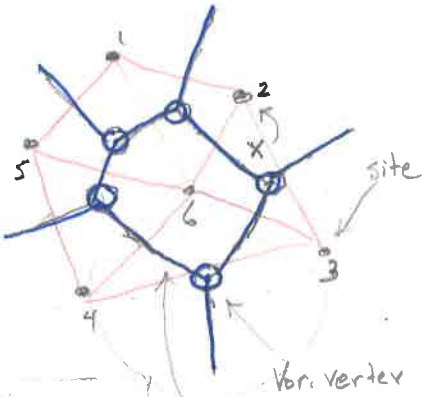
(L10) Descartes 1644 # Voronoi Diagrams + Delaunay triangulations

Post office problem (or closest charging station)
Given a point x, find the closest post office

Applications: Skipping, image dation (web)



Voronoi diagram using cones (web)

Voronoi diagram of post office sites $P = \{p_1, p_2, \dots p_n\}$

Voronoi region for p_i

$$V(p_i) = \{ x \in \mathbb{R}^{d=2} \mid d(x,p_i) \le d(x,p_j)\ \forall j \}$$

$$Vor(P) = V(p_1)\ V(p_2) \dots V(p_n)$$

points closer to p_i than to p_j for all $j = 1, 2, \dots$

Vor Region is a convex polygon

Unbounded iff site is on convex hull of sites

Voronoi vertex has degree 3 if no 4 co-circular sites

Vor. vertex is center of an empty circle that goes through its 3 (or more) closest sites

Add vertex at infinity

$|F| = \#\text{sites} = n$

All vertices have degree ≥ 3

$$2|E| = \sum_{v \in V} \deg v \ge 3|V|$$

$$2 = V - E + F \le V - \tfrac{3}{2}V + n$$

$$\Rightarrow V \le 2(n-2)$$

$$E = V + F - 2 \le 3(n-2)$$

vor. vertices $< 2(n-2)$

vor. edges $< 3(n-2)$