CS 516 3 Nov 2022 (1)

Point location & Range Query

L17

In 1-D

Find interval containing query point.

Preprocessing	$O(n \log n)$	using binary search in sorted array
Space	$O(n)$	or balanced binary search tree
query	$O(\log n)$	(to handle updates in $O(\log n)$)

In 2D

→ Find region of plane containing query point

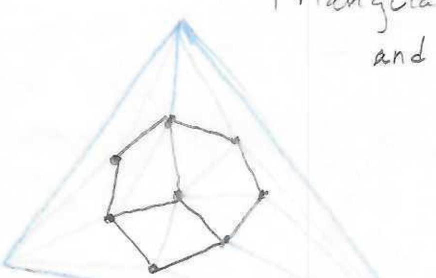
Invent the slab method

vertical lines through points
creates $n-1$ nontrivial slabs
$\equiv$ $n-1$ 1-D problems in y
(comparisons are functions of x)

preprocessing	$O(n^2)$
space	$O(n^2)$
query	$O(\underbrace{\log n}_{x} + \underbrace{\log n}_{\text{search } y})$

Kirkpatrick's triangulation refinement 1983

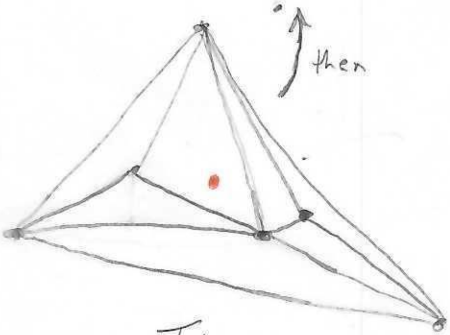
Triangulate planar subdivision $O(n \log n)$ or $O(n)$
and add a bounding $\triangle$



T_1

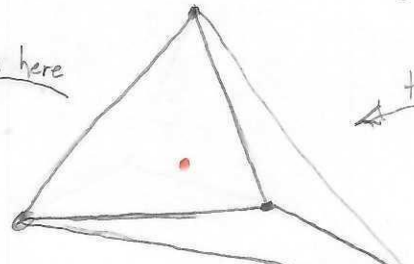
Idea . Make a sequence of coarser triangulations by deleting vertices — and $\triangle$ating holes
Find point in coarsest triangulation and then in next coarsest ... until located in original triangulation.

then



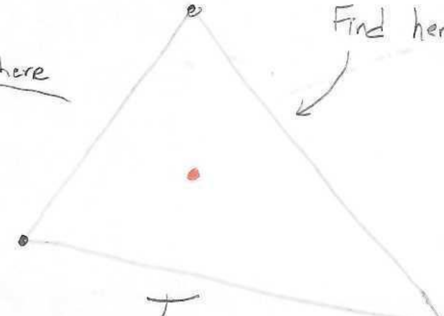
T_{h-2}

then here



T_{h-1}

then here



T_h

Find here

Want

① Small sequence of coarse Δtions $T_1\, T_2 \ldots T_h$

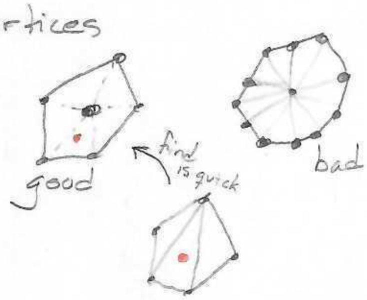
remove many vertices (constant fraction) $\Leftarrow$ ↳ remove many vertices in each step

② Easy to find query point in T_{i-1} given its location in T_i

↳ each Δ in T_i should intersect few Δ in T_{i-1}

remove low degree vertices (constant deg.) $\Leftarrow$ ↳ choose vertices to remove that create holes with few vertices

remove independent set of vertices $\Leftarrow$ ↳ don't remove two vertices that create one big hole



Lemma Every triangulated planar graph on $n \geq 4$ vertices has independent set of size $\geq n/18$ in which each vertex has degree ≤ 8.

$\Rightarrow$ total $\boxed{\text{Size}}$ of all Δtions $= \sum_{i=1}^{h+1} n(1 - 1/18)^{i-1} = O(n)$

and $\#\Delta$tions $\equiv h \in O(\log n)$

and $\boxed{\text{Query time}}$ is $h \times O(8) = O(\log n)$

Wow!

Proof $\sum_{v \in T} \deg(v) = 2(\#\text{edges}) = 2(3n-6) < 6n$

At least $n/2$ vertices have degree ≤ 8

otherwise $\geq n/2$ verts have deg. ≥ 9 and the others have deg ≥ 3

So $\sum \deg(v) \geq 9\, n/2 + 3\, n/2 = 6n$

$O(n)$ time

Alg Mark all vertices w/ deg ≥ 9
While $\exists$ unmarked vertex v [add v to indep set, mark v and its nbrs]

Start with $n/2$ unmarked, mark ≤ 9 in each round $\Rightarrow$ $n/18$ in indep set

$\boxed{\text{Preprocessing time}} = O(n \log n)$ Note: Retriangulate holes to get from T_{i-1} to T_i takes $O(n)$ ...