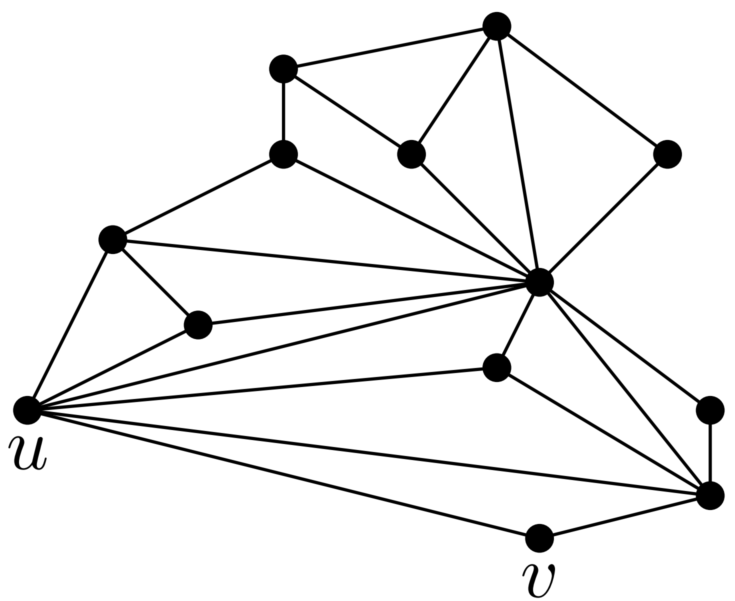
Drawing a Planar Graph on a Grid [de Fraysseix, Pach, Pollack 90]

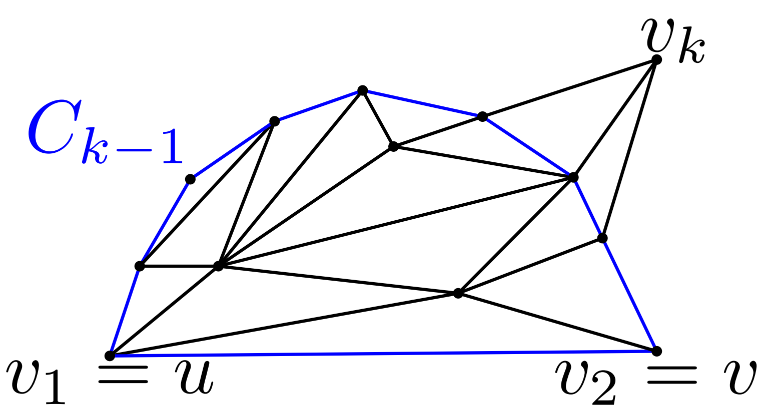
Lemma. Let G be a simple planar graph with an embedding and $u = u_1, u_2, \ldots, u_k = v$ be a cycle of G. Then there exists a vertex w on the cycle, different from u and v and not adjacent to any inside chord.

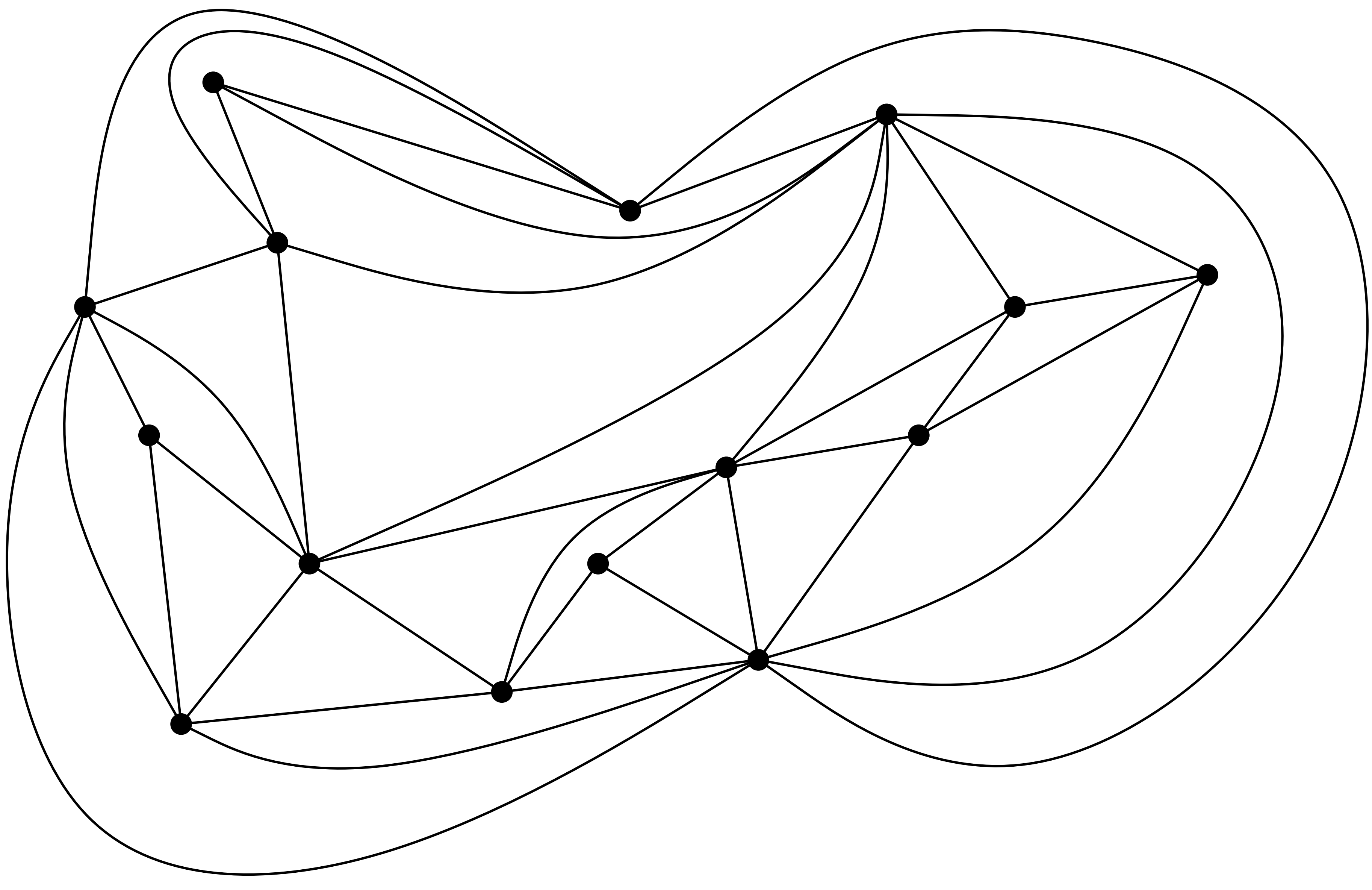


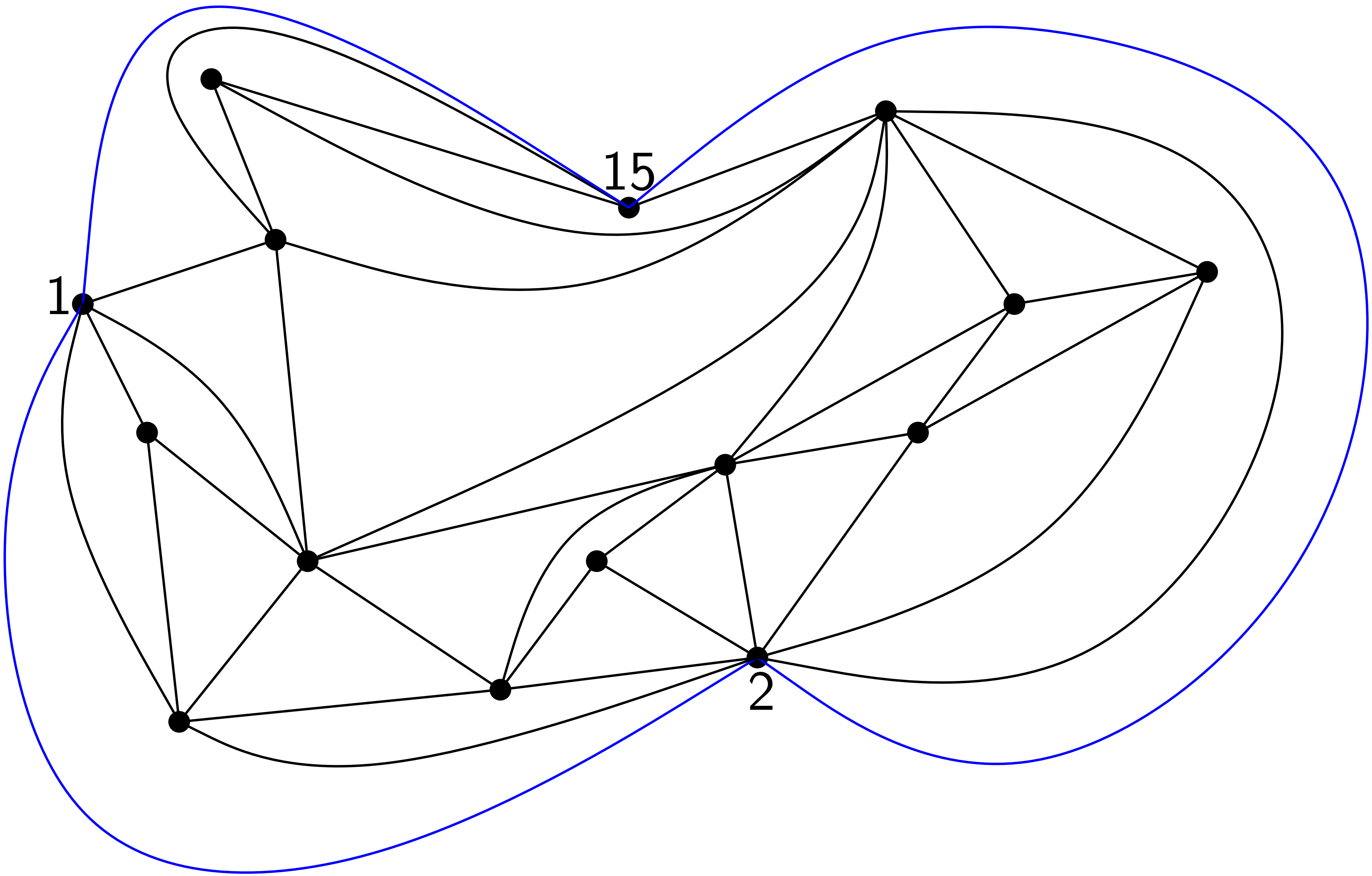
Canonical Ordering. If $G = (V, E, \mathcal{E})$ is a maximal plane graph with exterior face u, v, w then there exists an ordering of V, $v_1 = u, v_2 = v, v_3, \ldots, v_n = w$ such that for all $4 \leq k \leq n$:

(i) The subgraph G_{k-1} induced by $v_1, v_2, \ldots, v_{k-1}$ is 2-connected with uv on its exterior cycle C_{k-1};

(ii) v_k is in the exterior face of G_{k-1}, and its neighbors in G_{k-1} are a length ≥ 1 subpath of the path $C_{k-1} - uv$.



