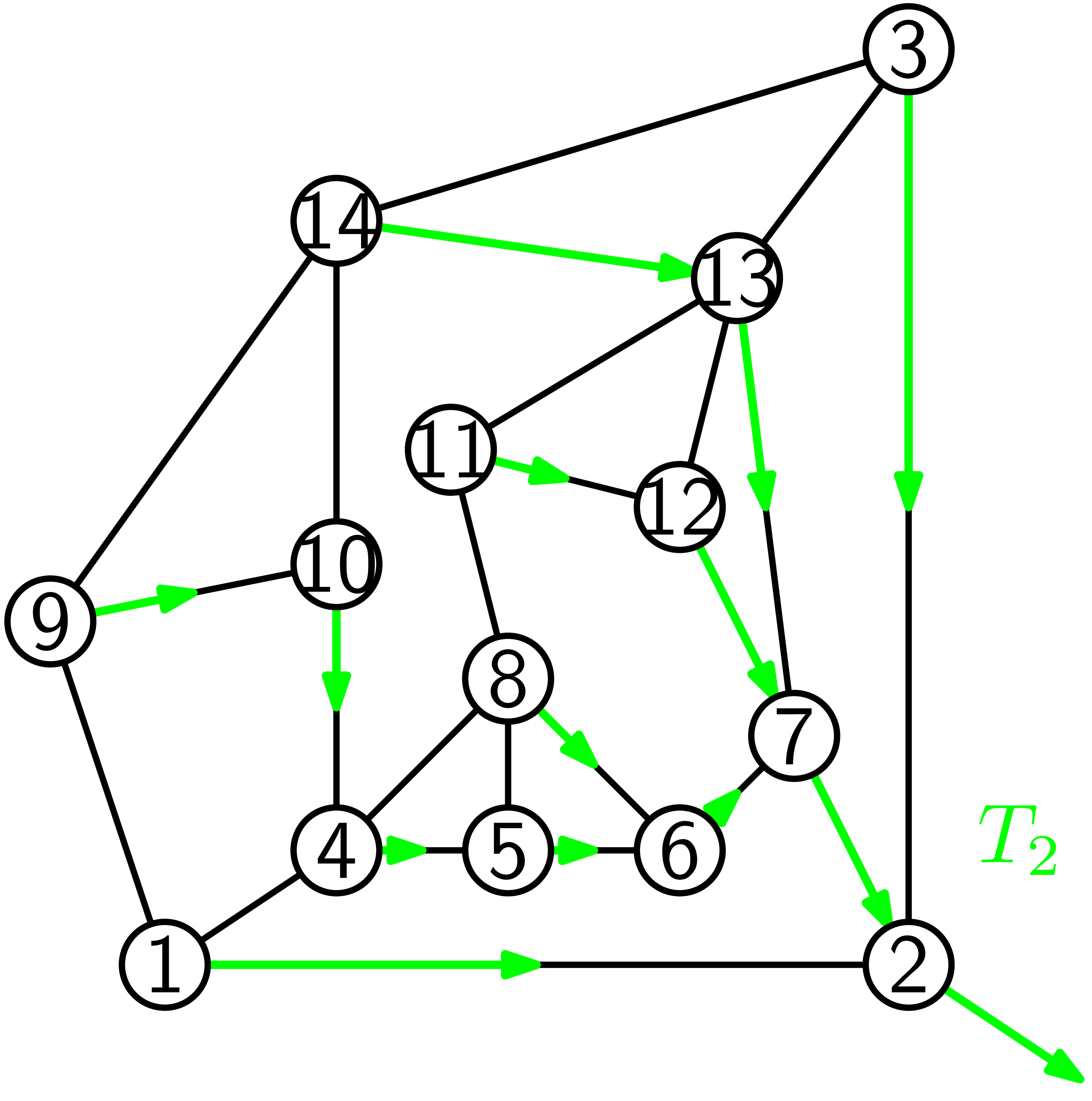

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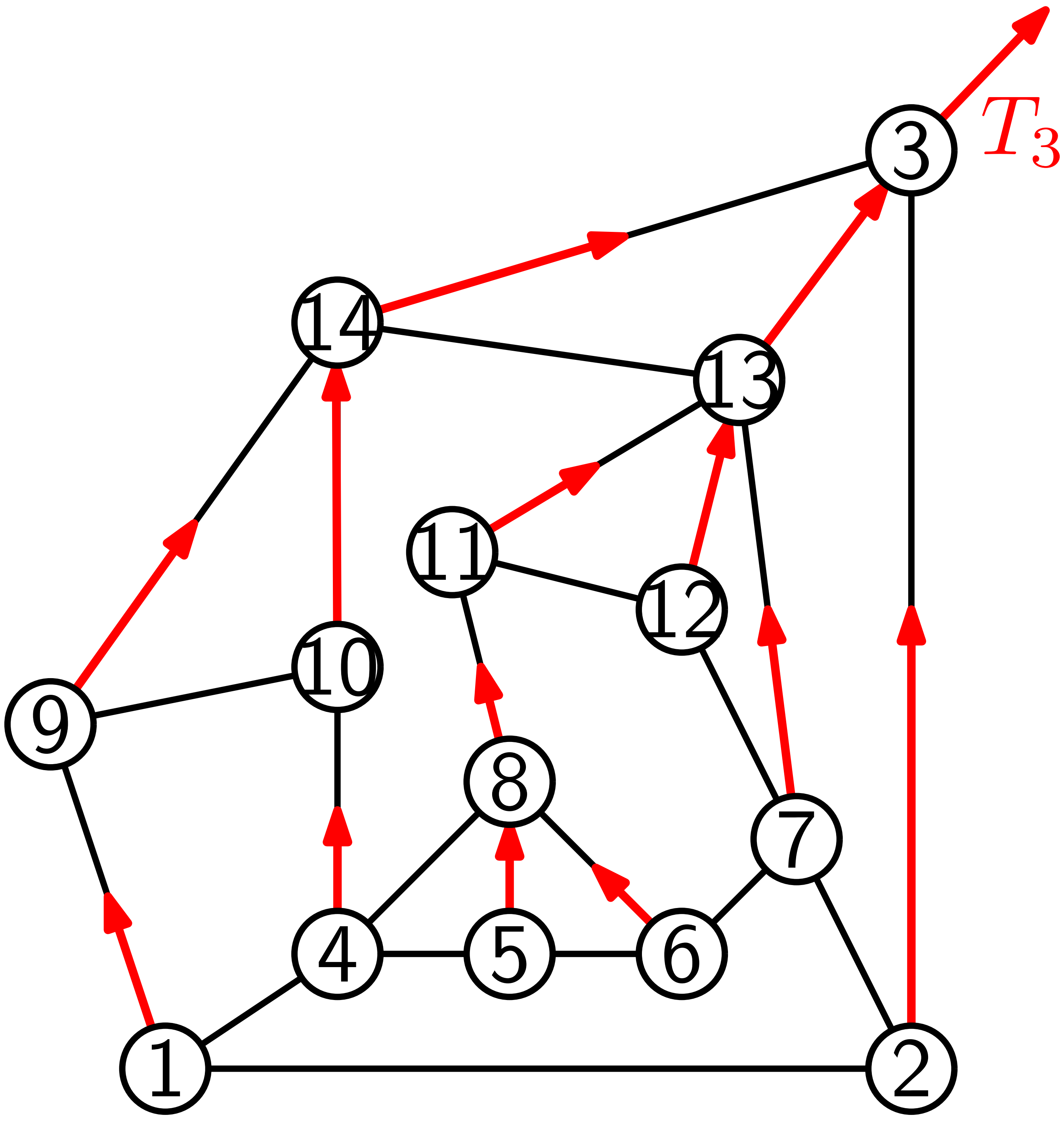
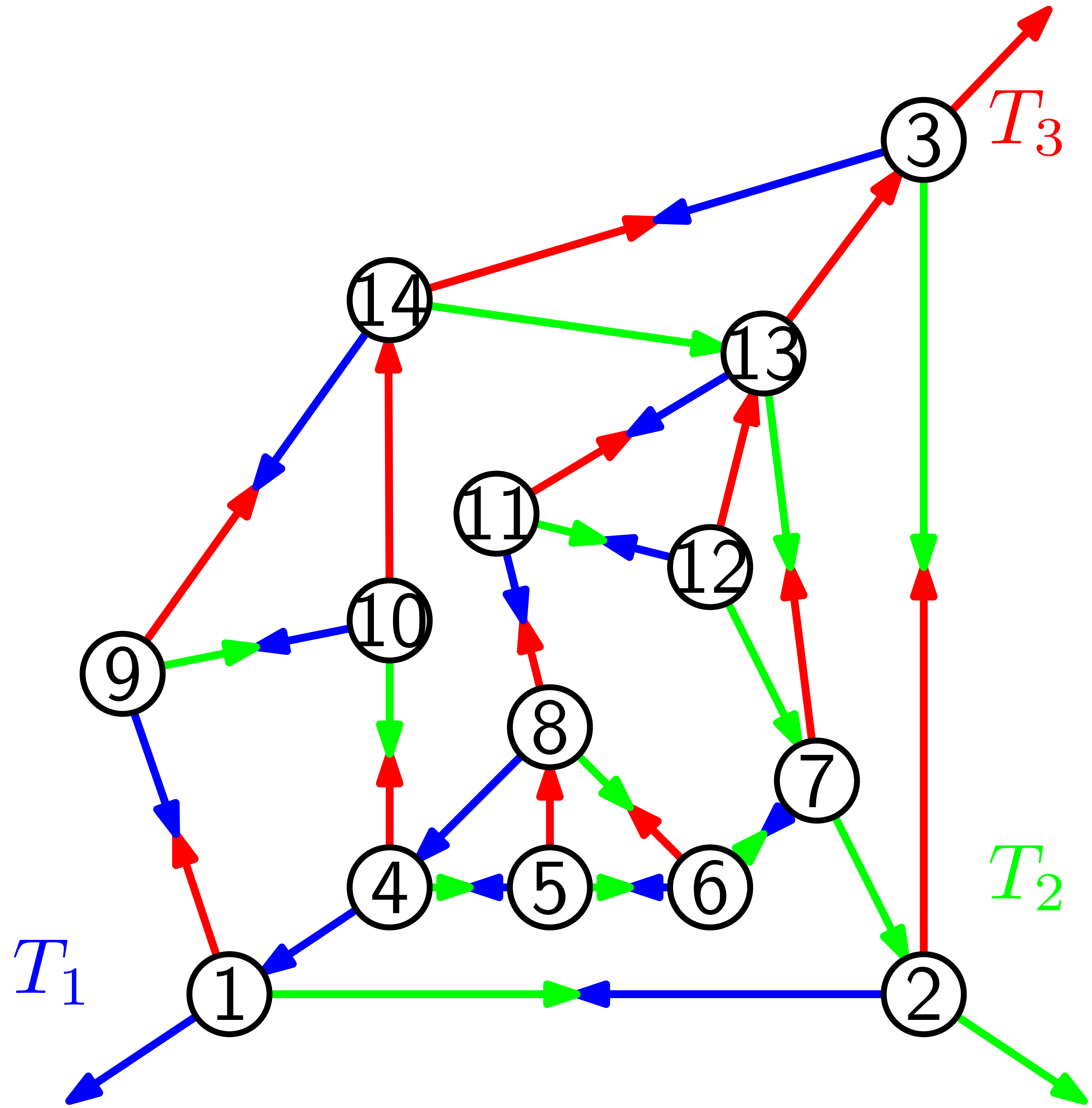


