(L12) Delaunay Triangulation

→ of convex polygon [Chew 1986]

Let S = ccw list of n vertices of convex poly P

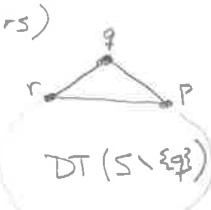
DT(S)

If $|S| = 3$ then return Δ with vertices S

Pick q at random from S (let p and r be its neighbors)

$T = DT(S \setminus \{q\}) + \Delta pqr$ ← this Δ and others may be bad (the circle of others might contain q)

return Flip(T, q, rp)



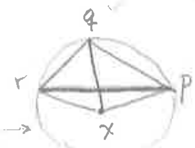
Flip(T, q, rp)

If $\overline{rp}$ is bad

remove $\overline{rp}$ from T add $\overline{qx}$ to T (x opposite rp from q)

Flip (T, q, rx)

Flip (T, q xp)



or find bad Δ's in T starting at Δpqr (They form a connected component in the planar dual of T)

What is number of bad Δ's in T ?

- Proportional to the degree of q in DT(S).

So expected runtime for Flip is O(average degree of vertex)

(Backwards Analysis)

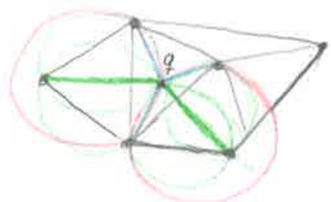
$$= \frac{\sum_{q \in S} \deg_{DT(S)}(q)}{n} = \frac{2(\#\text{edges in } DT(S))}{n} = \frac{2(2n-3)}{n} = 4 - 6/n$$

Thus expected runtime is $O(n)$

Incremental Delaunay △tion (of arbitrary 2D point set)

Same idea but ...

Adding q to $DT(S \setminus \{q\})$ requires finding △ in $DT(S \setminus \{q\})$ that contains q



Option 1 Maintain a search structure for $DT(S \setminus \{q\})$

Option 2 Rebucket remaining points to be added into newly created △s

[Both add expected $O(\log n)$ time to the cost to add q.

What is probability that a point x is rebucketed when $|S| = i$?

= prob. △ containing x in $DT(S)$ is created by adding q

$\cong 3/i$

$$\Rightarrow E[\#\text{ rebuckets for } x] \leq \sum_{i=1}^{n} 3/i = O(\log n)$$

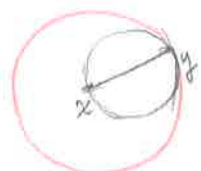
Relatives of Delaunay △tions.

① Nearest Neighbor graph of S   NN(S)

Draw edge $x \to y$ if y is closest to x $x, y \in S$

Claim $NN(S) \subseteq DT(S)$

$xy \in NN(S) \Rightarrow xy \in DT(S)$

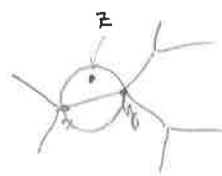


Big circle empty $\Rightarrow$ Small circle empty

② Minimum Spanning tree of S   MST(S)

Claim $MST(S) \subseteq DT(S)$

$\Leftarrow$ circle with diameter $xy \in MST(S)$ is empty



proof if z in this circle

remove $\overline{xy}$ from $MST(S)$

x and y now in two separate trees

z in one of these already (say x's)

then add $\overline{yz}$ to connect trees.

③ Relative Nbrhood Graphs   RNG(S)

add $\overline{xy}$ if lens is empty

④ Gabriel Graph   GG(S)

$$NN(S) \subseteq MST(S) \subseteq RNG(S) \subseteq GG(S) \subseteq DT(S)$$