It is remarkable that a science which began with the consideration of games of chance should become the most important object of human knowledge . . . The most important questions of life are, for the most part, really only problems of probability . . .

The theory of probabilities is at bottom nothing but common sense reduced to calculus.

– Pierre Simon de Laplace [1812]

Learning Objectives

At the end of the class you should be able to:

- justify the use and semantics of probability
- know how to compute marginals and apply Bayes' theorem
- identify conditional independence
- build a belief network for a domain

Using Uncertain Knowledge

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- An agent needs to reason about its uncertainty.

Why Probability?

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- Acting is gambling: agents who don't use probabilities will lose to those who do — Dutch books.
- Probabilities can be learned from data.
  Bayes' rule specifies how to combine data and prior knowledge.

Numerical Measures of Belief

- Belief in proposition, f, can be measured in terms of a number between 0 and 1 — this is the probability of f.
  - The probability f is 0 means that f is believed to be definitely false.
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- Probability is a measure of an agent's ignorance.
- Probability is *not* a measure of degree of truth.

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$$\omega \models \alpha \land \beta \text{ if } \omega \models \alpha \text{ and } \omega \models \beta$$

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(Sometimes the range of a random variable is called it's domain – it is the domain of a function on random variables.)

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- $P(\alpha) = \mu(\{\omega \mid \omega \models \alpha\})$.

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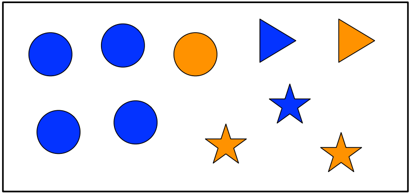

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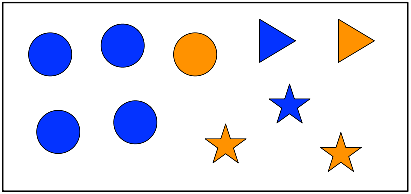

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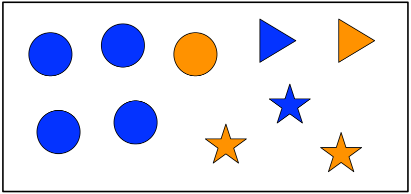

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- What is the probability of circle?
- What us the probability of star?

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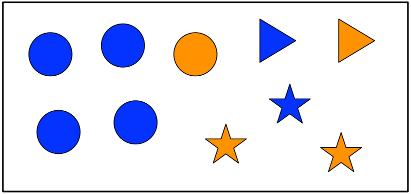

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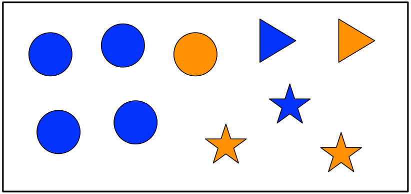

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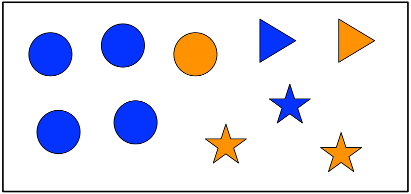

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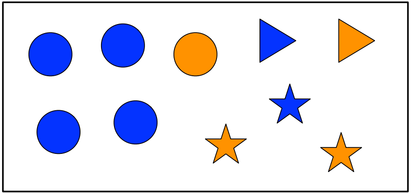

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Note that $P(\alpha \wedge \beta)$ is not a function of $P(\alpha)$ and $P(\beta)$.

Expected Value

- The expected value of numerical random variable X with respect to probability P is

$$\mathbb{E}_P(X) = \sum_{v \in domain(X)} v * P(X{=}v)$$

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- When the domain is continuous, the sum becomes an integral.
- If α is a proposition, treating *true* as 1 and *false* as 0:

$$\mathbb{E}_P(\alpha) = P(\alpha)$$

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  This gives the prior probability.
- All other information must be conditioned on.
- If evidence e is the all of the information obtained subsequently, the conditional probability $P(h \mid e)$ of h given e is the posterior probability of h.