Schnyder Wood and Ordered Path Partition

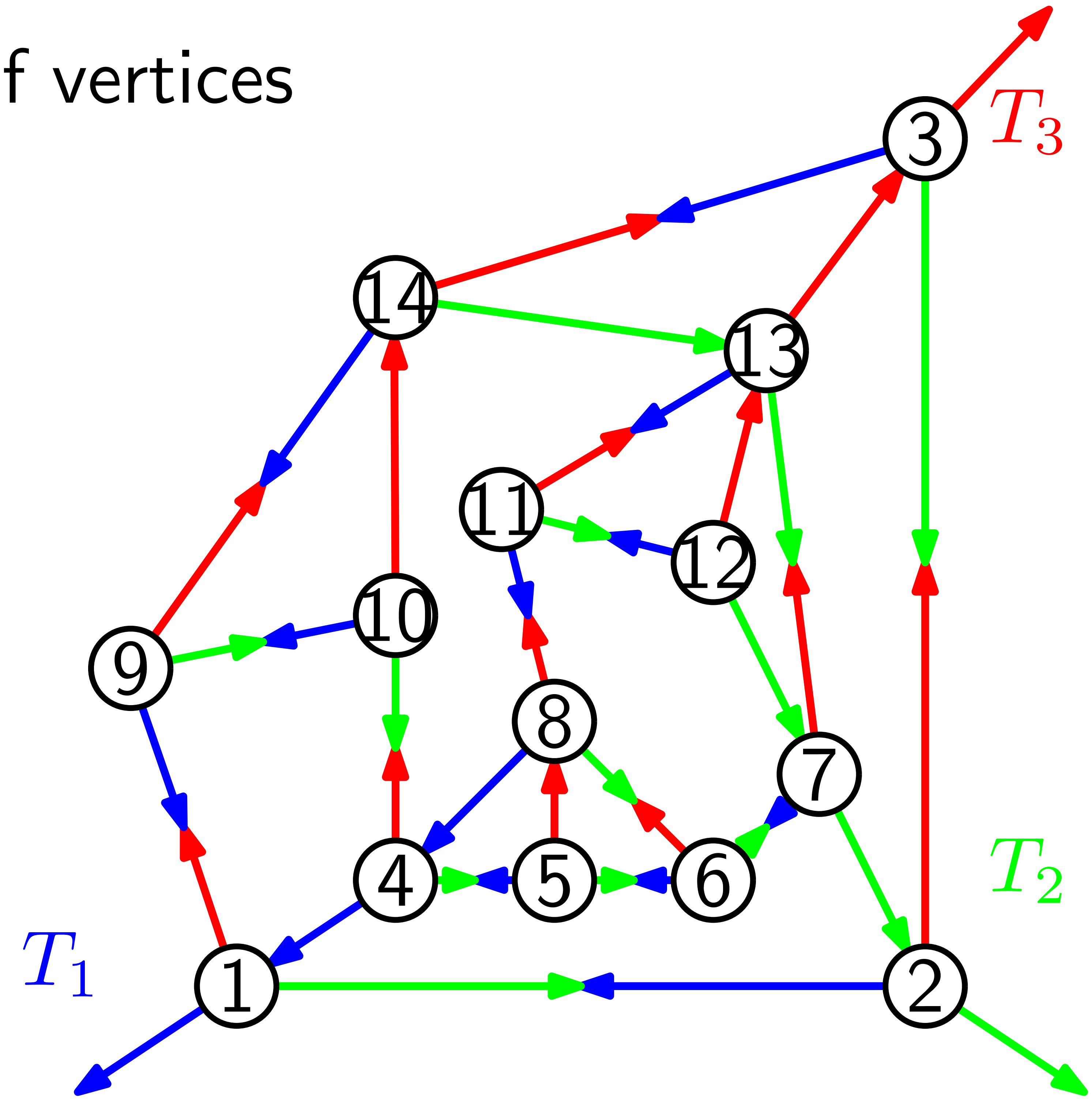
Partition graph into paths according to T_1 T_2 T_3