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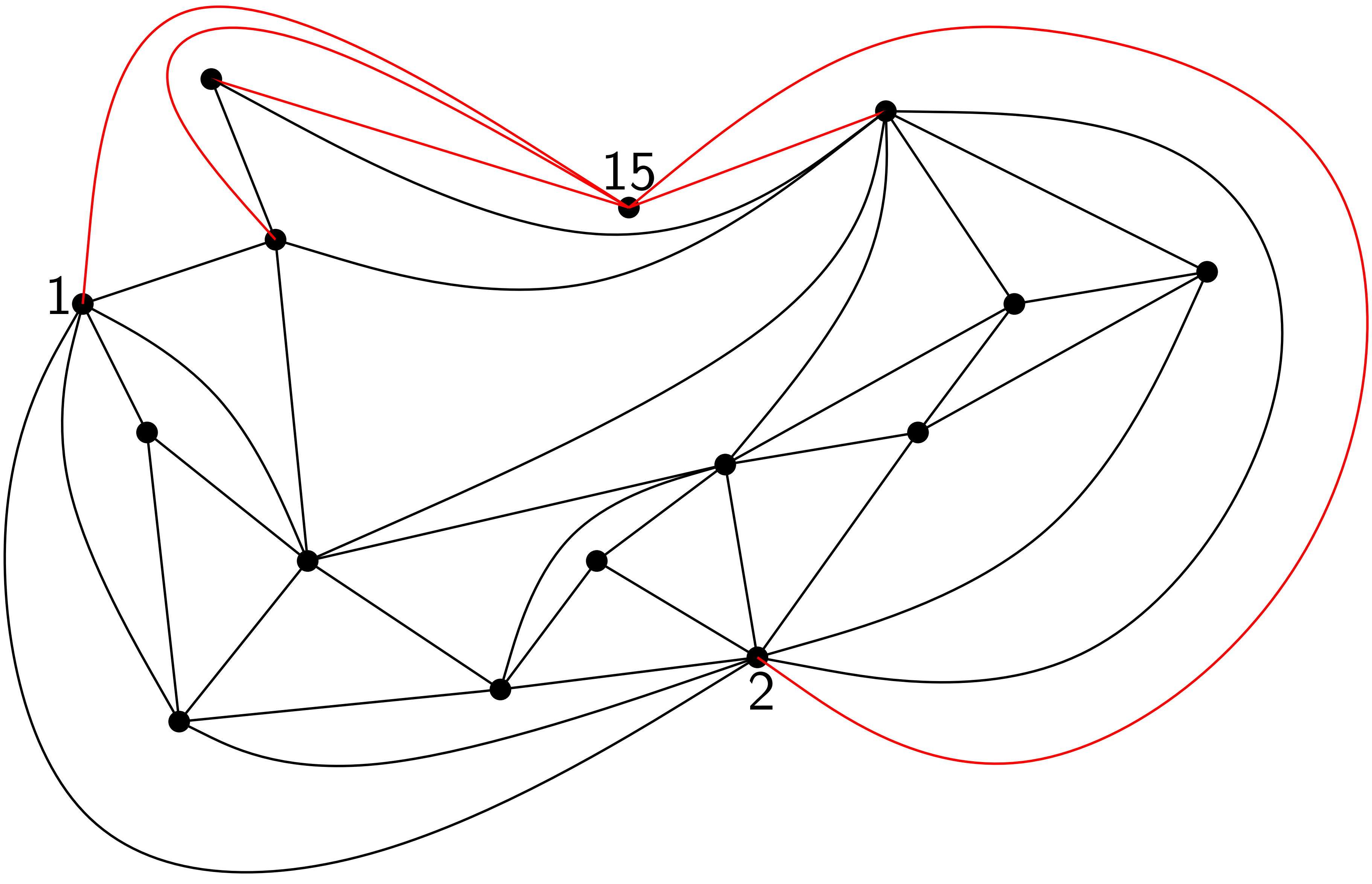
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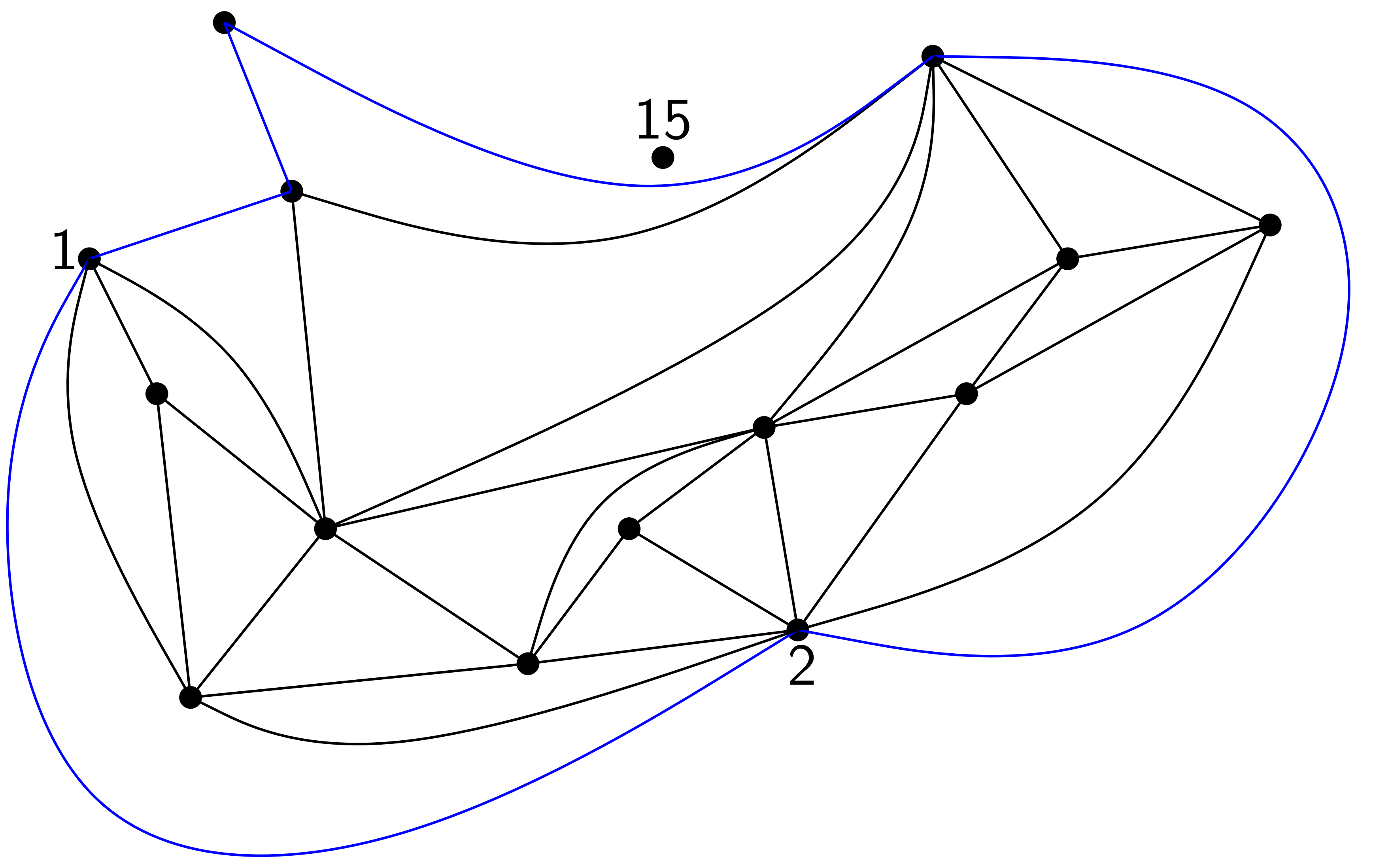
Use Lemma to find next vertex.



