CPSC 516 Sept 15, 2022

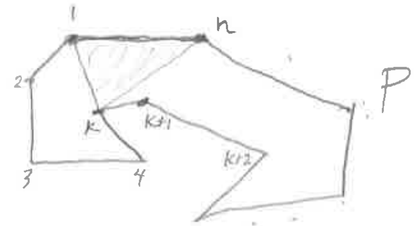
L3

Best Δtion or Partition of simple polygon P

(1) Optimal Δtion = Δtion with minimum total chord length

Dynamic Programming

- Break big problem into smaller problems
- Solve problems from smallest to biggest



(A) Edge $v_1 v_n$ is part of some $\Delta v_1 v_k v_n$ (try all values of k)

(B) This splits problem into (at most) two subproblems:
$OPT(v_1 v_2 .. v_k)$ and $OPT(v_k v_{k+1} .. v_n)$

(C) Solve these "recursively"

General form of subproblem $OPT(v_i v_{i+1} \dots v_j)$
$< n^2$ subproblems!

For $i<j$ $$OPT(v_i v_{i+1} .. v_j) = \min_k \underbrace{|v_i v_k|_p}_{\text{length in } P} + |v_j v_k|_p + OPT(v_i v_{i+1} .. v_k) + OPT(v_k v_{k+1} \cdots v_j)$$

length in P $= \infty$ if outside P

$O(n^3)$ in Real RAM model : calculate distance btwn points in constant time

$$|pq| = \sqrt{\underbrace{(p_x - q_x)^2 + (p_y - q_y)^2}_{\text{integer if integer coords}}}$$

AND compare $\sqrt{a_1} + \sqrt{a_2} \dots + \sqrt{a_s}$
vs $\sqrt{b_1} + \sqrt{b_2} \dots + \sqrt{b_t}$
(not integer comparison)

(2) Partion simple polygon into <u>convex</u> pieces

<u>Optimally</u> (min # convex pieces) M. Keil 1985

$O(n^3 \log n)$ using dyn. prog.

<tricky>

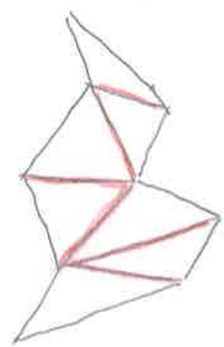
<u>Approximately Optimally</u> Hertel and Mehlhorn 1983

$O(n \log n)$

Finds partition with $< 4 \cdot |OPT|$ convex pieces

(A) Triangulate P

(B) Remove any chord that leaves only convex pieces (one at a time)



$\boxed{\# \text{ chords} = \# \text{ pieces} - 1}$

surv. chords $\leq 2r$ $\langle r = \#\text{reflex vtcs} \rangle$

- Every reflex vertex can be that endpoint for ≤ 2 surv. chords why?

→ Every surv. chord would leave reflex angle (if removed) at ≥ 1 of its endpoints WHY?

surv. chords $\leq 2r$ $\Rightarrow$ $\boxed{\overset{\text{ALG.}}{\#\text{ pieces}} \leq 2r + 1}$

Every reflex vertex touches a chord in OPT and one chord can touch ≤ 2 reflex vertices

So # OPT chords $\geq \lceil r/2 \rceil$ $\Rightarrow$ OPT pieces $\geq \lceil r/2 \rceil + 1$

Thus # ALG pieces ≤ 4 OPT