Schnyder Wood and Ordered Path Partition

Partition graph into paths according to T_1 T_2 T_3

Partially order groups of vertices to respect T_1^{-1} T_2^{-1} T_3



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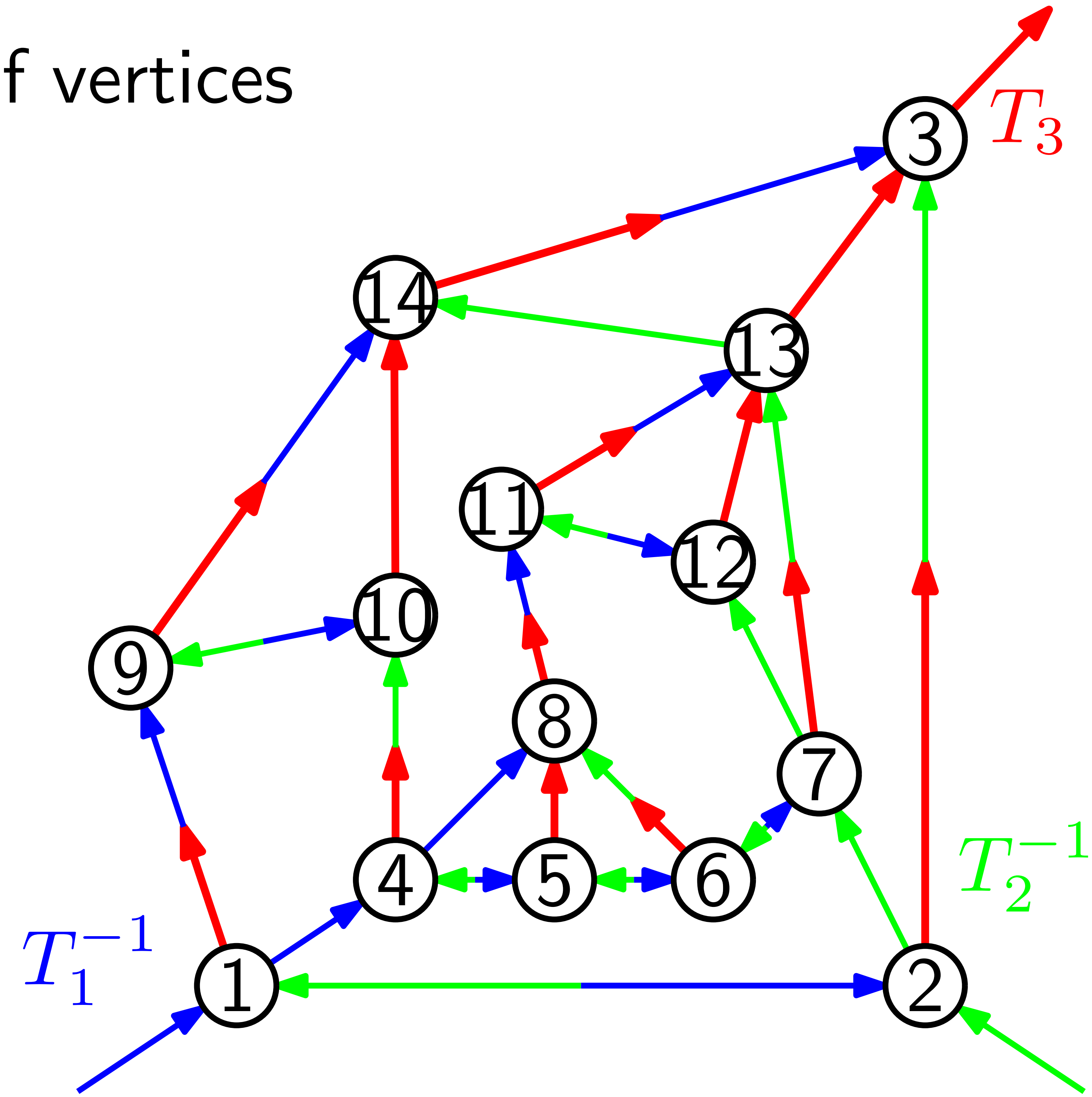
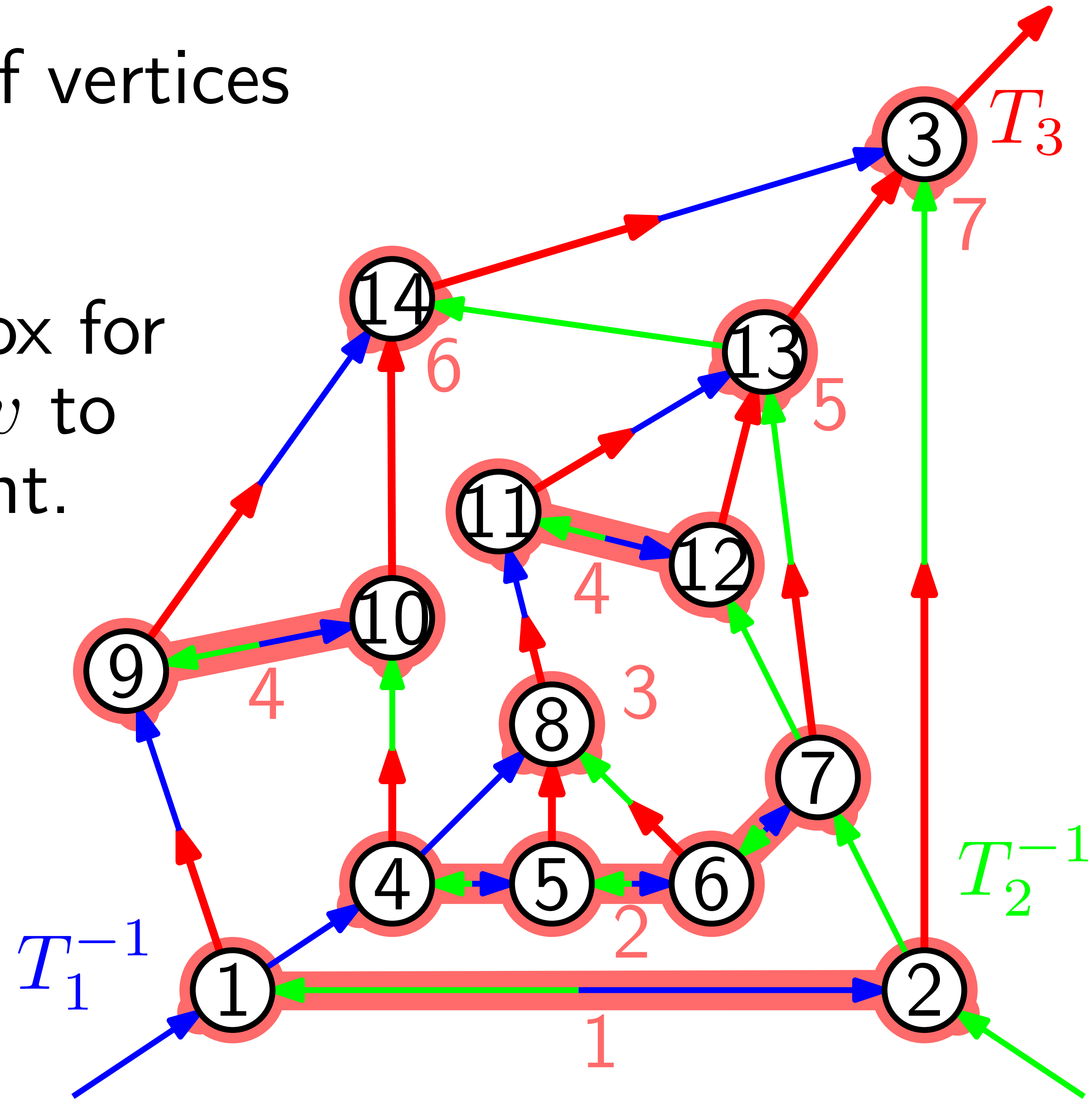


Schnyder Wood and Ordered Path Partition

Partition graph into paths according to T_1 T_2 T_3

Partially order groups of vertices to respect T_1^{-1} T_2^{-1} T_3

The z-interval of the box for vertex v is the level of v to the level of v's T_3 parent.