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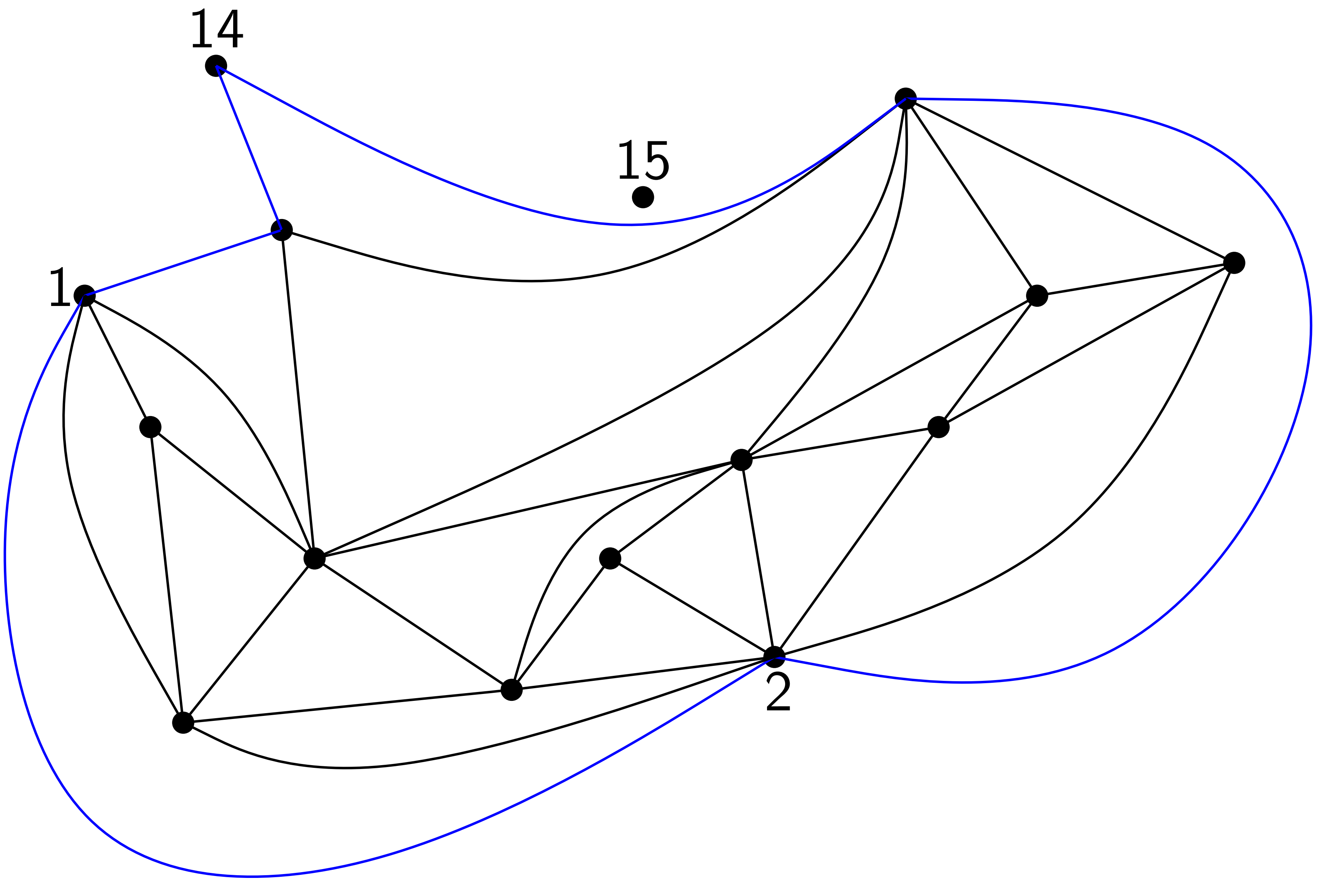
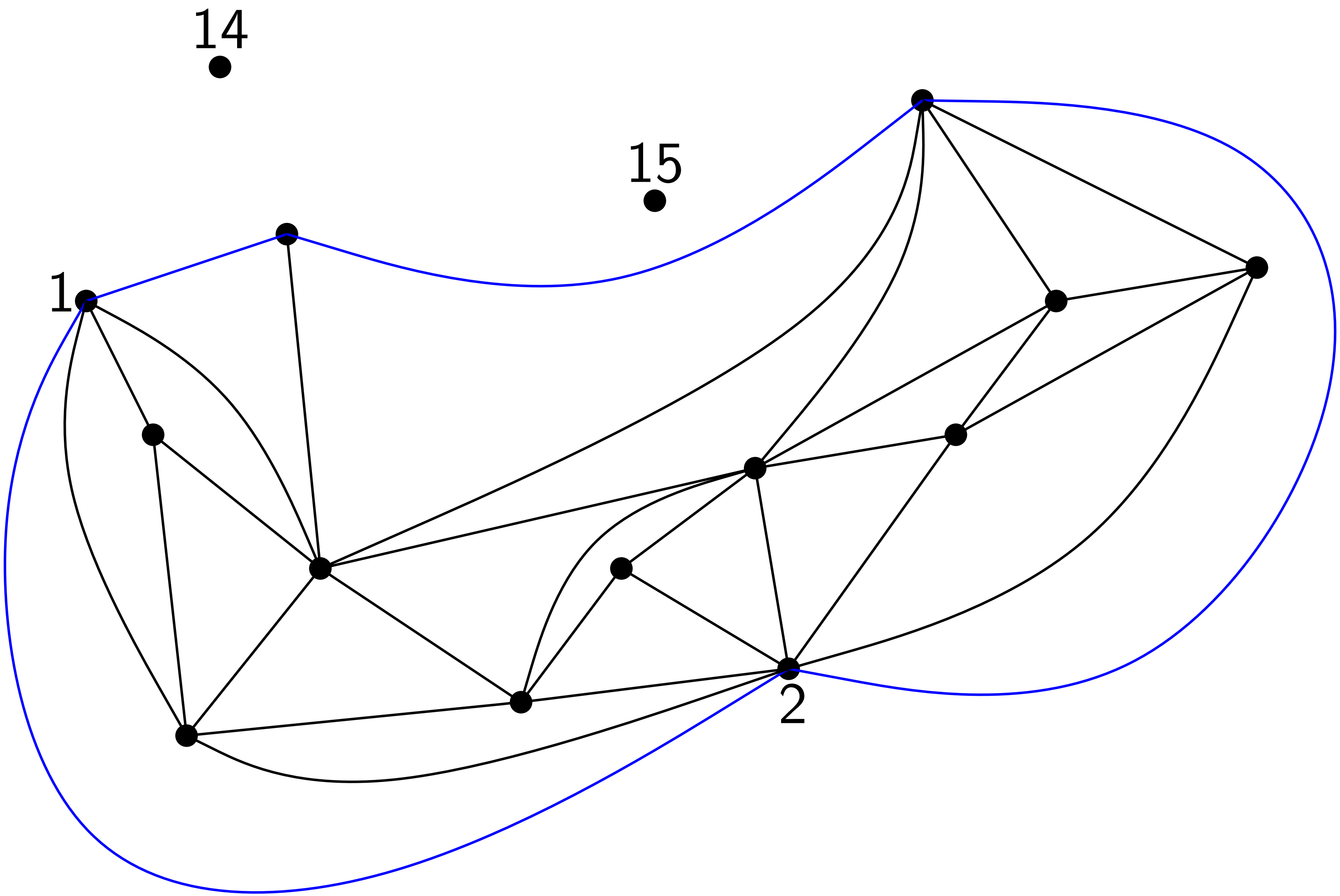
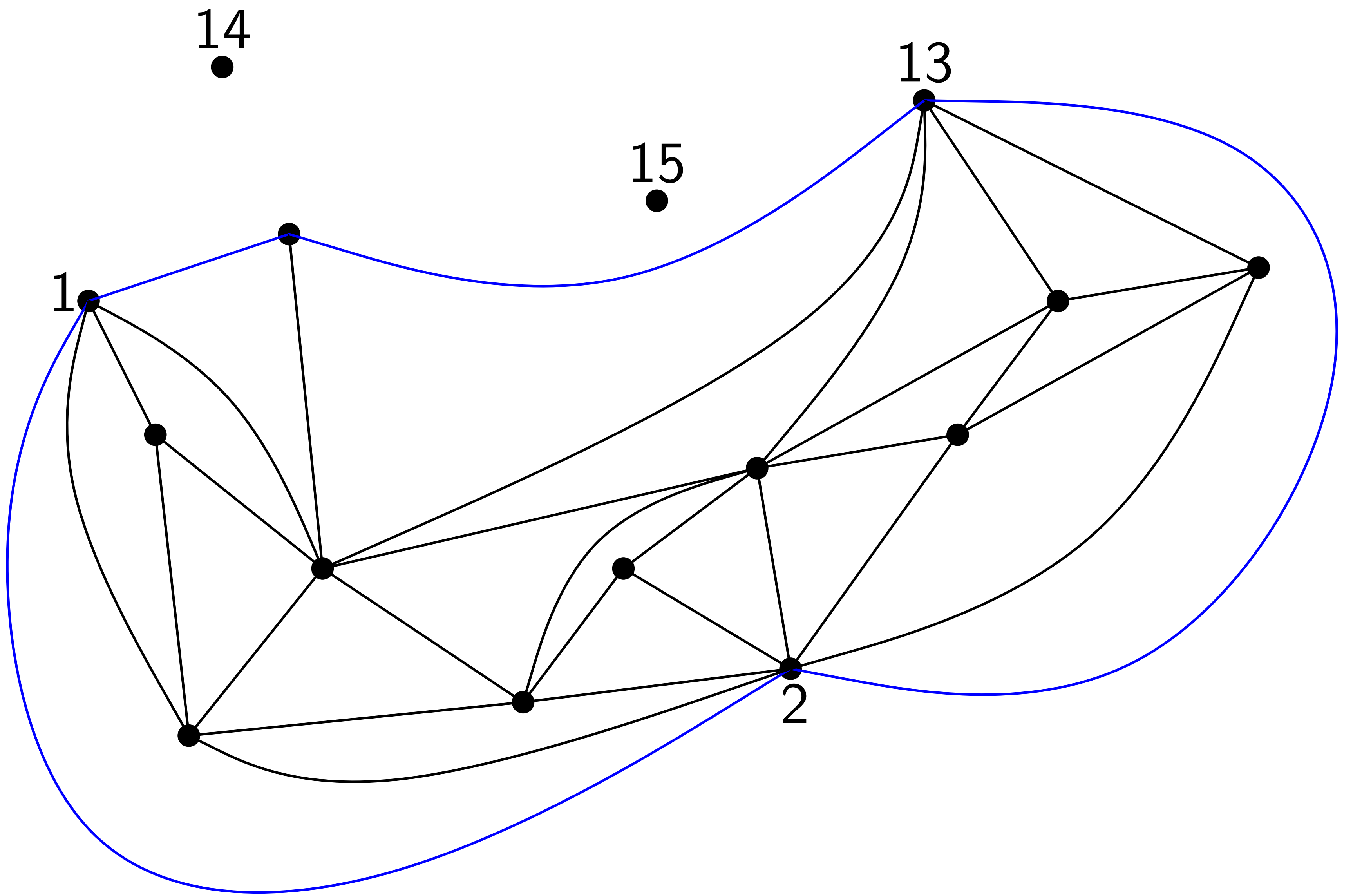


