

Stat521A Spring 2009: homework 1

1 Conditional independence

(Source: Koller)

Are the following properties true? Prove or disprove. Note that we are not restricting attention to distributions that can be represented by a graphical model.

- Is the following true?

$$(X \perp W|Z, Y) \wedge (X \perp Y|Z) \Rightarrow (X \perp Y, W|Z) \quad (1)$$

- Is the following true?

$$(X \perp Y|Z) \wedge (X \perp Y|W) \Rightarrow (X \perp Y|Z, W) \quad (2)$$

2 Marginalizing a node in a DGM

(Source: Koller)

Consider the DAG G in Figure 1. Assume it is a minimal I-map for $p(A, B, C, D, E, F, X)$. Now consider marginalizing out X . Construct a new DAG G' which is a minimal I-map for $p(A, B, C, D, E, F)$. Specify (and justify) which extra edges need to be added.

3 Bayes Ball

(Source: Jordan)

Here we compute some global independence statements from some directed graphical models. You can use the “Bayes ball” algorithm, the d-separation criterion, or the method of converting to an undirected graph (all should give the same results).

1. Consider the DAG in Figure 2(a). List all variables X s.t., $X \perp A|B$, i.e., variables that are independent of A given evidence on B .
2. Consider the DAG in Figure 2(b). List all variables X s.t., $X \not\perp A|J$, i.e., variables which depend on A given evidence on J .

4 Markov blanket for a DGM

Prove that the full conditional for node i in a DGM is given by

$$p(X_i|X_{-i}) \propto p(X_i|Pa(X_i)) \prod_{Y_j \in ch(X_i)} p(Y_j|Pa(Y_j)) \quad (3)$$

where $ch(X_i)$ are the children of X_i and $Pa(Y_j)$ are the parents of Y_j .

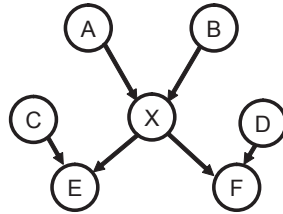


Figure 1: A small DGM.

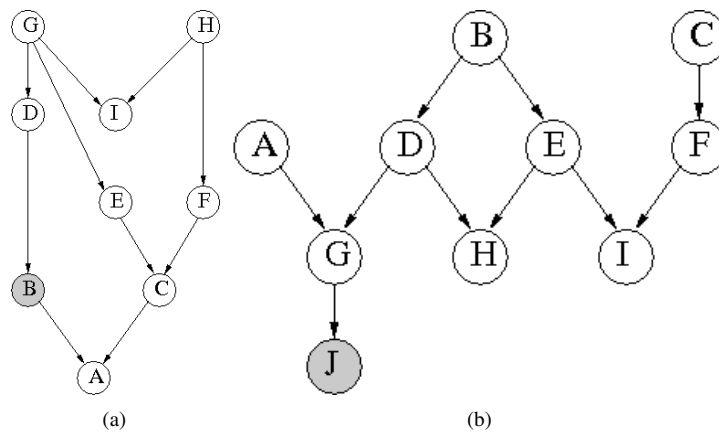


Figure 2: Bayes nets

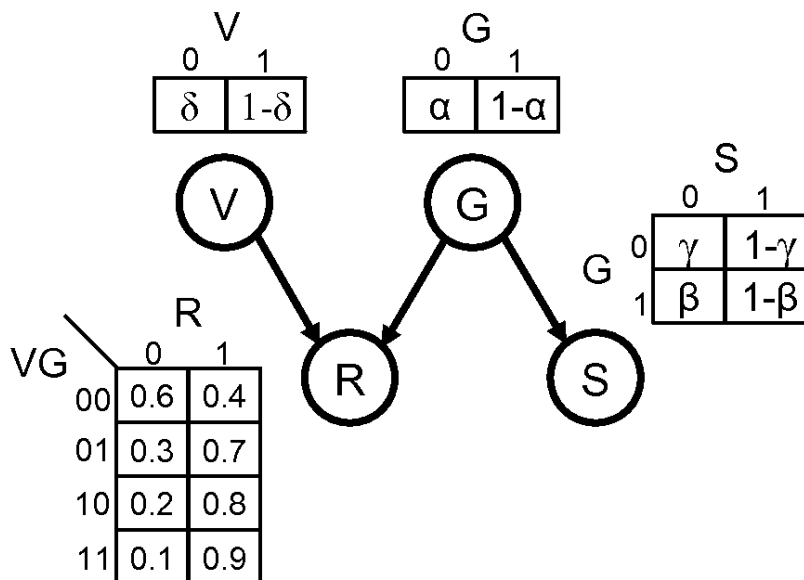


Figure 3: Weather BN.

5 Bayes nets for a rainy day

(Source: de Freitas)

In this question you must model a problem with 4 binary variables: G = "gray", V = "Vancouver", R = "rain" and S = "sad". Consider the directed graphical model describing the relationship between these variables shown in Figure ??.