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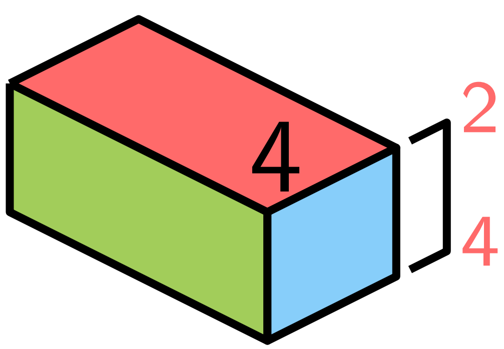
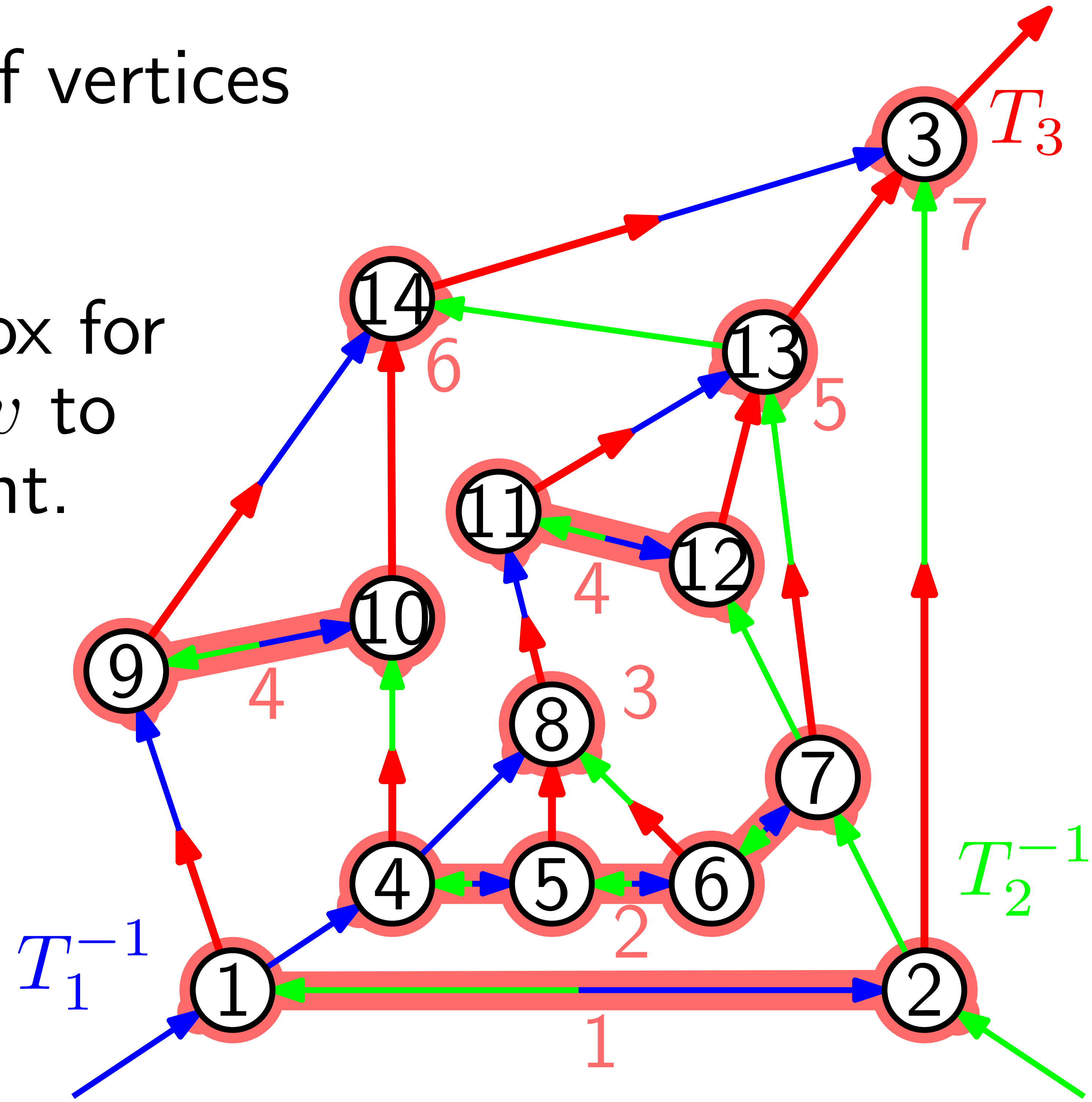
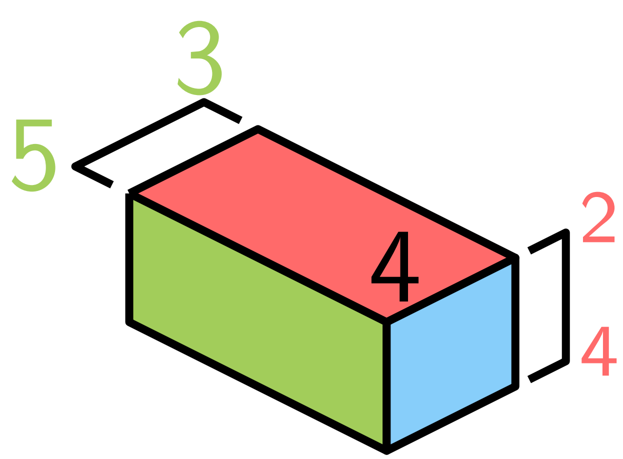