Repeat

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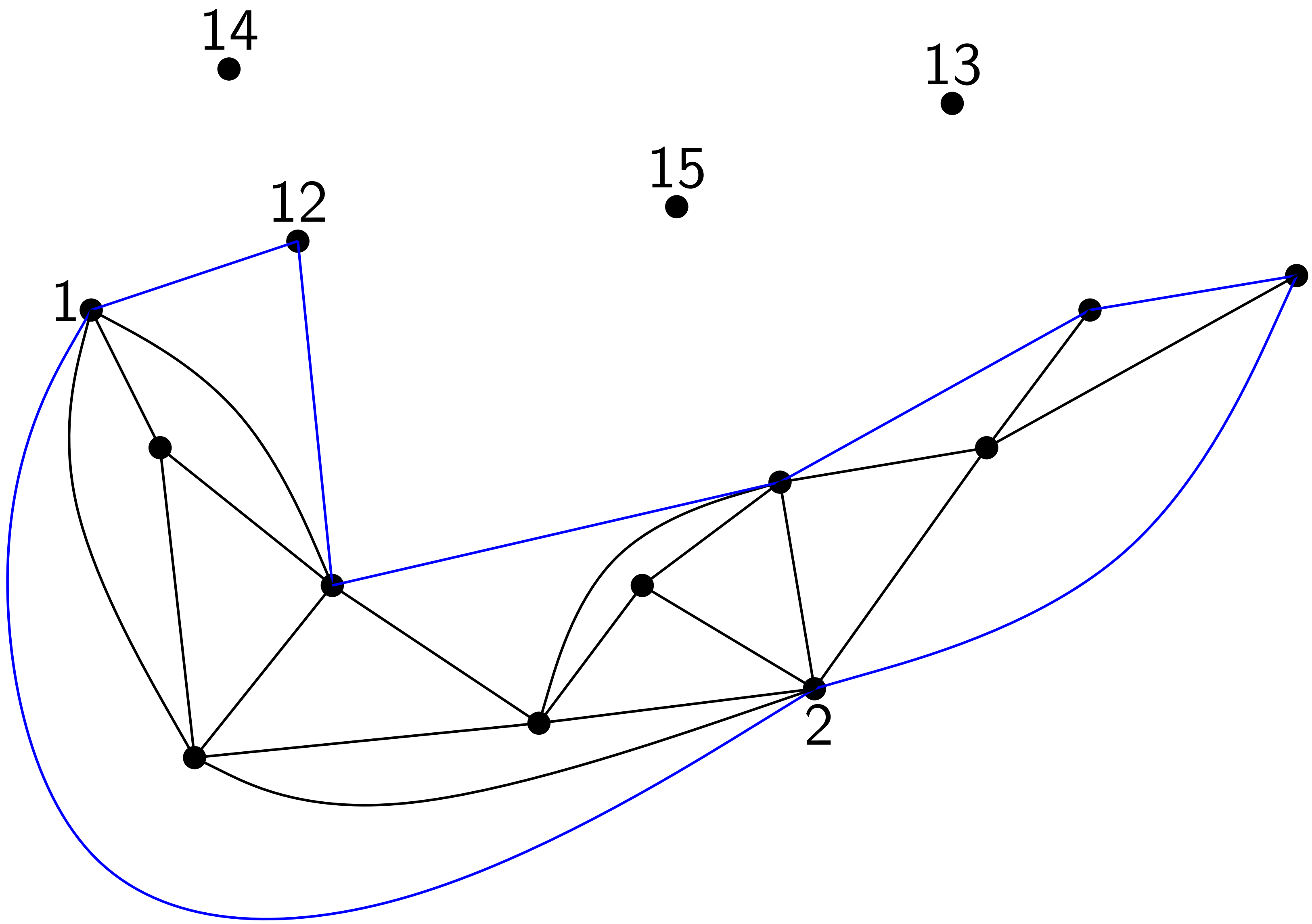
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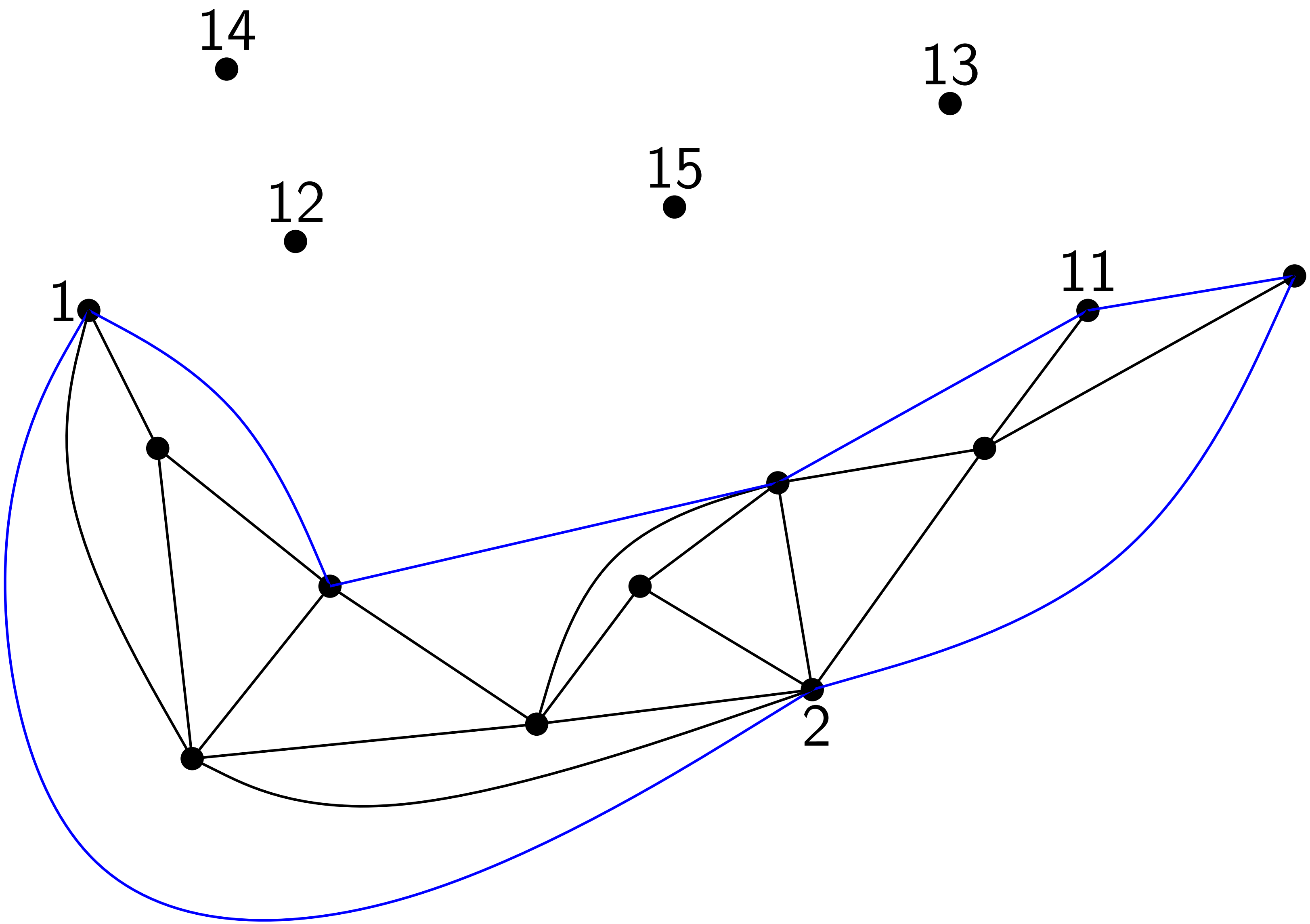
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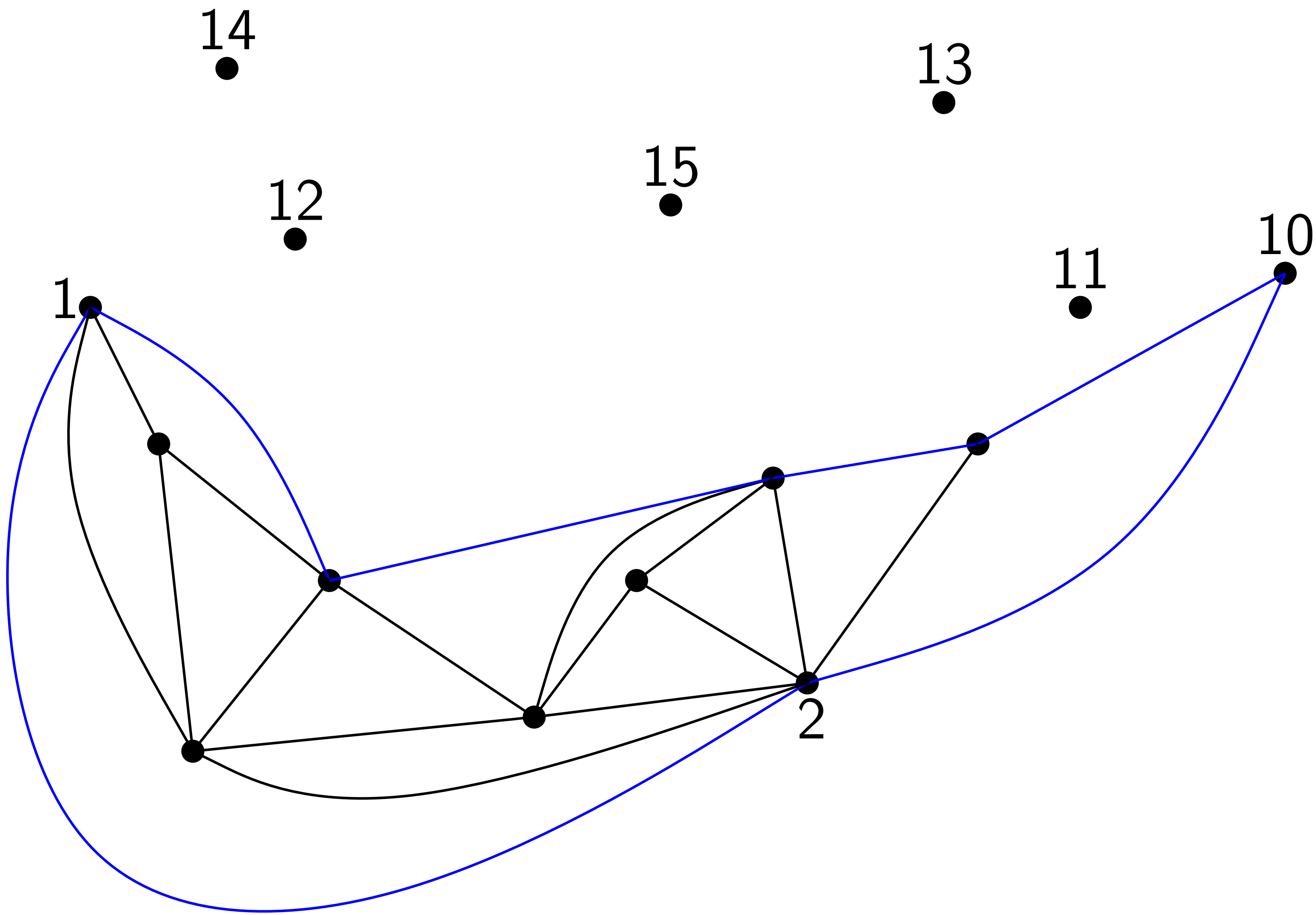