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Conditioning

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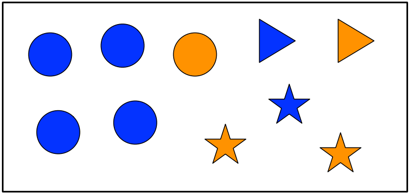

$P(Shape{=}star) = 0.3$

$P(Shape{=}circle) = 0.5$

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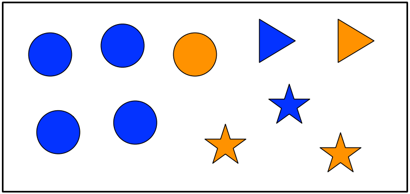

Observe $Color{=}orange$:

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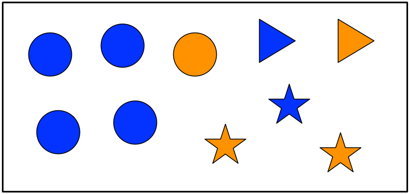

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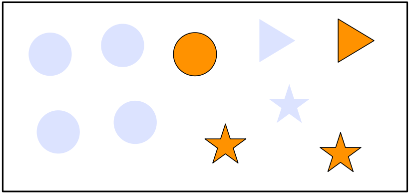

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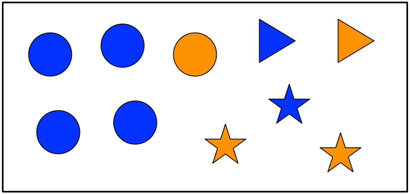

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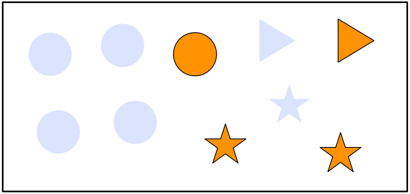

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$$P(Shape{=}star \mid Color{=}orange) = 0.5$$

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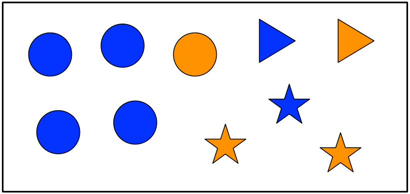

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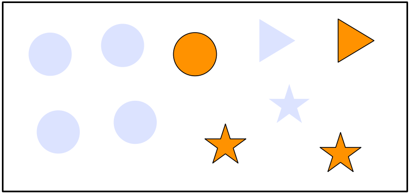

$$P(Shape{=}star \mid Color{=}orange) = 0.5$$
$$P(Shape{=}circle \mid Color{=}orange) = 0.25$$

Clicker Question

Flu	$Sneeze$	$Snore$	μ
true	true	true	0.064
true	true	false	0.096
true	false	true	0.016
true	false	false	0.024
false	true	true	0.096
false	true	false	0.144
false	false	true	0.224
false	false	false	0.336

What is:

(a) $P(flu \wedge sneeze)$

A: 0.04
B: 0.16
C: 0.24
D: 0.4
E: 0.8

Clicker Question

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Clicker Question

Flu	*Sneeze*	*Snore*	μ
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true	true	false	0.096
true	false	true	0.016
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A: 0.04
B: 0.16
C: 0.24
D: 0.4
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What is:

(a) $P(flu \wedge sneeze)$ 0.16

(b) $P(flu \wedge \neg sneeze)$

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Chain Rule: probability of conjunctions

$$P(h \mid e) = \frac{P(h \wedge e)}{P(e)}$$

Therefore

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= \;& \prod_{i=1}^{n} P(f_i \mid f_1 \wedge \cdots \wedge f_{i-1})
\end{aligned}
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Bayes' theorem

The chain rule and commutativity of conjunction ($h \wedge e$ is equivalent to $e \wedge h$) gives us:

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This is Bayes' theorem.

Why is Bayes' theorem interesting?

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  $P(\textit{a tree is in front of a car} \mid \textit{image looks like}$ $)$

Exercise

A cab was involved in a hit-and-run accident at night. Two cab companies, the Green and the Blue, operate in the city. You are given the following data:

- 85% of the cabs in the city are Green and 15% are Blue.
- A witness identified the cab as Blue. The court tested the reliability of the witness in the circumstances that existed on the night of the accident and concluded that the witness correctly identifies each one of the two colors 80% of the time and failed 20% of the time.

What is the probability that the cab involved in the accident was Blue?

[From D. Kahneman, Thinking Fast and Slow, 2011, p. 166.]