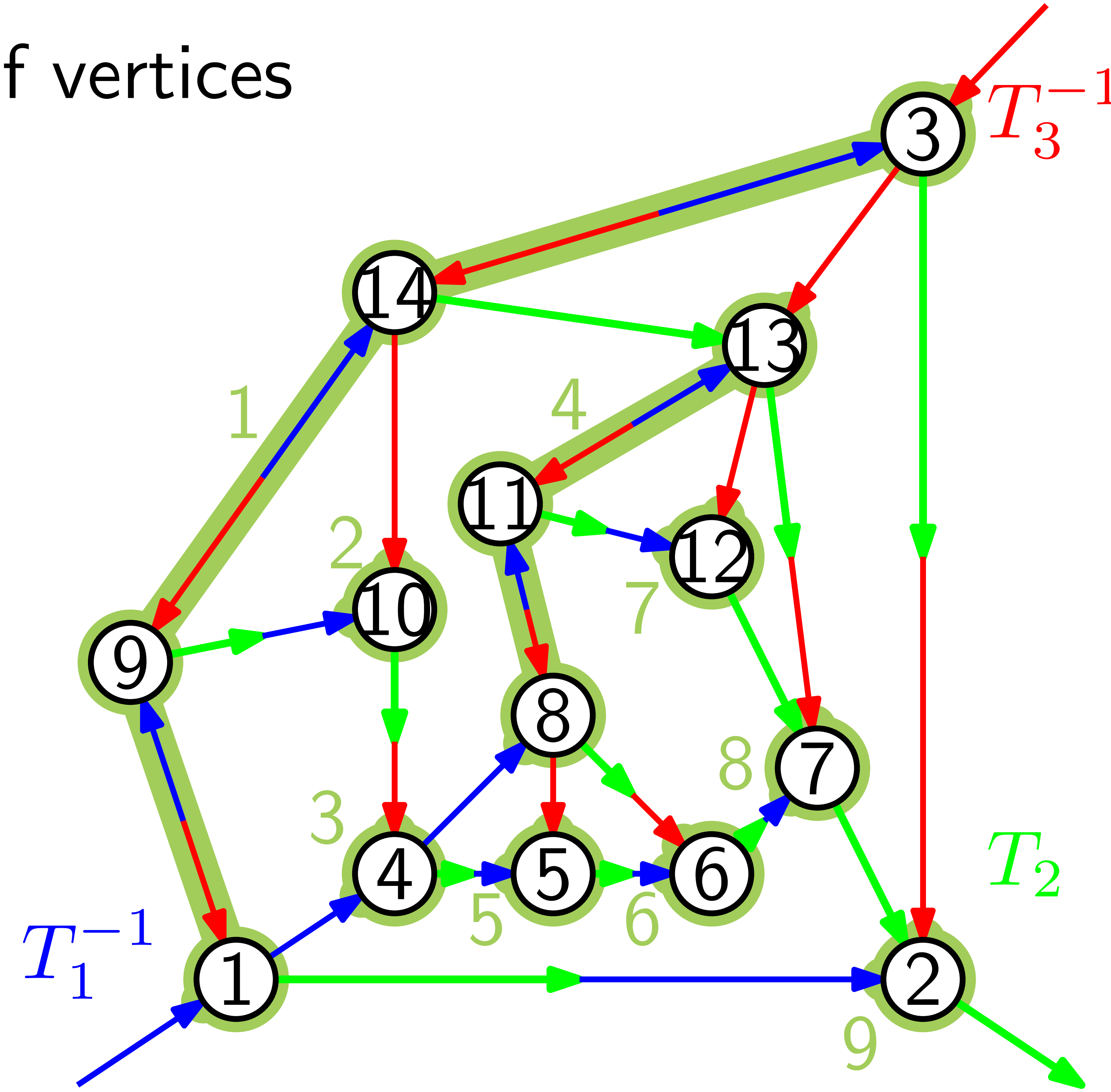


Schnyder Wood and Ordered Path Partition

Partition graph into paths according to T_1 T_2 T_3

Partially order groups of vertices to respect T_1^{-1} T_2 T_3^{-1}





Schnyder Wood and Ordered Path Partition

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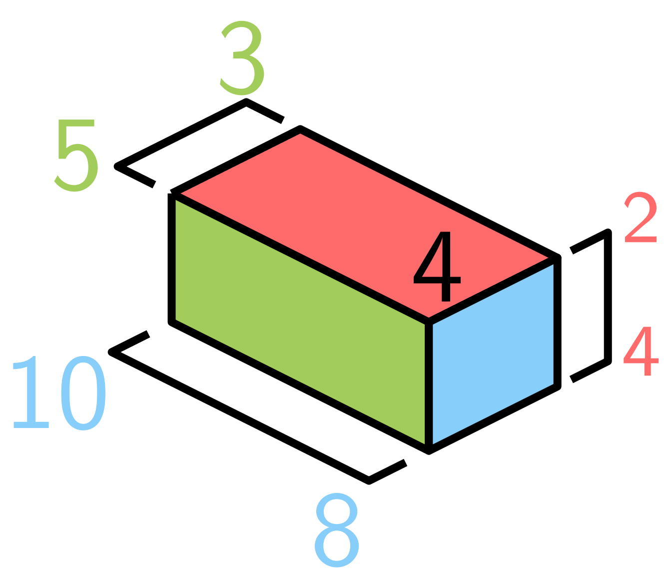
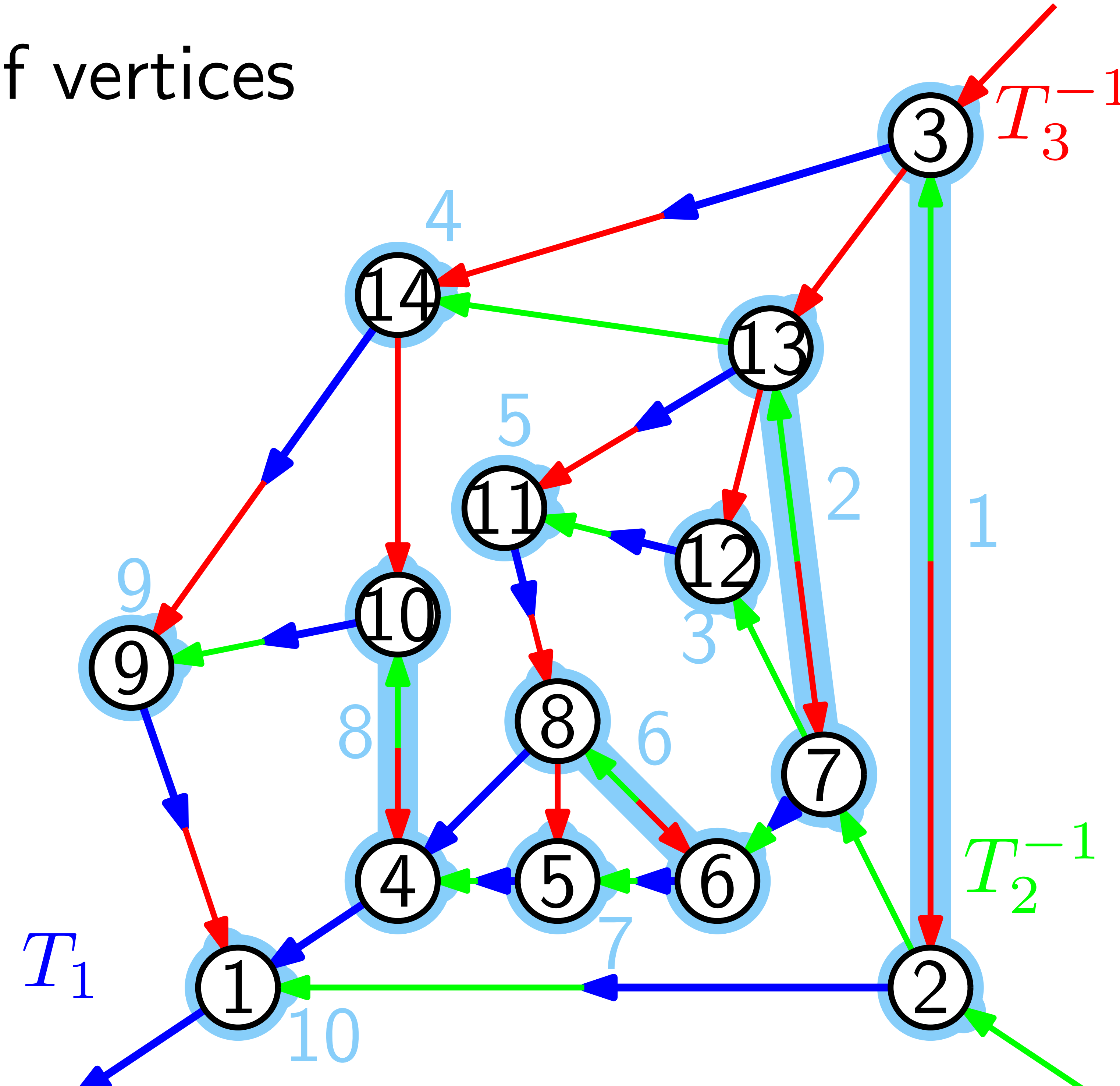
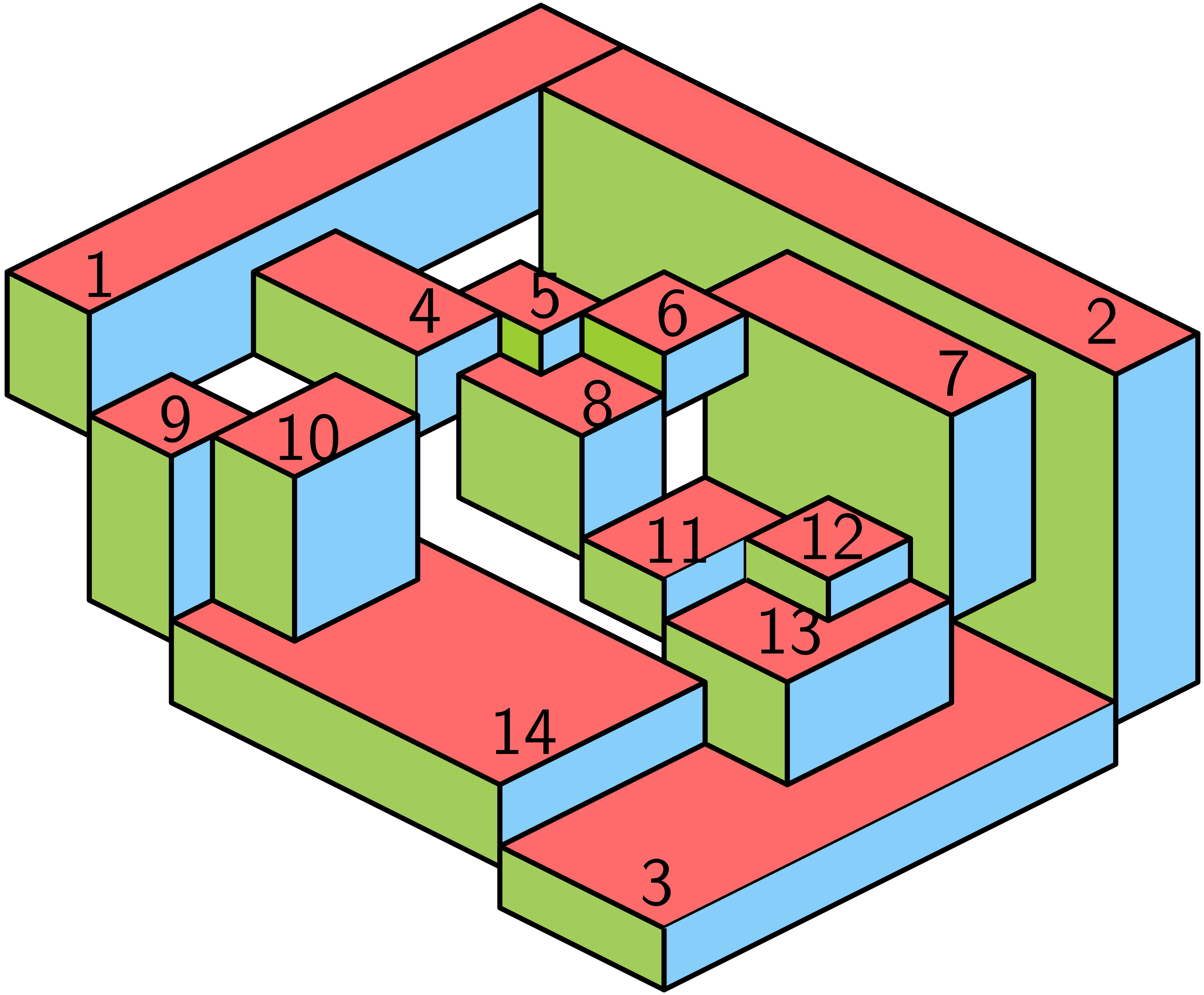


