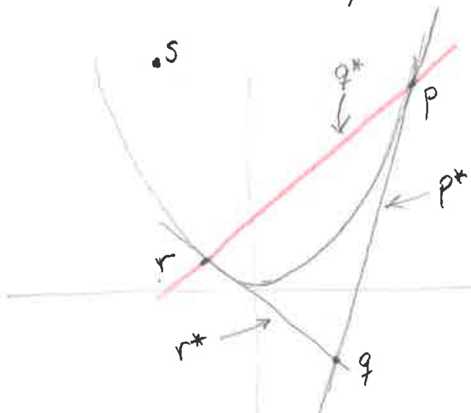
CPSC 516 27 Oct 2022 ①

(L15) Remember duality from last time...



Dual of point p on parabola?

Dual of point q below parabola?

incidence property (twice)

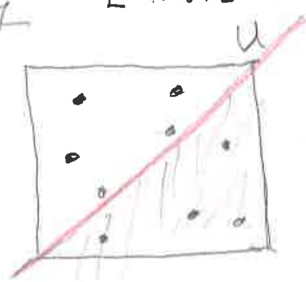
$q \in p^* \Rightarrow p \in q^*$

$q \in r^* \Rightarrow r \in q^*$

now we know two points on q^*

Dual of point s above parabola?

Discrepency [what's a good sample?]



Given a set S of n points in unit square U

what is the maximum (supremum) over all half planes h of

$$|\mu(h) - \mu_S(h)|$$

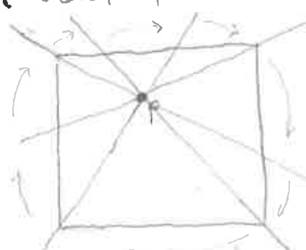
fraction of U in h

fraction of S in h

$$\boxed{\begin{aligned}&|\mu(h) - \mu_S(h)| \\ &= |1/2 - 4/9|\end{aligned}}$$

we could shift h slightly to make this difference larger until h touches $p \in S$.

① If h touches only one point p of S, rotating h about p



in each wedge, at most two extremes of $\mu(h)$

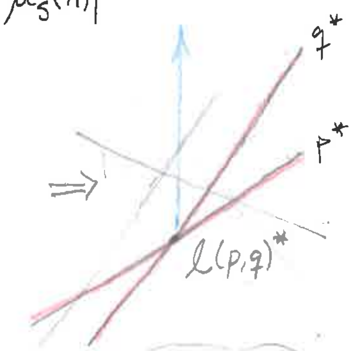
at most 8 wedges

Find Max in Linear time for all p

② max $|\mu(h) - \mu_S(h)|$

h touching two points of S p, q

$\Rightarrow$



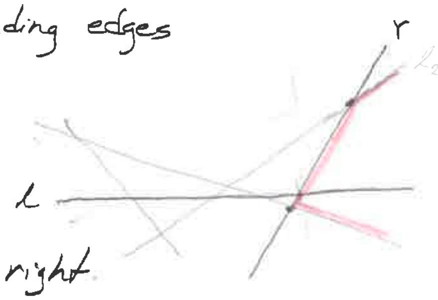
points below $\ell(p,q)$ = # lines above $\ell(p,q)^*$

(we also need # points above and on)

Using the level of every vertex in line arrangement we can calculate this

level = # lines above a vertex in $A(L)$

We prove that the number of left bounding edges of faces in zone of ℓ is $\leq 5n$



by induction on n. $n=1$ true

Let $r \in L$ be line intersecting ℓ farthest to right

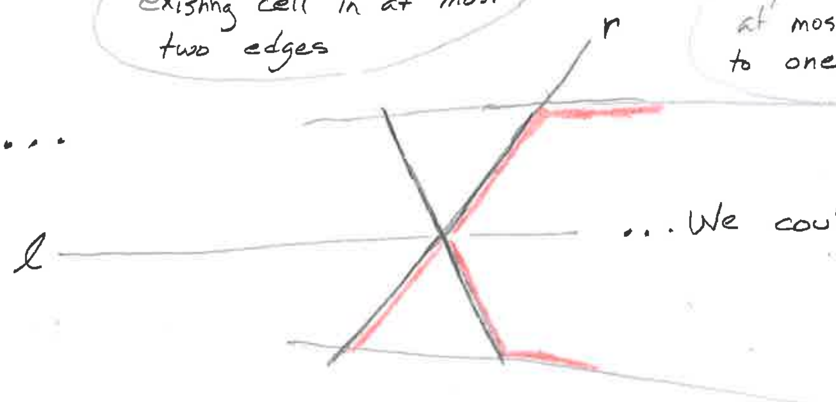
By induction the zone of ℓ in $A(L \setminus \{r\})$ has $\leq 5(n-1)$ left bounding edges.

Adding r creates new left-bounding edges on r (exactly 1) and splits old left-bounding edges (at most 2)

any line intersects one existing cell in at most two edges

any line contributes at most one bounding edge to one cell why?

But ...



... We could get 5.

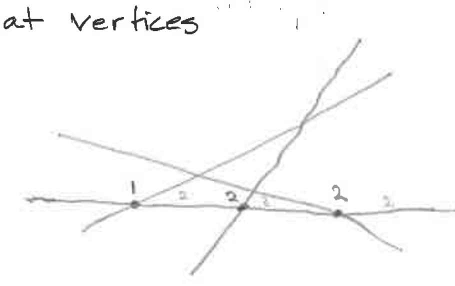
$5(n-1) + 5 \leq 5n$ $\boxtimes$

Incremental construction of arrangement takes

$$O\left(\sum_{i=1}^{n} i\right) = O(n^2) \text{ time.}$$

Can calculate level of each vertex in $O(n^2)$ time

[Walk along each line from left to right
Update level at vertices]



$\Rightarrow$ $O(n^2)$ time discrepency alg.