L8

Convex Hulls in d-dimensions ($d > 3$)

Clarkson + Shor 1989 Seidel 1991

Why can't we make a 3D movie of 4D CH?

Let S be set of n points in $\mathbb{R}^d$

[no $d+1$ points lie in a hyperplane]

If P is a simplicial d-polytope with vertex set V of size m

Fact (A) P can have at most $O(m^{\lfloor d/2 \rfloor})$ faces (including 0-faces, 1-faces, ..., $(d-2)$ faces = ridges, $(d-1)$ faces = facets; dimension)

Every facet is uniquely identified by d-tuple of its vertices

" ridge " $(d-1)$-tuple "

Every ridge is contained in two facets.

Let $G(P)$ be the facet graph of P with facets of P as nodes and an edge between facets that share a ridge

Fact $G(P)$ is regular of degree d

A facet F of P is visible from p iff F's hyperplane separates P from p.

A face X of P is visible from p iff X is only in facets visible from p.

X is a horizon face wrt p iff X is in a facet visible from p and a facet invisible from p

Incremental Algorithm Step (add a new point p to existing CH P to get $P' = \text{conv}(P \cup \{p\})$)

1. No visible (from p) face of P is a face of P'
2. All invisible and horizon faces of P are faces of P'
3. For each horizon face X of P, the pyramid $\text{conv}(G \cup \{p\})$ is a face of P'

These are all the faces of P'.

① Determine $Vis(p, P)$, visible facets of P from p (if none are visible, P contains p and done)

② Determine visible and horizon ridges of P from $Vis(p, P)$ (check visibility of facets sharing ridge)

Delete visible facets and ridges

③ For each horizon ridge X generate the new facet $conv(X \cup \{p\})$ for P'

④ Generate new ridges for P'

Step ② takes time $O(|Vis(p, P)|)$ but all these faces are deleted and never reappear so charge cost to their creation.

Step ③ # new facets created = # facets of P' that contain p
$= deg(p, P')$

Step ④ # new ridges created = $(d-1)\, deg(p, P')/2$
to add them as edges of $G(P')$
Find $d-1$ ridges for each new facet
Radix sort them [$(d-1)$-tuples of vertex indices] to match them up
Time = $n + deg(p, P')$

Total time without step 1
is $n + deg(p, P')$ (or $deg(p, P')$ for $d=2,3$)
↑ but this could be big