The Particle Level Set Method

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Joint work with
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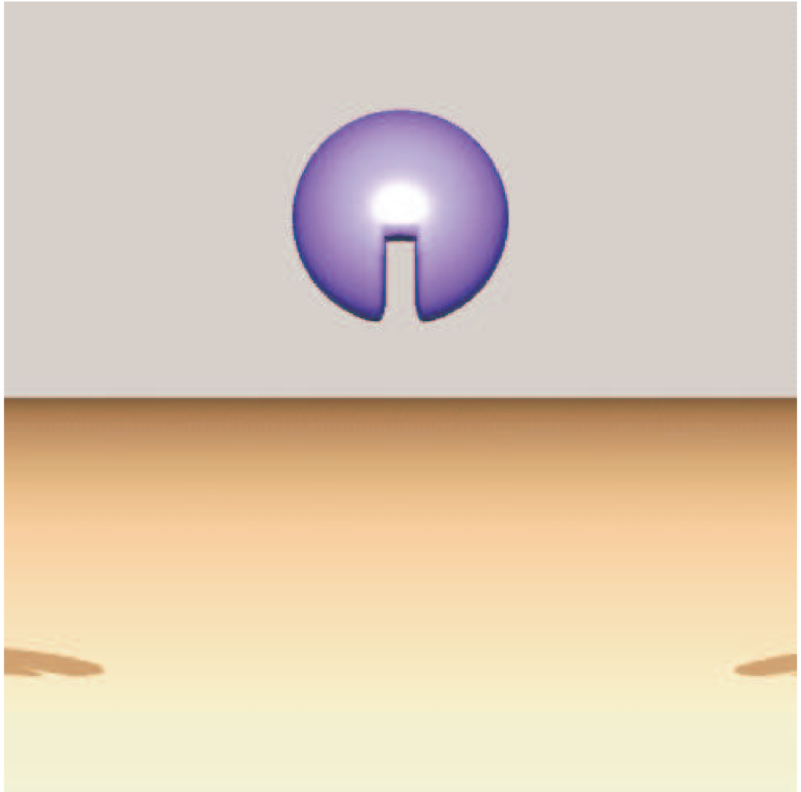
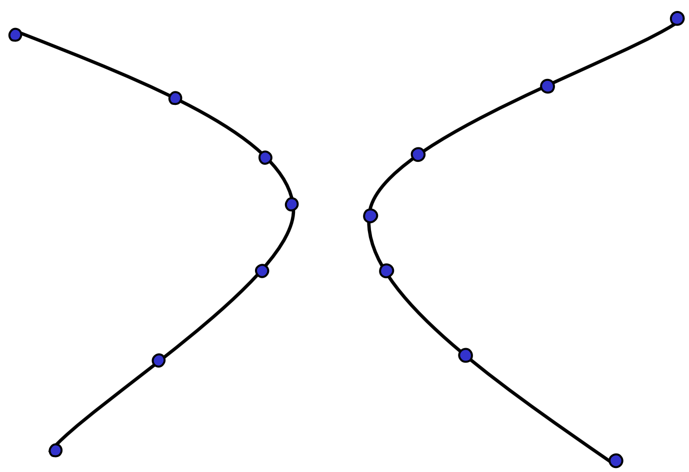
Level Set Methods and Boundaries

- Level sets are just one method of tracking interfaces
- Advantages
  - Geometric information easy to extract
  - Handles merging and breaking interfaces
  - Easy to implement in 3D
- Disadvantages
  - Volume loss

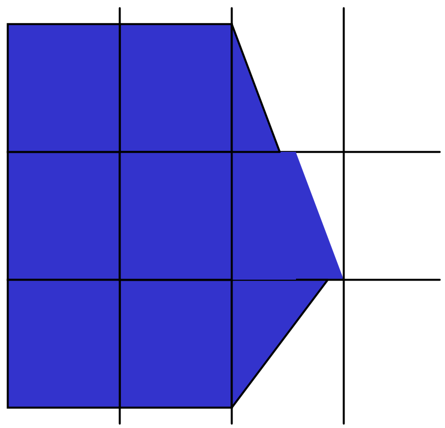


Characterizing Boundaries

- Other ways to model moving boundaries
  - Discretization of front with particles (Tryggvason)
  - Marker particles (Harlow & Welch, Raad & Chen)
  - Characteristic function / Volume of Fluid (Noh & Woodward, Hirt & Nichols, Brackbill, Pilliod & Puckett)



