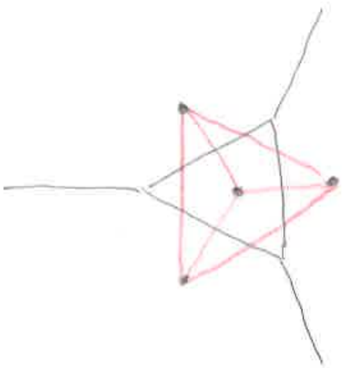
CPSC 516 13 October 2022 (1)

L 11

Delaunay Triangulation



Connect two sites iff they share a Voronoi Edge

i.e. $D(P)$ is the planar dual of $V(P)$

Properties (1) $p_i p_j$ is Delaunay edge iff (by def)

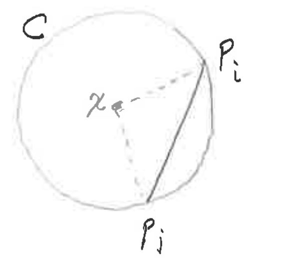
$V(p_i)$ and $V(p_j)$ share Vor. Edge iff

$\exists$ point x (not a site) that is closer (and equal distance) to p_i and p_j than any other site iff

$\exists$ Circle C (with center x) through p_i and p_j that is empty of other sites

Note: $\overline{x p_i}$ is in $V(p_i)$

$\overline{x p_j}$ is in $V(p_j)$



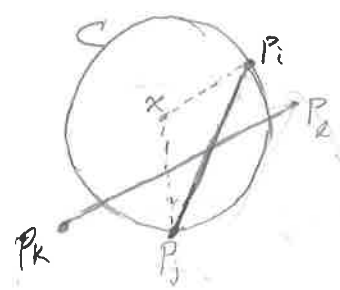
(2) No Delaunay Edges cross.

Suppose $p_i p_j$ and $p_k p_\ell$ cross

p_k and p_ℓ must be outside circle C

$\Rightarrow$ p_k and p_ℓ outside $\triangle p_i x p_j$

$\Rightarrow$ $\overline{p_k p_\ell}$ intersects $\overline{x p_i}$ or $\overline{x p_j}$



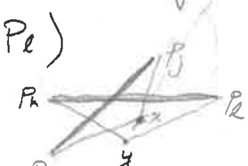
Similarly $\overline{p_i p_j}$ intersects $\overline{y p_k}$ or $\overline{y p_\ell}$ (for empty circle with center y through p_k and p_ℓ)

$\Rightarrow$ one of $\overline{x p_i}$ or $\overline{x p_j}$ intersects $\overline{y p_k}$ or $\overline{y p_\ell}$

say...

But $\overline{x p_i} \in V(p_i)$ and $\overline{y p_k} \in V(p_k)$

$\Rightarrow\Leftarrow$



Hard to draw b/c impossible.

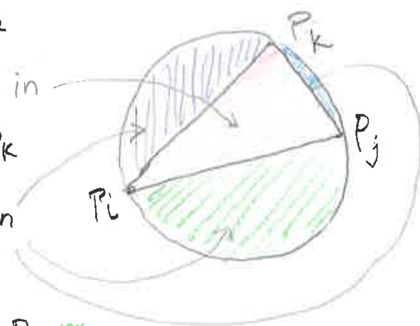
(3) [Sites $p_i\, p_j\, p_k$ form a face in Del. Δtion iff there is an empty circle through them]

$\Leftarrow$ empty circle implies $\overline{p_i p_j}$, $\overline{p_j p_k}$, $\overline{p_k p_i}$ are Del. Edges and nothing can cross them so its a face

$\Rightarrow$ if $p_i\, p_j\, p_k$ is a face then $\bigcirc p_i\, p_j\, p_k$ is empty since

no sites in $\Delta p_i p_j p_k$

and no site in



otherwise $p_i p_j$ / $p_j p_k$ / $p_k p_i$ would not be Del Edge

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Alternative definition of Delaunay triangulation

[All triangles* whose bounding circle contains no other sites]

* assuming no 4 colinear points

Fortune's Sweep parabolic front

Incremental Delaunay