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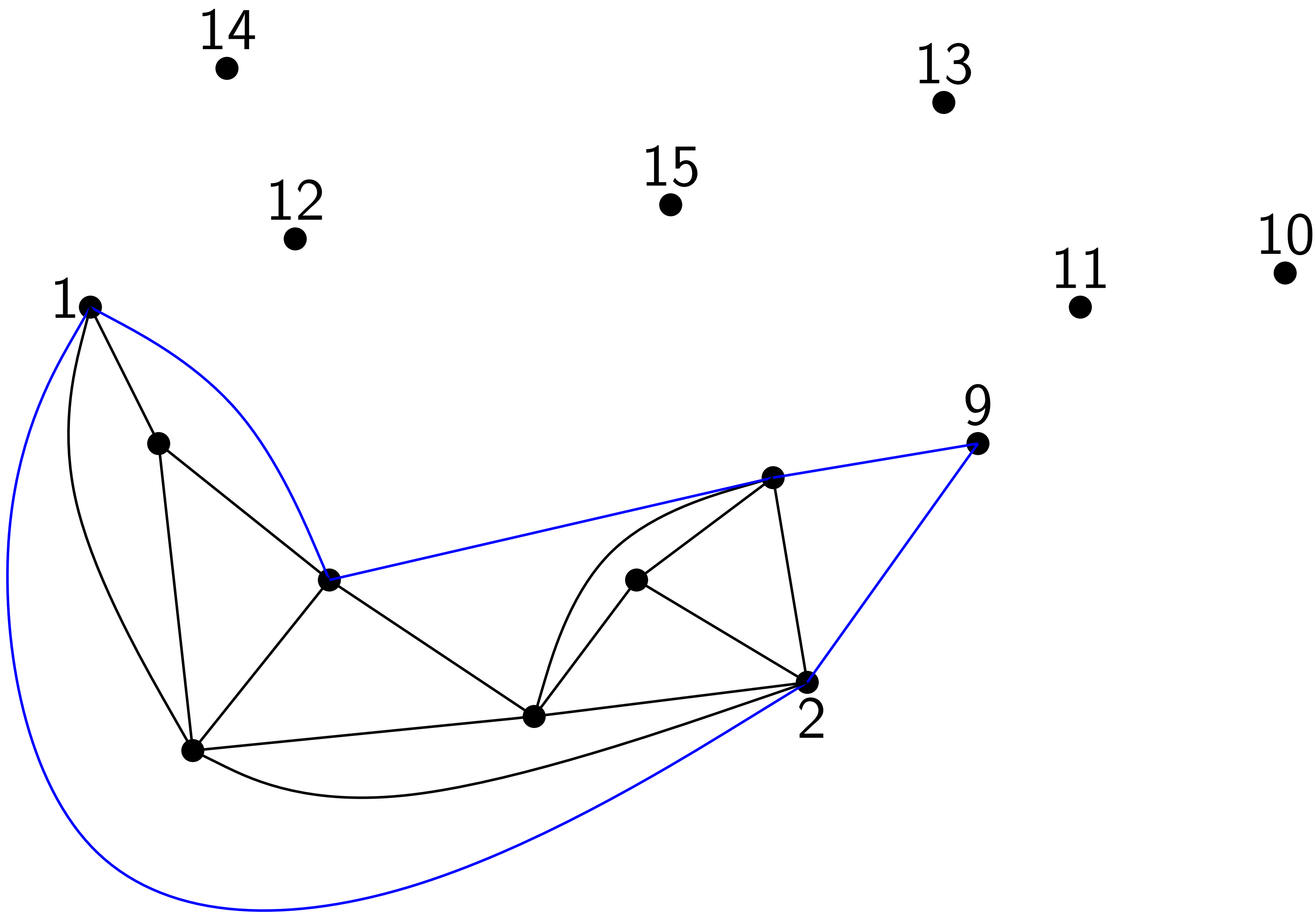
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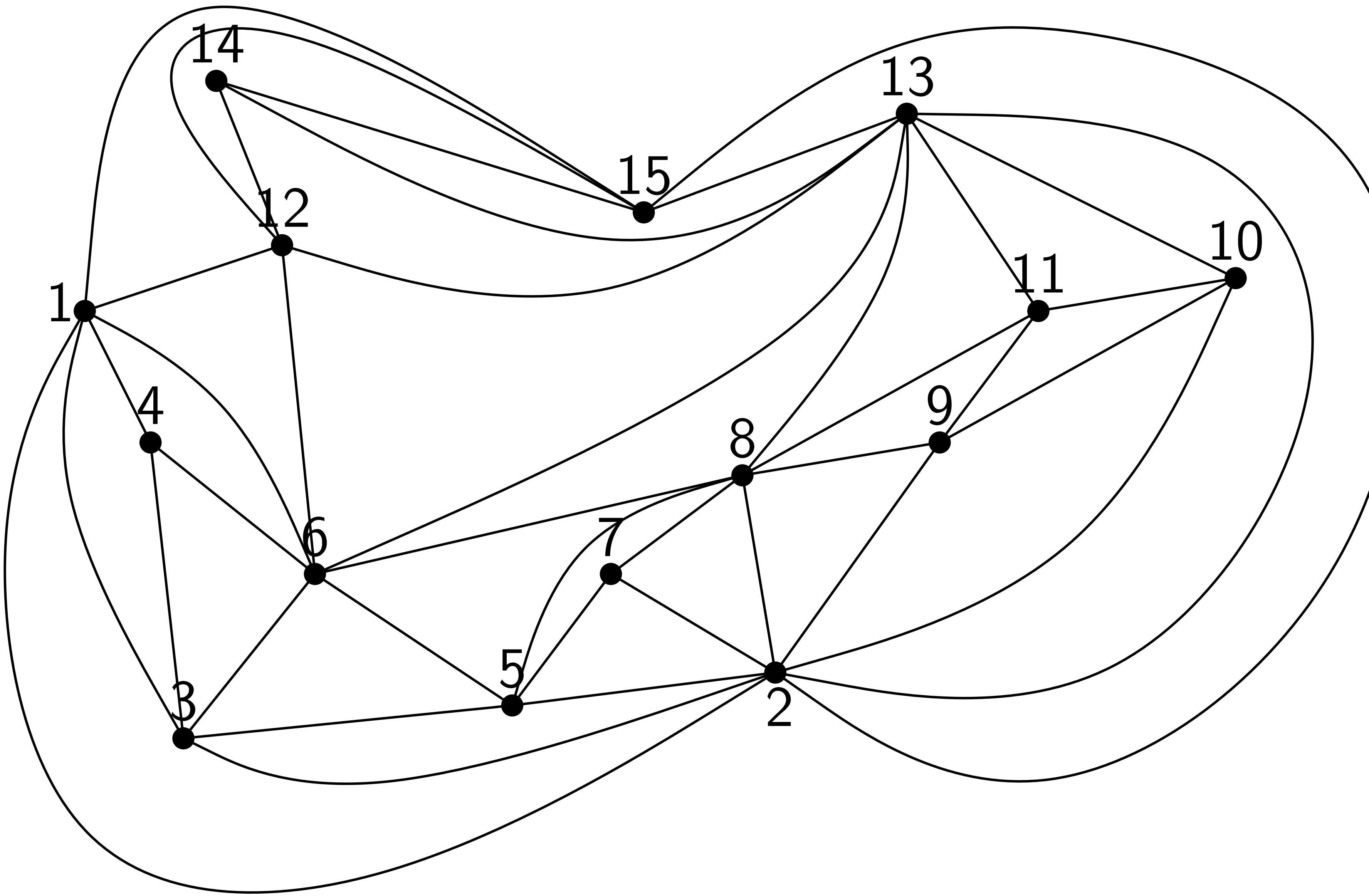
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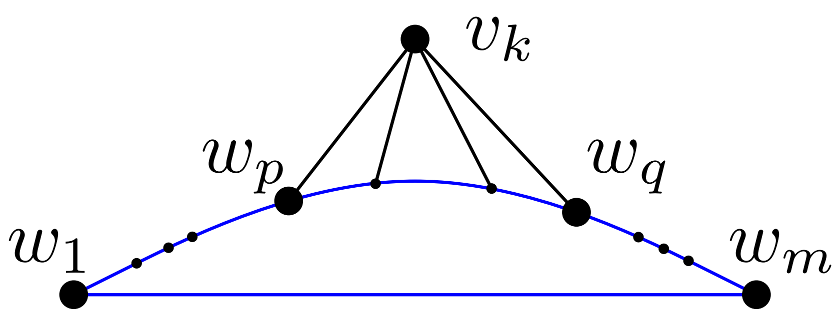