1. Write down an expression for $P(S = 1|V = 1)$ in terms of $\alpha, \beta, \gamma, \delta$.
2. Write down an expression for $P(S = 1|V = 0)$. Is this the same or different to $P(S = 1|V = 1)$? Explain why.

6 Full conditional in an Ising model

Consider an Ising model

$$p(x_1, \dots, x_n | \theta) = \frac{1}{Z(\theta)} \prod_{\langle ij \rangle} \exp(J_{ij} x_i x_j) \prod_{i=1}^n \exp(h_i x_i) \quad (4)$$

where $\langle ij \rangle$ denotes all unique pairs (i.e., all edges), $J_{ij} \in \mathbb{R}$ is the coupling strength (weight) on edge $i - j$, $h_i \in \mathbb{R}$ is the local evidence (bias term), and $\theta = (\mathbf{J}, \mathbf{h})$ are all the parameters.

If $x_i \in \{0, 1\}$, derive an expression for the full conditional

$$p(x_i = 1 | \mathbf{x}_{-i}, \theta) = p(x_i = 1 | \mathbf{x}_{nb_i}, \theta) \quad (5)$$

where nb_i are the neighbors of i in the graph. Hint: your answer should use the sigmoid/ logistic function $\sigma(z) = 1/(1 + e^{-z})$. Now suppose $x_i \in \{-1, +1\}$. Derive a related expression for $p(x_i | \mathbf{x}_{-i}, \theta)$ in this case. (This result can be used when applying Gibbs sampling to the model.)

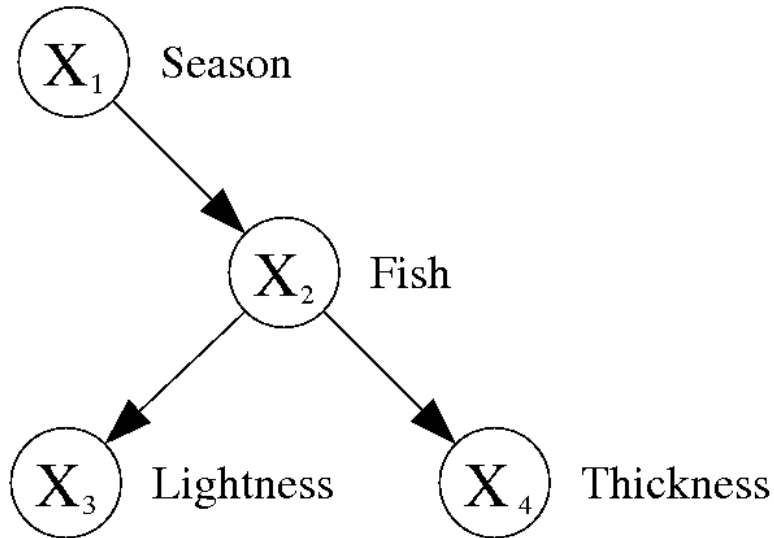


Figure 4: Fishing Bayes net.

7 Fishing nets

(Source: ?)

Consider the Bayes net shown in Figure ???. Here, the nodes represent the following variables

$X_1 \in \{\text{winter, spring, summer, autumn}\}$

$X_2 \in \{\text{salmon, sea bass}\}$

$X_3 \in \{\text{light, medium, dark}\}$

$X_4 \in \{\text{wide, thin}\}$

The corresponding conditional probability tables are

$$p(x_1) = (.25 \ .25 \ .25 \ .25)$$

$$p(x_2|x_1) = \begin{pmatrix} .9 & .1 \\ .3 & .7 \\ .4 & .6 \\ .8 & .2 \end{pmatrix}$$

$$p(x_3|x_2) = \begin{pmatrix} .33 & .33 & .34 \\ .8 & .1 & .1 \end{pmatrix}$$

$$p(x_4|x_2) = \begin{pmatrix} .4 & .6 \\ .95 & .05 \end{pmatrix}$$

Note that in $p(x_4|x_2)$, the rows represent x_2 and the columns x_4 (so each row sums to one and represents the child of the CPD). Thus $p(x_4 = \text{thin}|x_2 = \text{sea bass}) = 0.05$, $p(x_4 = \text{thin}|x_2 = \text{salmon}) = 0.6$, etc.

Answer the following queries. You may use matlab or do it by hand. In either case, show your work.

1. Suppose the fish was caught on December 20 — the end of autumn and the beginning of winter — and thus let $p(x_1) = (.5, 0, 0, .5)$ instead of the above prior. (This is called **soft evidence**, since we do not know the exact value of X_1 , but we have a distribution over it.) Suppose the lightness has not been measured but it is known that the fish is thin. Classify the fish as salmon or sea bass.

2. Suppose all we know is that the fish is thin and medium lightness. What season is it now, most likely? Use $p(x_1) = (.25 \ .25 \ .25 \ .25)$