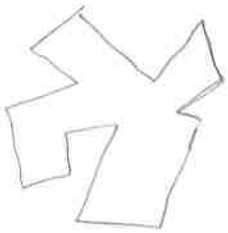
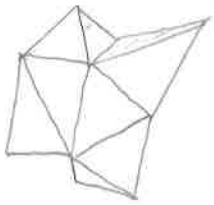
(L6) Convex Hulls in higher dimensions



2D polygon
vertices, edges
0-dim 1-dim

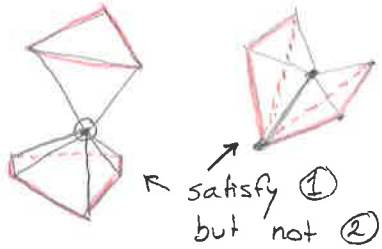


3D polyhedron
vertice, edges, faces
0-dim 1-dim 2-dim — polygons or triangles (since we can △ late any polygon)

Polyhedron (O'Rourke 1993)

other definitions exist

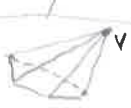
① Faces share a vertex, an edge and 2 vertices, or nothing



↖ satisfy ① but not ②

② Neighborhood of every point is "topologically" an open disk

≡ the link of every vertex is a simple, closed polygonal path



v — set of edges opposite vertex v in all triangles incident to v

donuts allowed!?

③ The surface is connected

≡ the 1-skeleton is connected

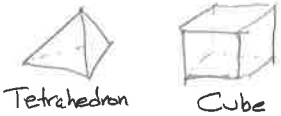
— set of vertices and edges

Regular Polyhedra

All faces are the same (Platonic solids) ← same for all faces

Let p = # vertices per face

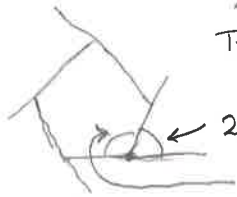
f = # faces that share vtx ← same for all vertices



Tetrahedron 3, 3 | Cube 4, 3 | Octahedron 3, 4 | Dodecahedron 5, 3 | Icosahedron 3, 5

Thm Only 5 Platonic Solids

Proof



$2\pi/p$ since ext. angles sum to π

$\pi - 2\pi/p = \pi(1 - 2/p)$

$f\,\pi(1-2/p) < 2\pi$

$\Rightarrow 1 - 2/p < 2/f$

$\Rightarrow pf < 2p + 2f$ (why?)

$\Rightarrow (p-2)(f-2) < 4$ and $p \geq 3$ and $f \geq 3$

☒