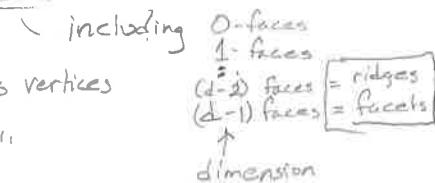


L8

Convex Hulls in d -dimensions ($d > 3$)

Clarkson + Shar 1989 Seidel 1991

Let S be set of n points in \mathbb{R}^d [no $d+1$ points lie in a hyperplane]why can't we
make a 3D movie
of 4D CH?If P is a simplicial d -polytope with vertex set V of size m Fact A P can have at most $O(m^{\lfloor d/2 \rfloor})$ facesEvery facet is uniquely identified by d -tuple of its vertices
 " ridge " $(d-1)$ -tuple "

Every ridge is contained in two facets.

Let $G(P)$ be the facet graph of P with facets of P as nodes
and an edge between facets that share a ridgeFact $G(P)$ is regular of degree d .A facet F of P is visible from p iff F 's hyperplane separates P from p .A face X of P is visible from p iff X is only in facets visible from p . X is a horizon face wrt p iff X is in a facet visible from p
and a facet invisible from p Incremental Algorithm Step (add a new point p to existing CH P
from p to get $P' = \text{conv}(P \cup \{p\})$)

- ① No visible face of P is a face of P'
- ② All invisible and horizon faces of P are faces of P'
- ③ For each horizon face X of P , the pyramid $\text{conv}(G \cup \{p\})$ is a face of P'

These are all the faces of P' .

- $\text{Vis}(P, P)$ from P
- ① Determine visible facets of P (if none are visible, P contains p and done)
 - ② Determine visible and horizon ridges of P from $\text{Vis}(P, P)$
(check visibility of facets sharing ridge)
 - ③ Delete visible facets and ridges
 - ④ For each horizon ridge X generate the new facet
 $\text{conv}(X \cup \{p\})$ for P'
 - ⑤ Generate new ridges for P'

Step ② takes time $O(|\text{Vis}(P, P)|)$ but all these faces are deleted and never reappear so charge cost to their creation.

Step ③ # new facets created = # facets of P' that contain p
 $\equiv \deg(p, P')$

Step ④ # new ridges created = $(d-1) \deg(p, P')/2$
 to add them as edges of $G(P')$
 Find $d-1$ ridges for each new facet
 Radix sort them [($d-1$)-tuples of vertex indices]
 to match them up
 Time = $n + \deg(p, P')$

Total time without step 1
 is $n + \deg(p, P')$ or $\deg(p, P')$ for $d=2,3$)
 ↗ but this could be big