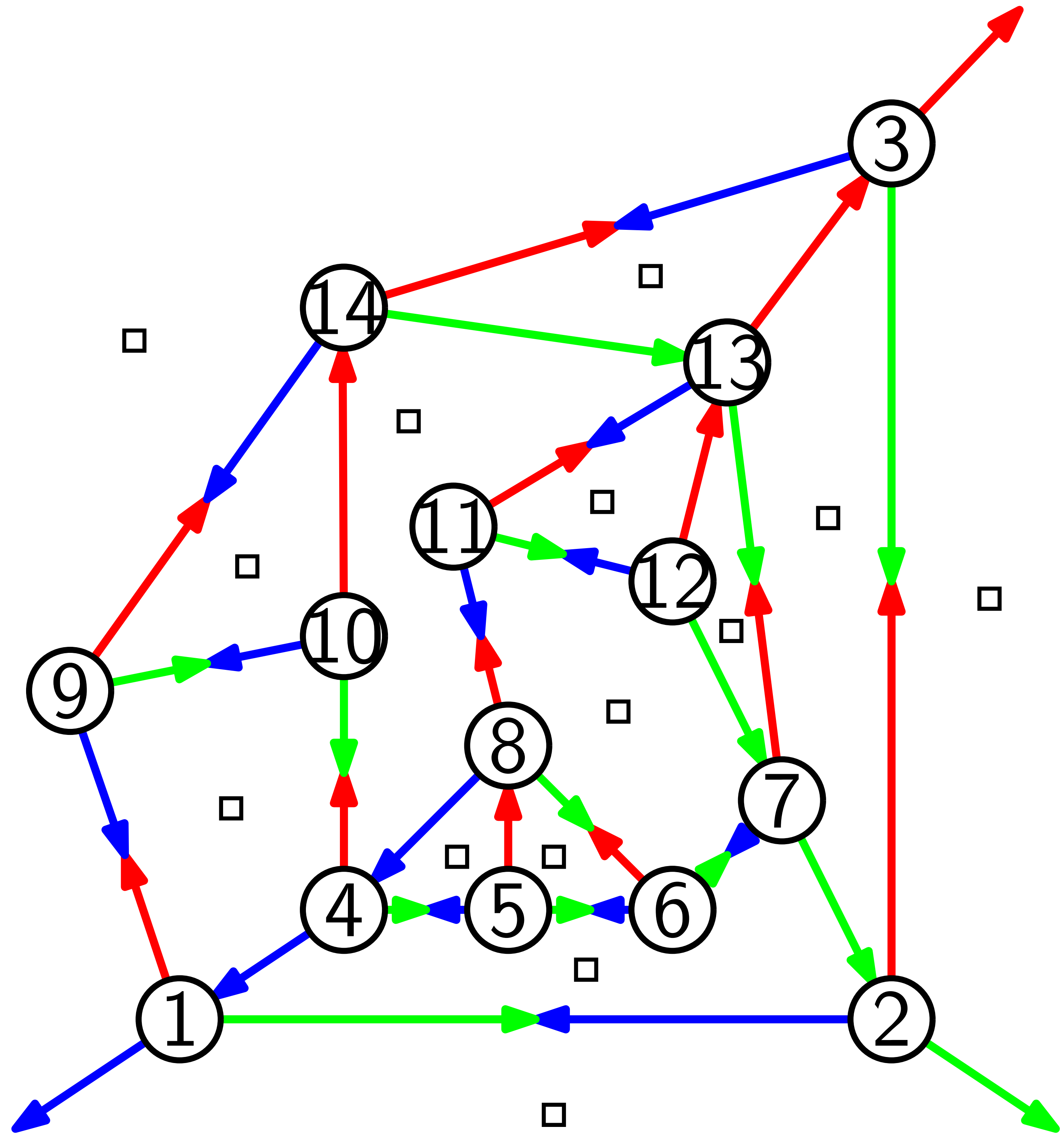


Box Contact Representation



Compatible Dual Schnyder Wood

Between an edge and its dual, all 3 colors appear.