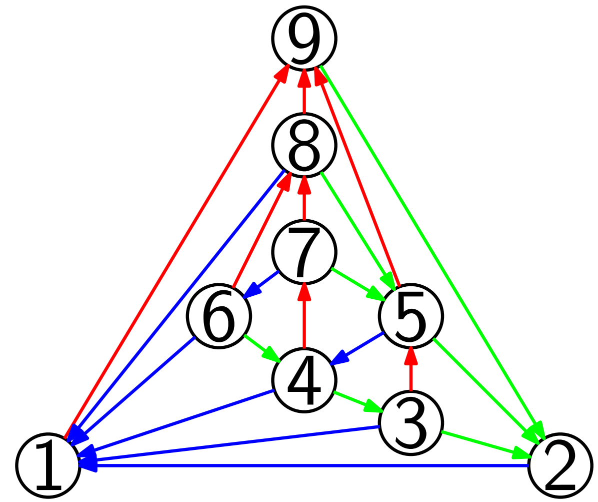

If $v \in \bigcup_{i=q}^{m} L(w_i)$ then $x(v) += 2$.

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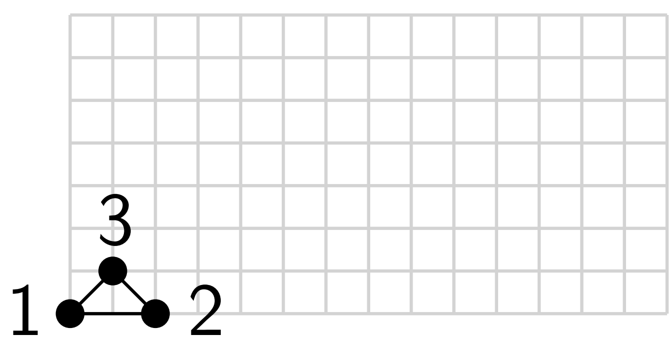

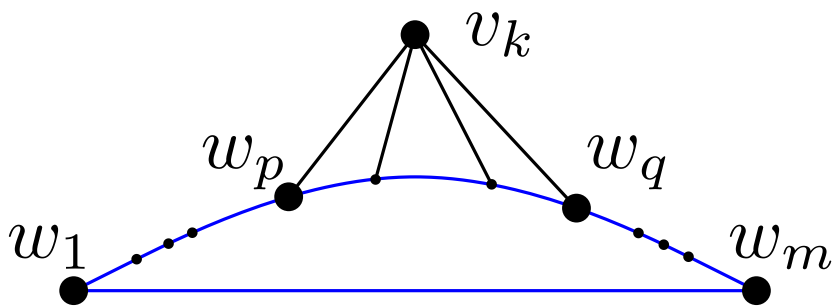

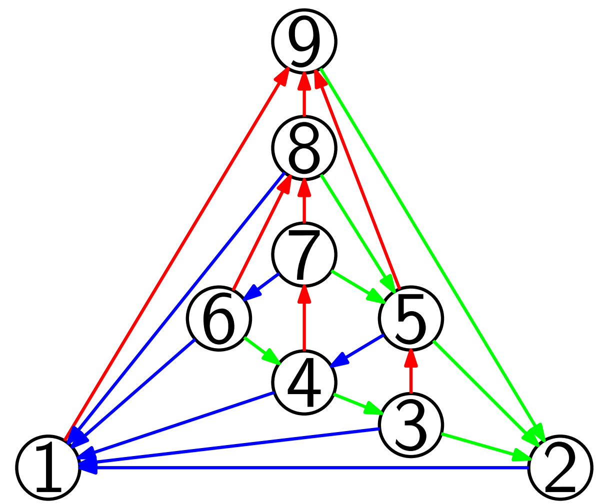

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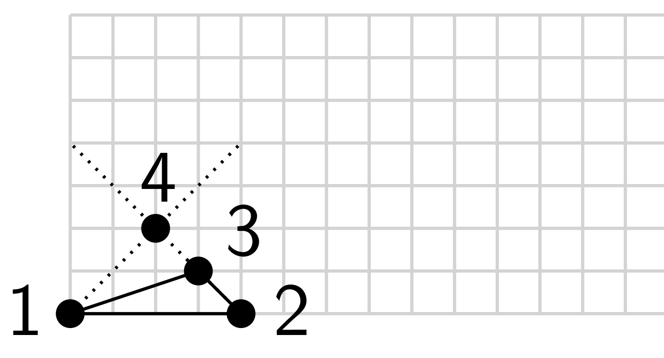

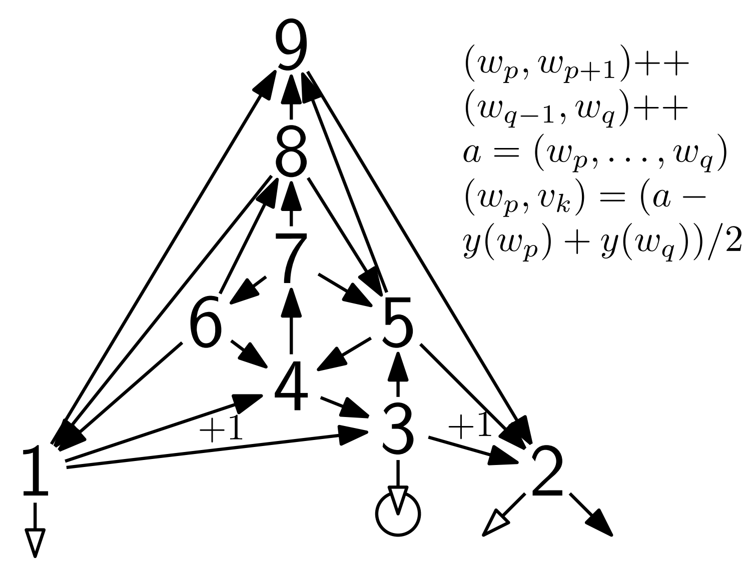

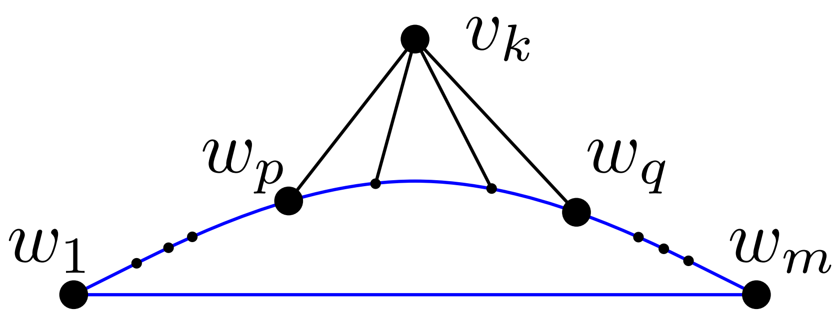

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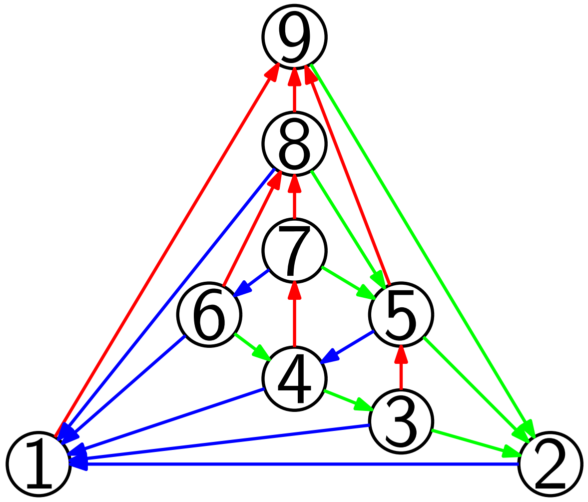

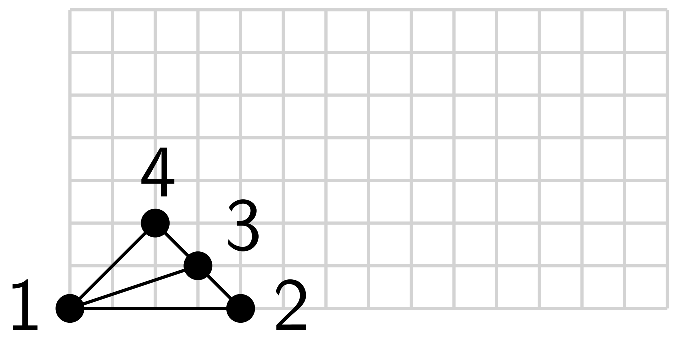

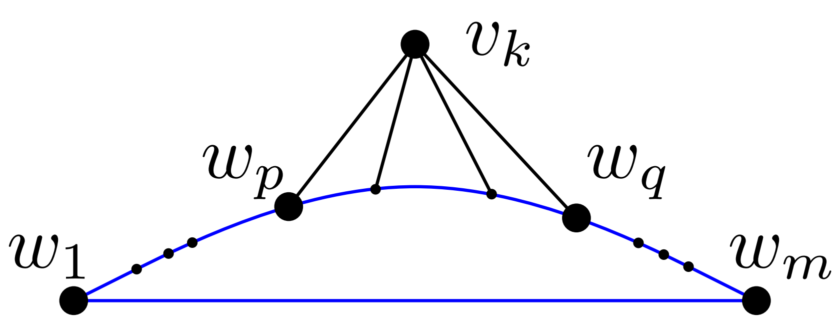

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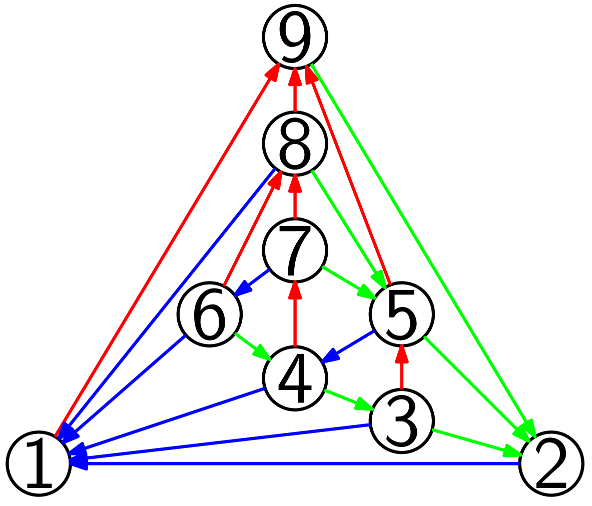

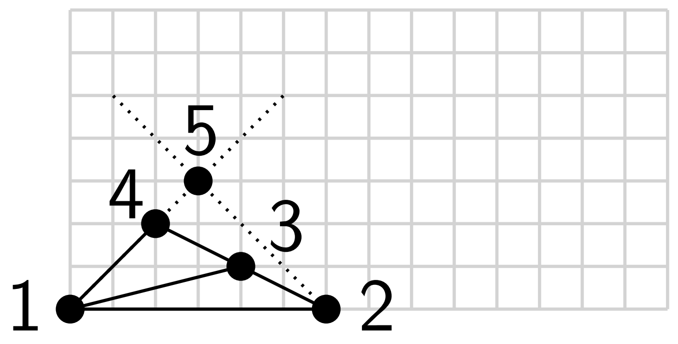

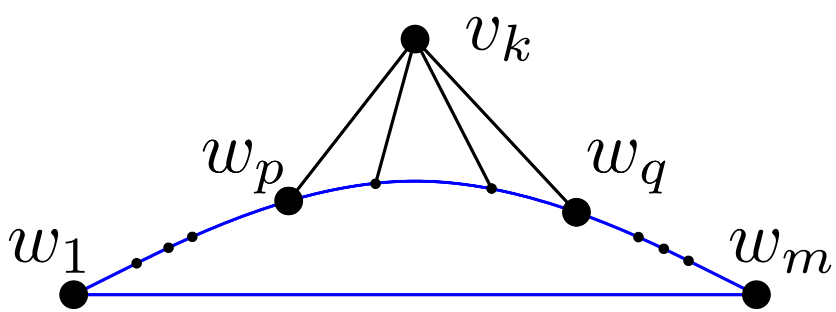

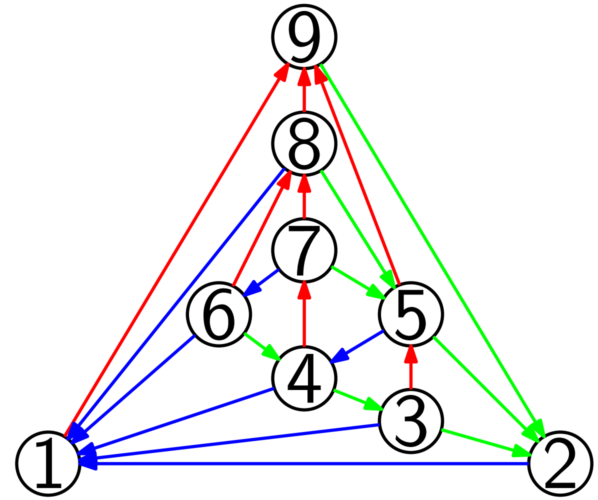

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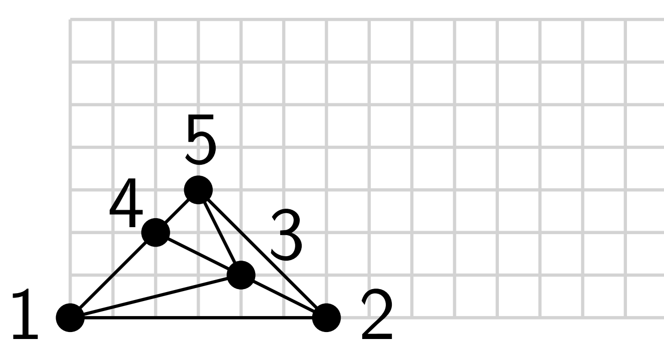

