

# Complete Knowledge Assumpt

- Sometimes you want to assume that a database is complete. Any fact not listed is false.
- **Example:** Assuming a database of *enrol* relations is complete, you can define *emp*
- The definite clause RRS is **monotonic:** a conclusion doesn't invalidate a previous conclusion.
- With the complete knowledge assumption, the RRS is **nonmonotonic:** a conclusion can be invalidated by adding more clauses.

## CKA: propositional ca

- Suppose the rules for atom  $a$  are

$$a \leftarrow b_1.$$

⋮

$$a \leftarrow b_n.$$

or equivalently:  $a \leftarrow b_1 \vee \dots \vee b_n$

- Under the CKA, if  $a$  is true, one of the  $b_i$

$$a \rightarrow b_1 \vee \dots \vee b_n.$$

- Under the CKA, the clauses for  $a$  mean

**Clark's completion:**

$$a \leftrightarrow b_1 \vee \dots \vee b_n$$

## CKA: Ground Databases

**Example:** Consider the relation defined by:

*student(mary).*

*student(john).*

*student(ying).*

The CKA specifies these three are the only students:

$$student(X) \leftrightarrow X = mary \vee X = john \vee X = ying$$

To conclude  $\neg student(alan)$ , you have to be able to prove:

$$alan \neq mary \wedge alan \neq john \wedge alan \neq ying$$

This needs the unique names assumption.

# Clark Normal Form

The **Clark normal form** of the clause:

$$p(t_1, \dots, t_k) \leftarrow B$$

is the clause

$$p(V_1, \dots, V_k) \leftarrow \\ \exists W_1 \dots \exists W_m V_1 = t_1 \wedge \dots \wedge V_k =$$

where  $V_1, \dots, V_k$  are  $k$  different variables that  
in the original clause.

$W_1, \dots, W_m$  are the original variables in the cl

## Clark normal form: example

➤ The Clark normal form of:

$$room(C, room208) \leftarrow cs\_course(C) \wedge enrollment(C, E)$$

is

$$room(X, Y) \leftarrow \exists C \exists E X = C \wedge Y = E \wedge cs\_course(C) \wedge enrollment(C, E)$$

# Clark's Completion of a Predicate

Put all of the clauses for  $p$  into Clark normal form with the same set of introduced variables:

$$p(V_1, \dots, V_k) \leftarrow B_1$$

$$\vdots$$

$$p(V_1, \dots, V_k) \leftarrow B_n$$

This is the same as:  $p(V_1, \dots, V_k) \leftarrow B_1 \vee \dots \vee B_n$ .

**Clark's completion** of  $p$  is the equivalence

$$p(V_1, \dots, V_k) \leftrightarrow B_1 \vee \dots \vee B_n,$$

That is,  $p(V_1, \dots, V_k)$  is true if and only if one

# Clark's Completion Example

Given the *mem* function:

$$mem(X, [X|T]).$$

$$mem(X, [H|T]) \leftarrow mem(X, T).$$

the completion is

$$mem(X, Y) \iff (\exists T Y = [X|T]) \vee \\ (\exists H \exists T Y = [H|T] \wedge mem(X, T))$$

## Clark's Completion of a

- **Clark's completion** of a knowledge base is the completion of every predicate symbol, along with axioms for equality and inequality.
- If you have a predicate  $p$  defined by no clauses in a knowledge base, the completion is  $p \leftrightarrow \text{false}$  and  $\neg p$ .
- You can interpret negations in the bodies of clauses to mean that  $p$  is false under the Complete Assumption. This is called **negation as failure**.



## Using negation as failure

Previously we couldn't define *empty\_course*(*C*) in a database of *enrolled*(*S*, *C*).

This can be defined using negation as failure:

$$\begin{aligned} \textit{empty\_course}(\mathbf{C}) \leftarrow \\ & \textit{course}(\mathbf{C}) \wedge \\ & \sim \textit{has\_Enrollment}(\mathbf{C}). \end{aligned}$$

$$\begin{aligned} \textit{has\_Enrollment}(\mathbf{C}) \leftarrow \\ & \textit{enrolled}(\mathbf{S}, \mathbf{C}). \end{aligned}$$

# Bottom-up NAF proof procedure

$C := \{\}$ ;

repeat

either select “ $h \leftarrow b_1 \wedge \dots \wedge b_m$ ”  $\in KB$  s.t.

$b_i \in C$  for all  $i$ , and  $h \notin C$ ;

$C := C \cup \{h\}$

or select  $h$  such that

for every rule “ $h \leftarrow b_1 \wedge \dots \wedge b_m$ ”

either for some  $b_i, \sim b_i \in C$

or some  $b_i = \sim g$  and  $g \in C$

$C := C \cup \{\sim h\}$

until no more selections are possible

## Negation as failure exam

$$p \leftarrow q \wedge \sim r.$$

$$p \leftarrow s.$$

$$q \leftarrow \sim s.$$

$$r \leftarrow \sim t.$$

$$t.$$

$$s \leftarrow w.$$

## Top-Down NAF Proceed

- If the proof for  $a$  fails, you can conclude  $\sim a$ .
- Failure and success can be defined recursively.

➤ Suppose you have rules for atom  $a$ :

$$a \leftarrow b_1$$

$$\vdots$$

$$a \leftarrow b_n$$

If each body  $b_i$  fails,  $a$  fails.

If one body,  $b_i$  succeeds,  $a$  succeeds.

➤ A body fails if one of the conjuncts in

A body succeeds if all of the conjunct

➤ If  $a$  fails,  $\sim a$  succeeds. If  $a$  succeeds

# Free Variables in Negation as

## Example:

$$p(X) \leftarrow \sim q(X) \wedge r(X).$$

$$q(a).$$

$$q(b).$$

$$r(a).$$

$$r(c).$$

There is only one answer to the query  $?p(X)$ ,  $p(a)$ .

For calls to negation as failure with free variables, the system

to **delay** negation as failure goals that contain free variables

until the variables become bound.

# Floundering Goals

If the variables never become bound, a negated goal **flounders**.

In this case you can't conclude anything about the goal.

**Example:** Consider the clauses:

$$p(X) \leftarrow \sim q(X)$$

$$q(X) \leftarrow \sim r(X)$$

$$r(a)$$

and the query

$$?p(X).$$