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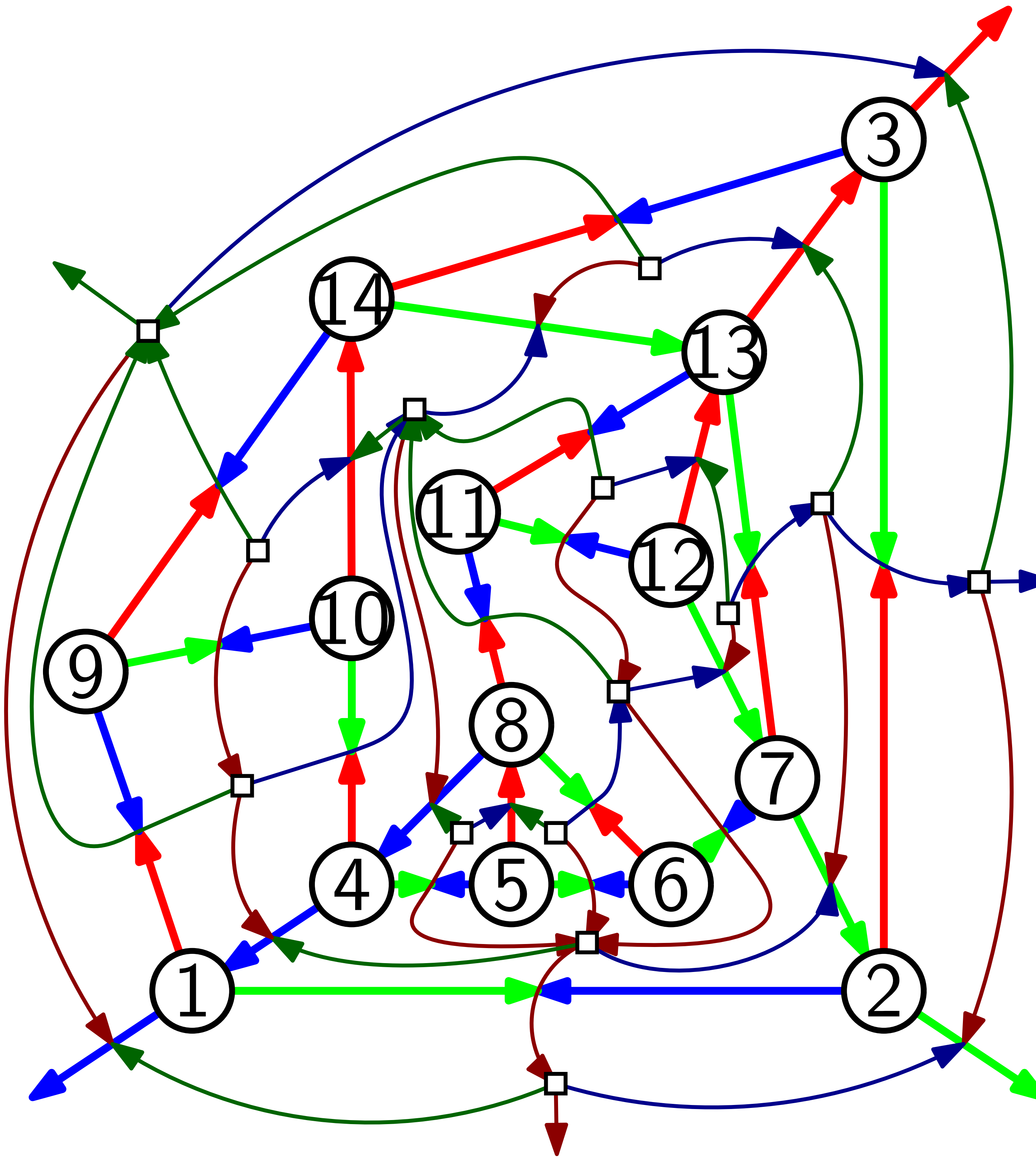
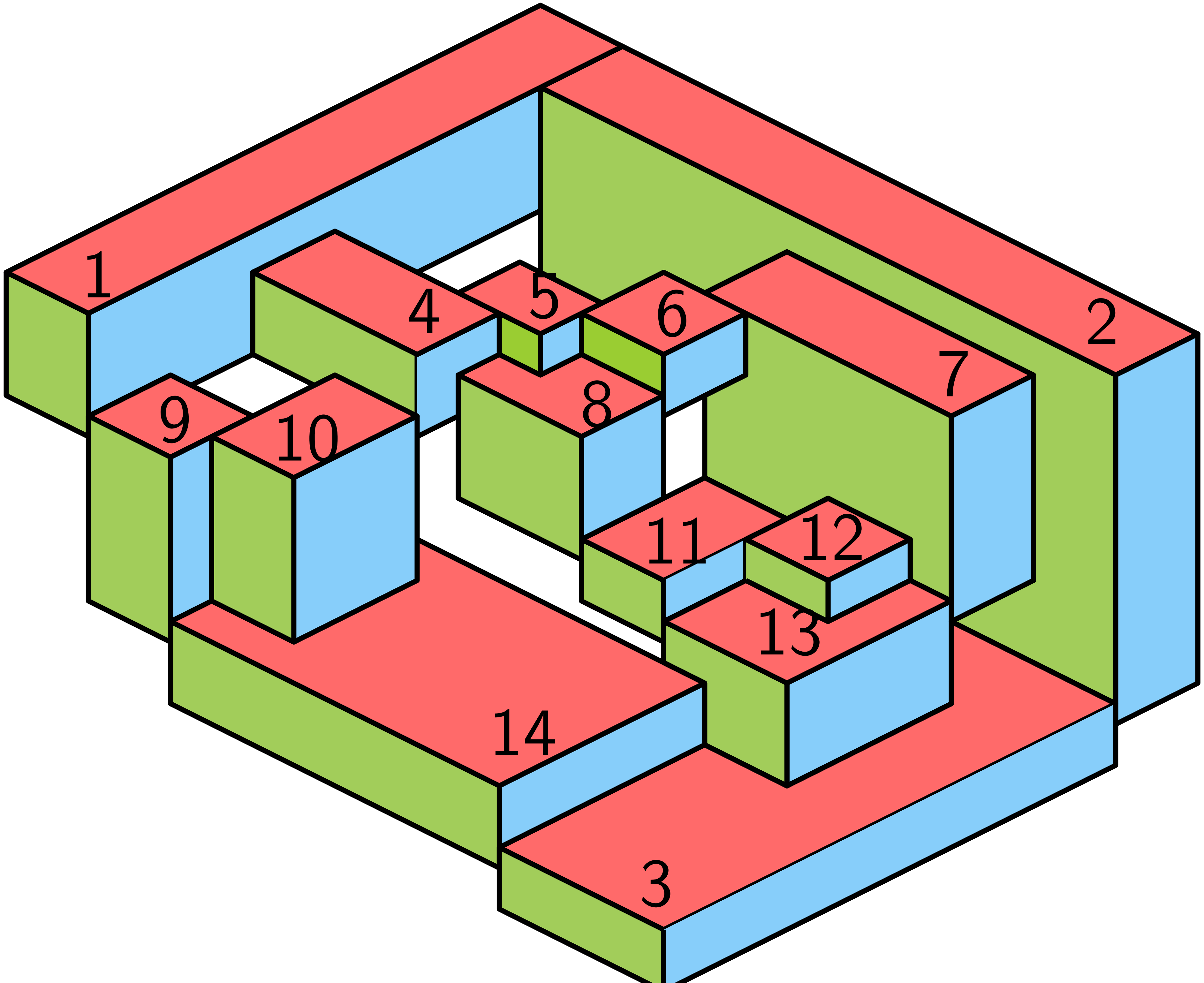