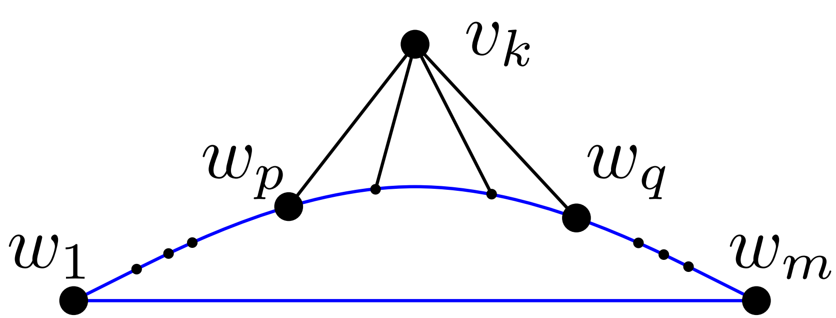

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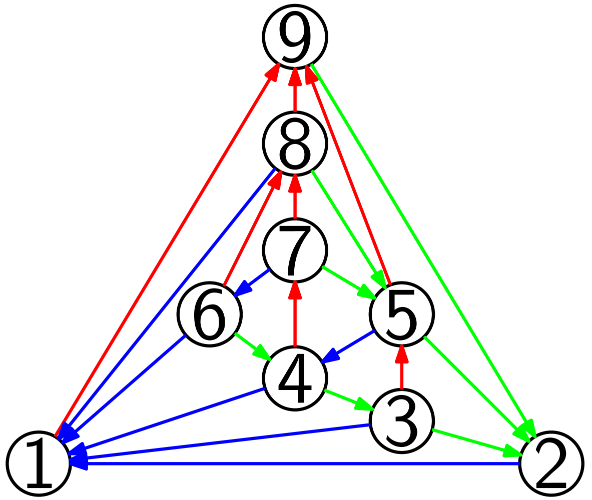

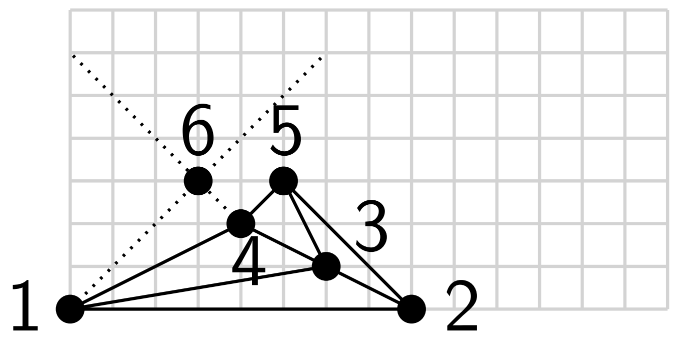

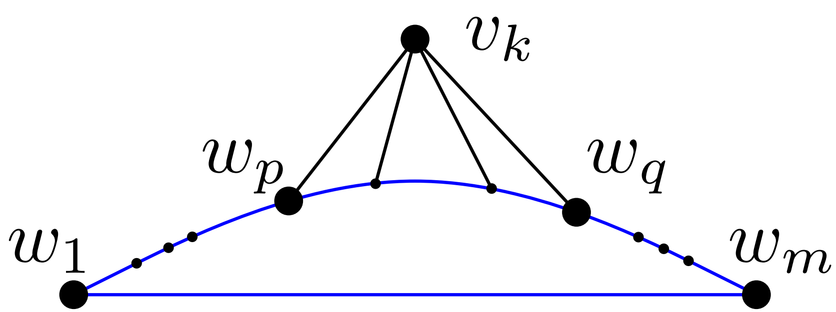

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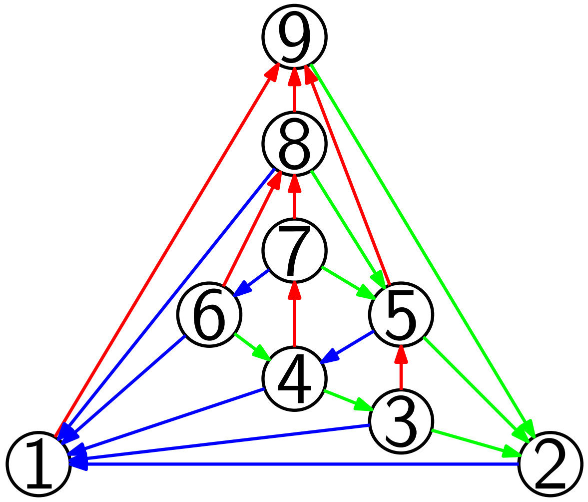

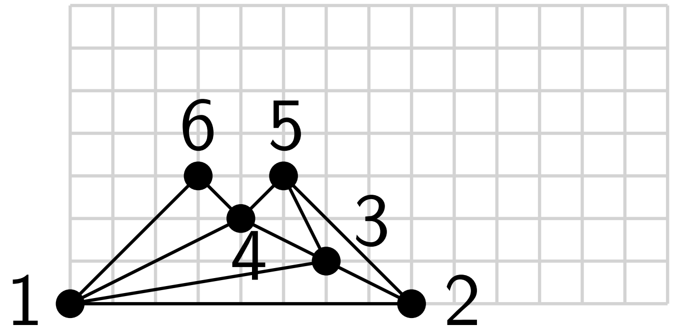

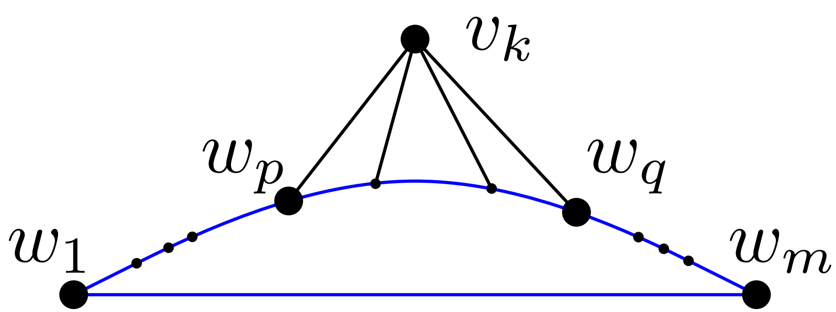

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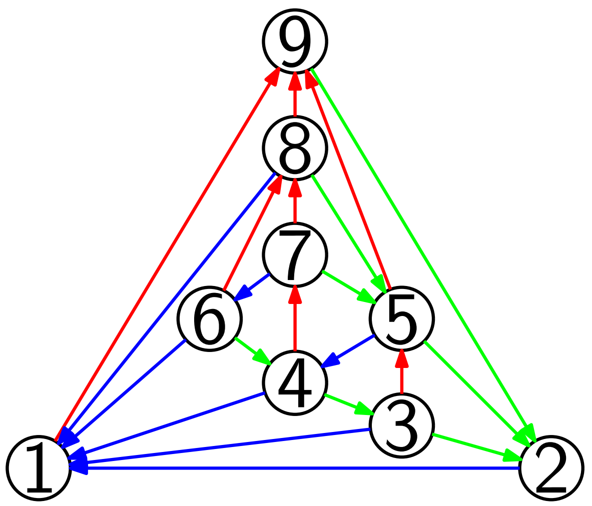

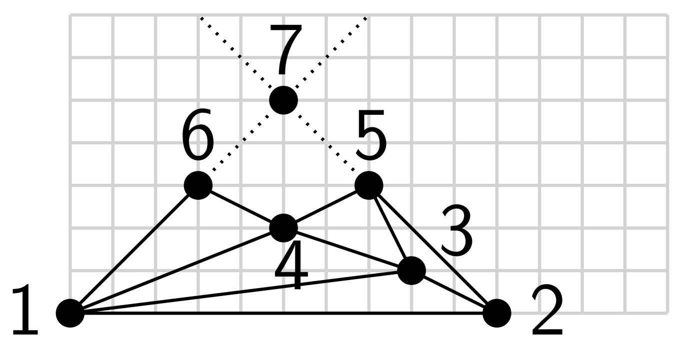

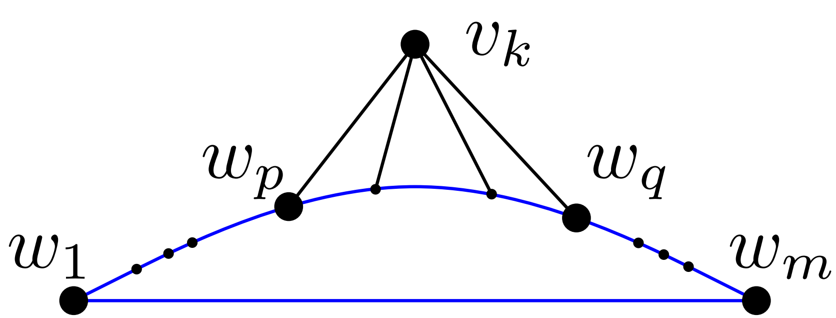

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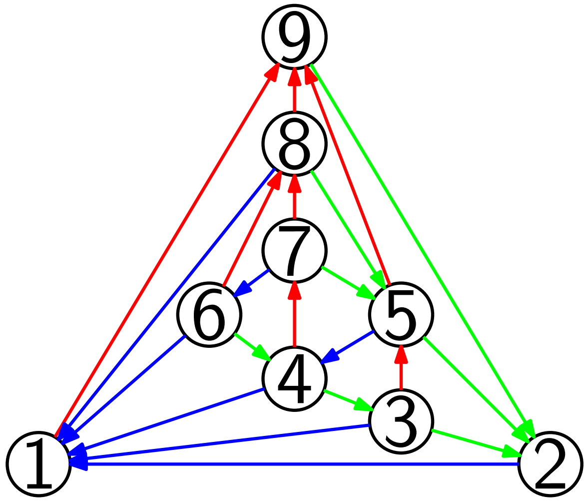

