CPSC 516 Problem Set 1 16 Sep 2022
W. Evans Due: 29 Sept 2022

Remember: Acknowledge any collaboration or resource.

1. Suppose we partition a simple polygon with n vertices into pieces by adding chords. Prove that the sum of the number of vertices in all of the pieces is $O(n)$.

2. (from CGAA Exercise 2.12) Let S be a set of n triangles in the plane. The boundaries of the triangles are disjoint, but it is possible that a triangle lies completely inside another triangle. Let P be a set of n points in the plane. Give an $O(n \log n)$ algorithm that reports each point in P lying outside all triangles. (Describe your algorithm in pseudocode, at a high level. You may use common data structures without specifying their implementation.)

3. (from O'Rourke "Computational Geometry in C" Exercise 2.2.4[7]) An *orthogonal polygon* is a polygon in which each pair of adjacent edges meets orthogonally. Without loss of generality, one may assume that the edges alternate between horizontal and vertical.

   An *orthogonal pyramid* P is an orthogonal polygon monotone with respect to the vertical, that contains one horizontal edge h whose length is the sum of the lengths of all the other horizontal edges. P consists of two "staircases" connected to h, as shown below.

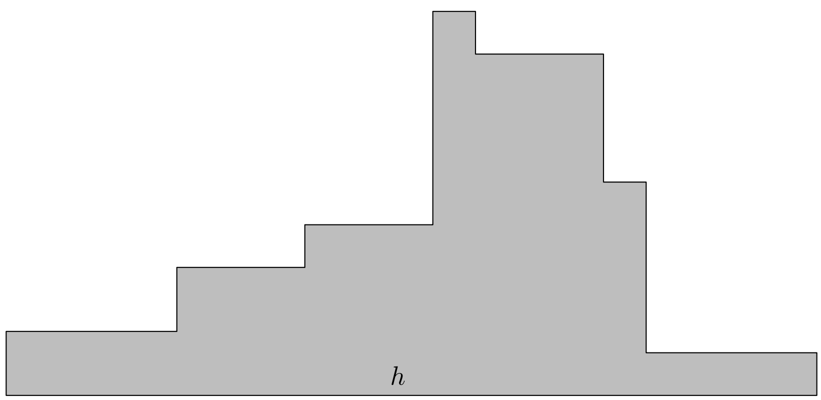

Figure 1: Orthogonal pyramid.

   (a) Prove that an orthogonal pyramid may be partitioned by chords into *convex* quadrilaterals.

   (b) Design an algorithm for finding such a partition. Try for linear-time complexity. Describe your algorithm in pseudocode, at a high level, ignoring data structure details and manipulations.

4. Describe an algorithm that takes as input a set of n points in the plane and outputs a simple polygon whose vertices are these n points. The output should be a list of the vertices of the polygon in counterclockwise order. What is the asymptotic running time of your algorithm? Argue that no asymptotically faster algorithm for this problem exists.