Primal-Dual Box Contact Representation



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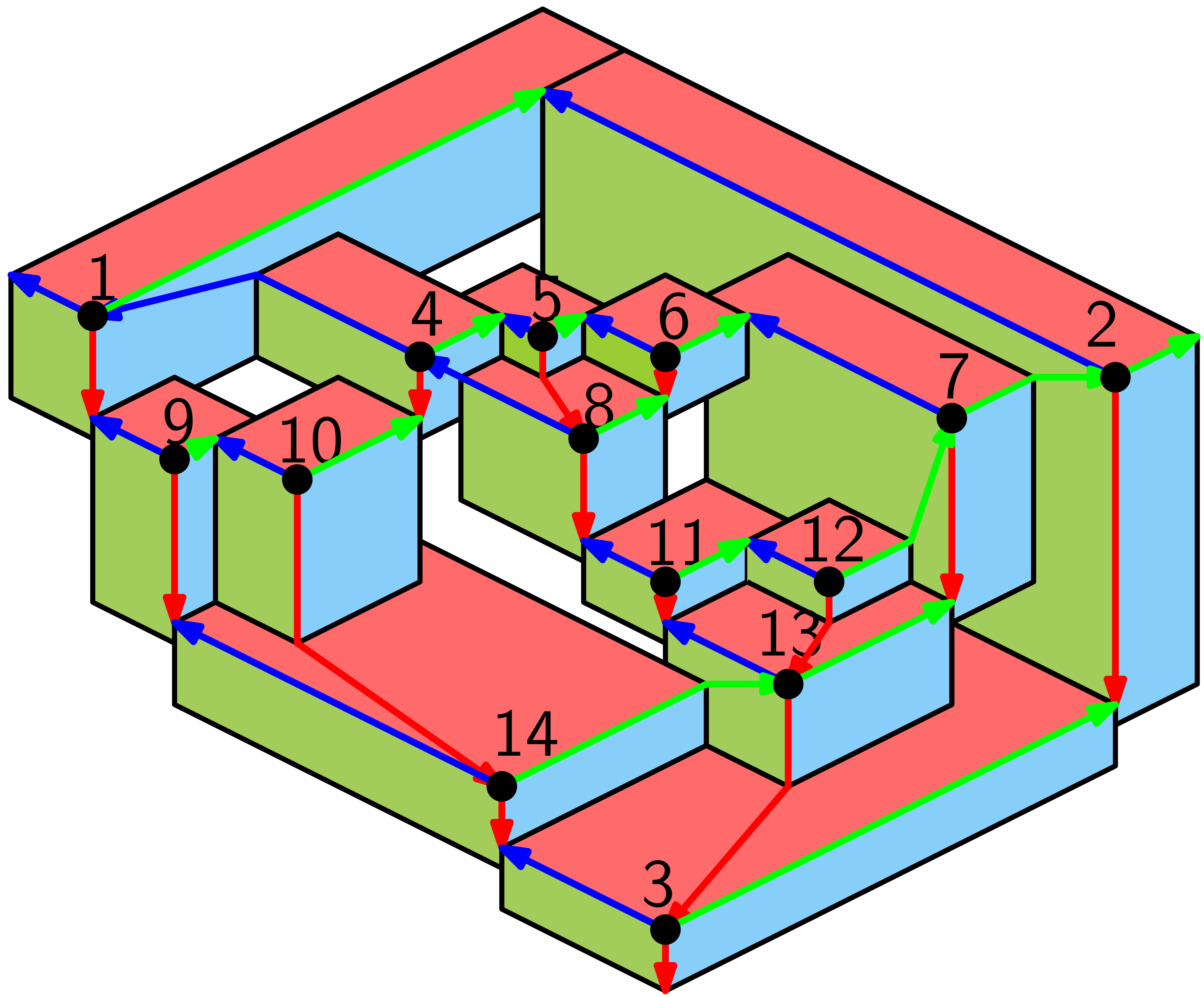
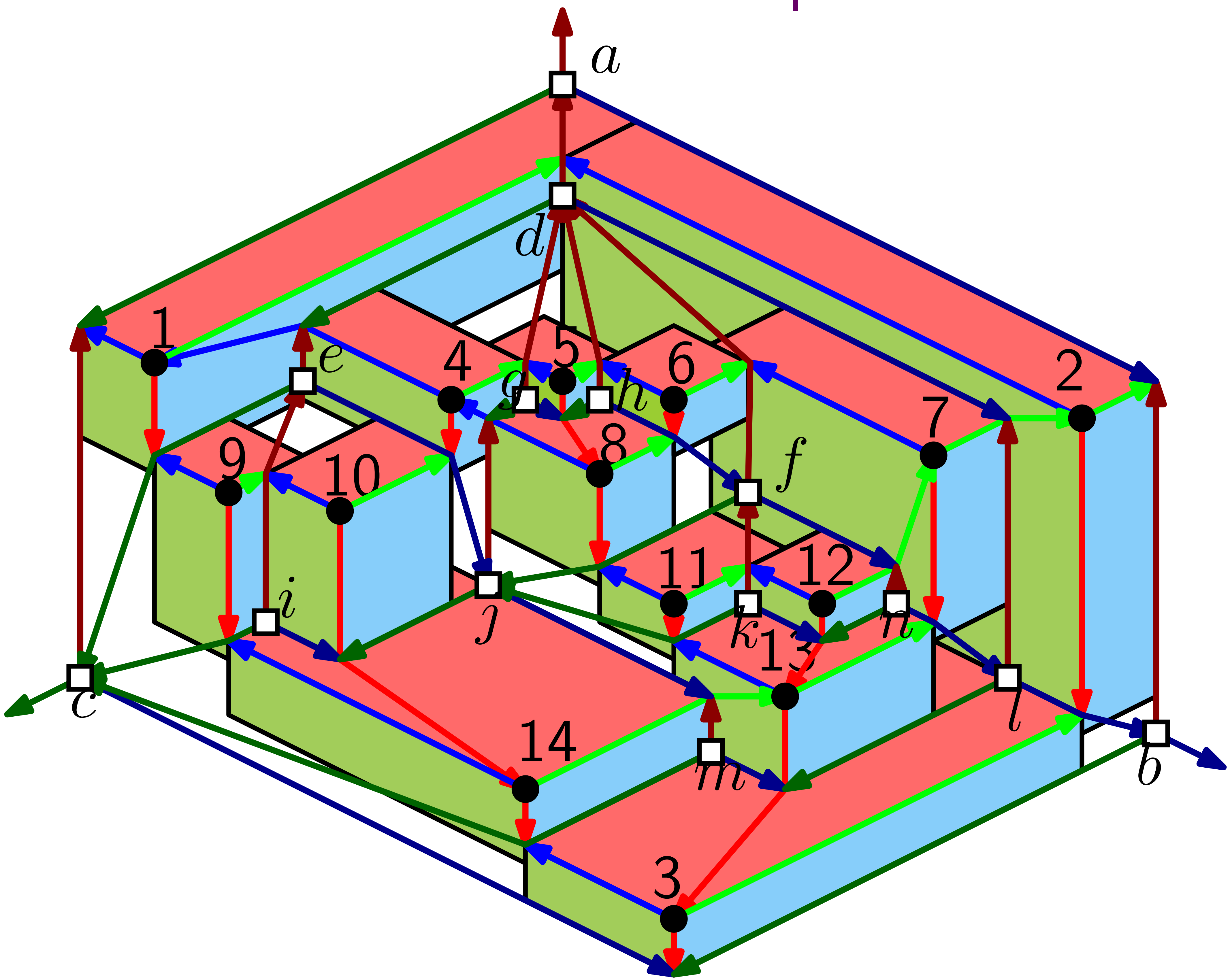


Primal-Dual Box Contact Representation



Open Problems

What graphs have 3D box-contact representations?

Do all planar graphs have proper 3D cube-contact representations?