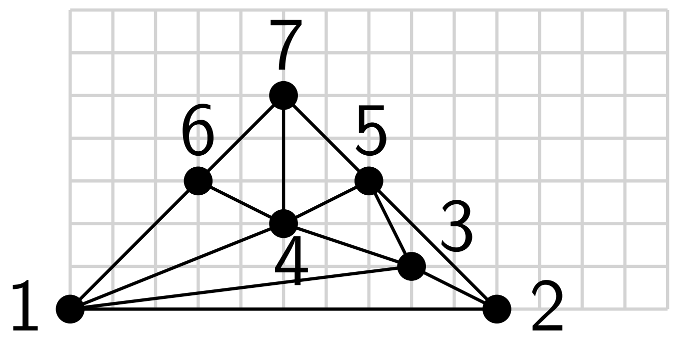

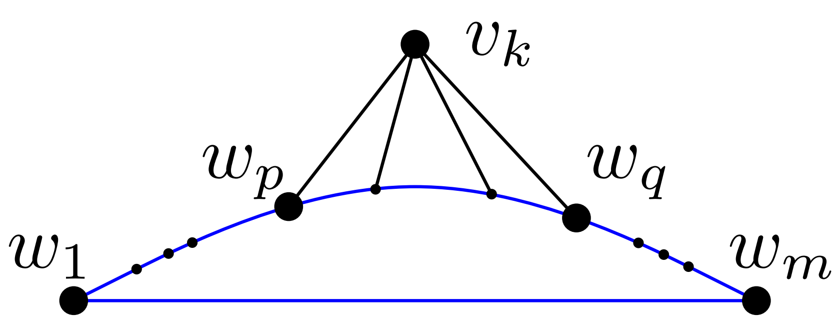

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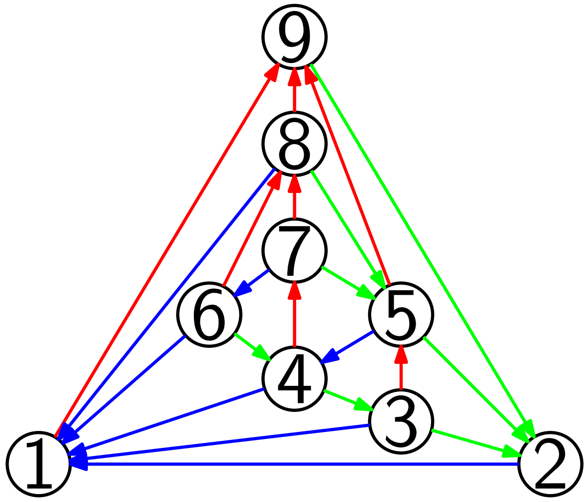

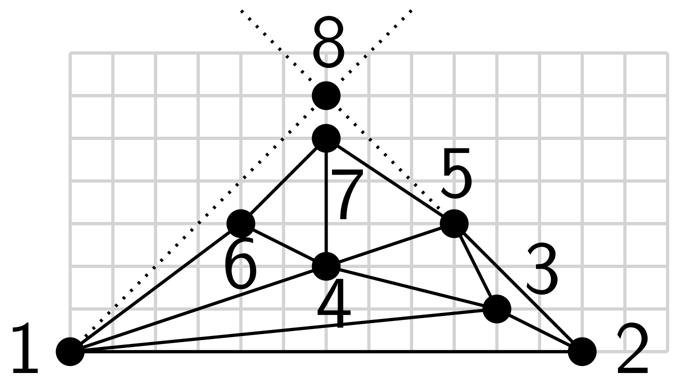

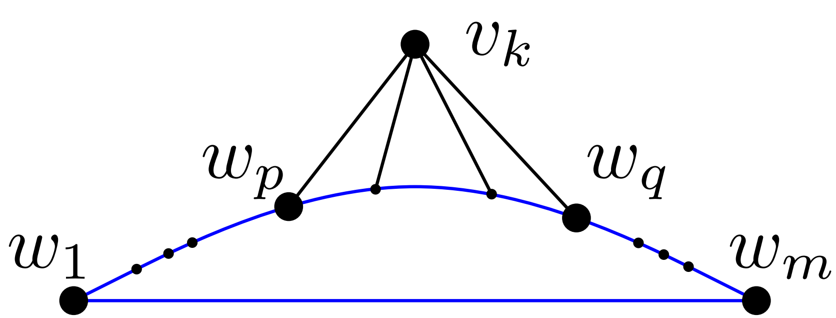

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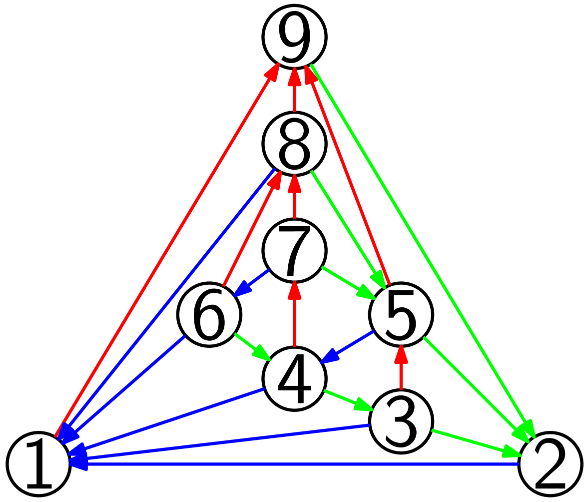

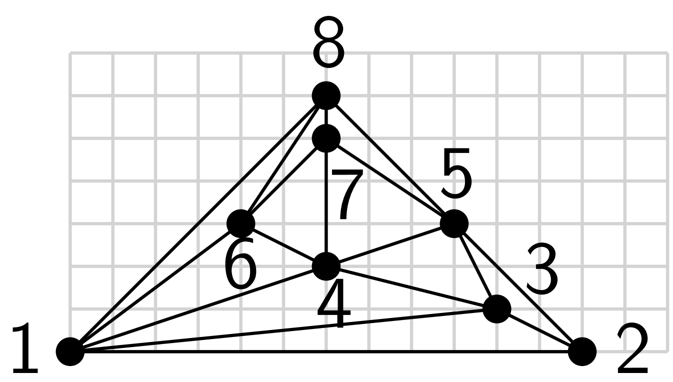

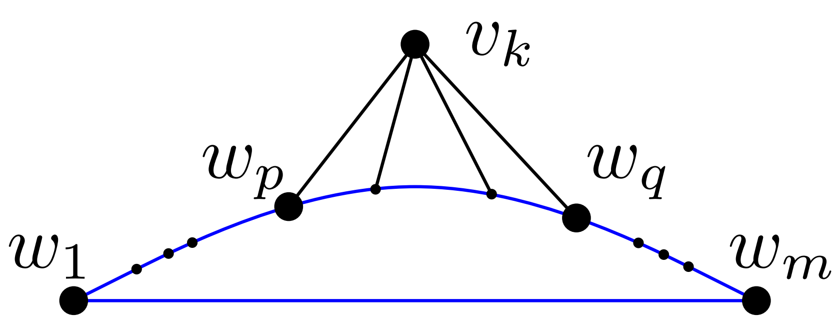

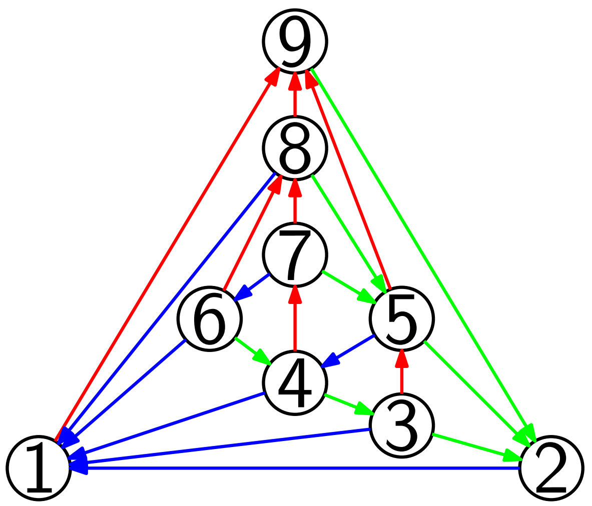

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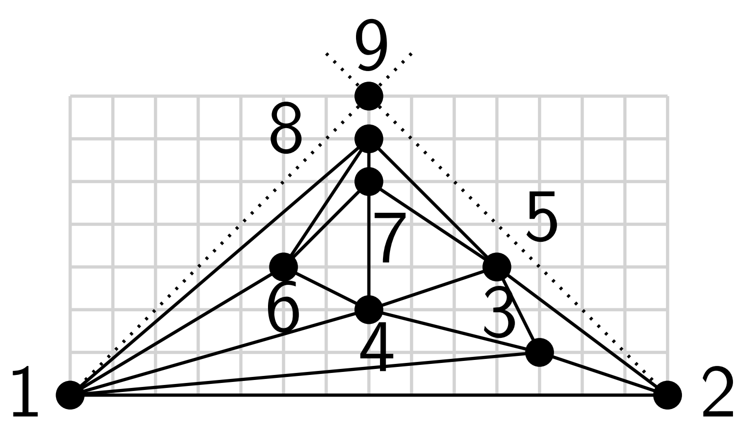

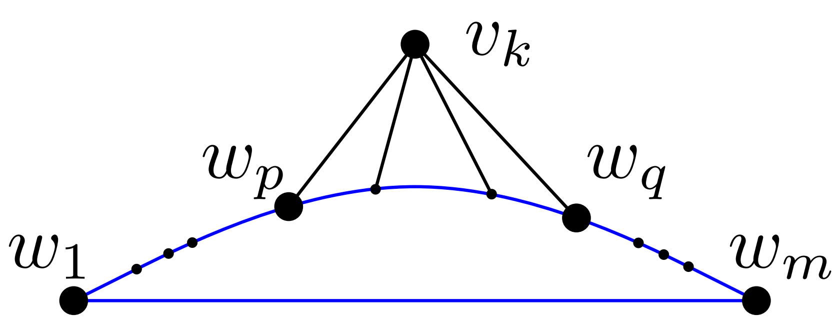

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