Marker Particle

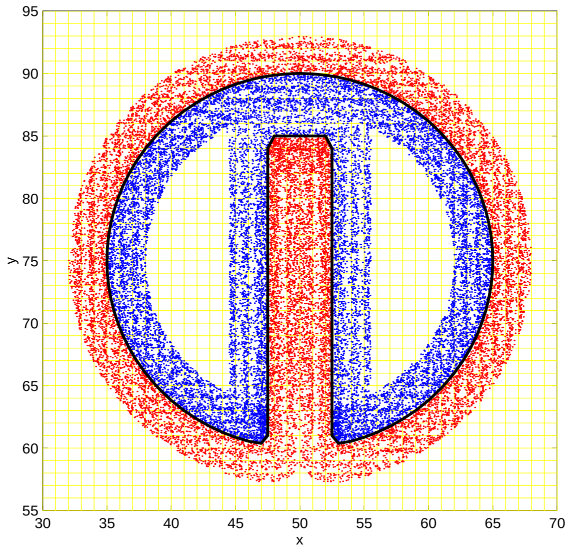


Volume of Fluid

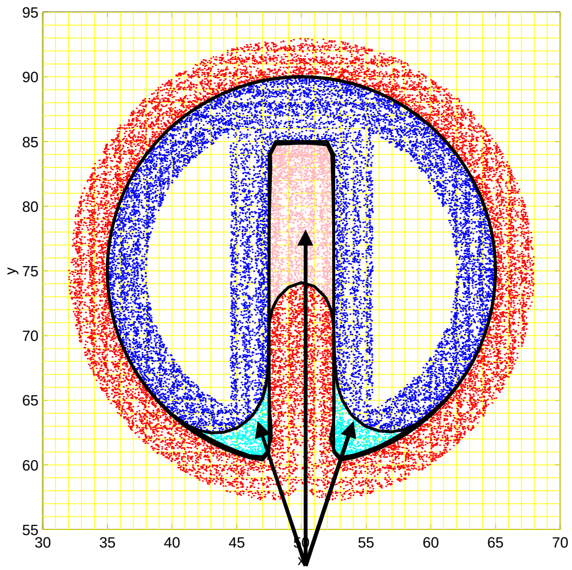
Particle Level Set Method

- Passively advected particles detect and correct errors in regions that the level set solution resolves poorly
  - [Enright, Fedkiw, Ferziger & Mitchell, JCP 2002]
- Example
  - 2D rigid body rotation of notched disk [Zalesak, 1979]
  - 100 x 100 grid, notch width 5, roughly 16 particles per cell

Initial conditions



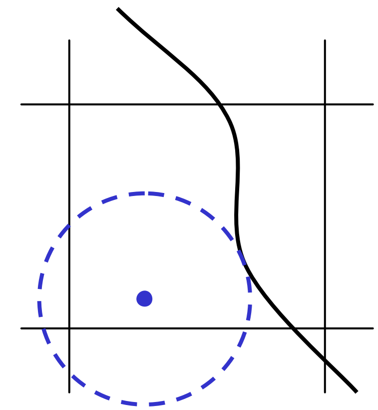
After one rotation



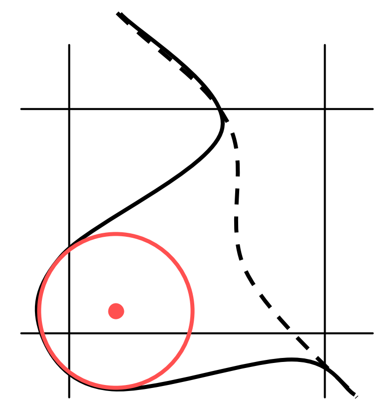
error in regular level set solution

A Mixture of Particles and Level Sets

- For each particle identify
  - Side of interface (inside or outside)
  - Distance to interface (radius)
- If no error, level set and particles are moved independently
  - Particle radii adjust, subject to maximum and minimum
- If a particle is on the wrong side of the interface
  - Adjust nearest node's level set value according to particle's local circular level set
- If multiple particles are on the wrong side
  - First, check particles from the same side and choose deepest node value
  - Second, check particles from opposite sides and choose shallowest node value



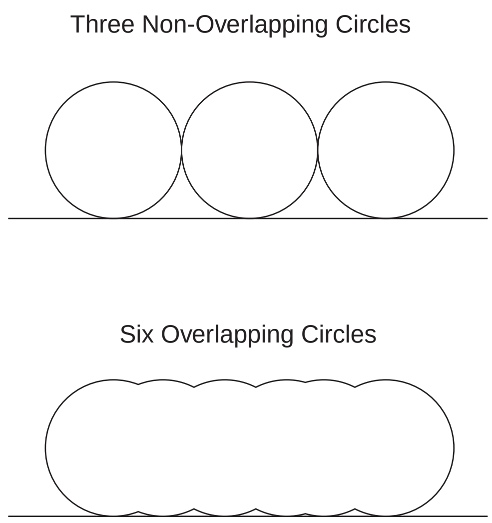
inside particle
inside level set



outside particle
inside level set

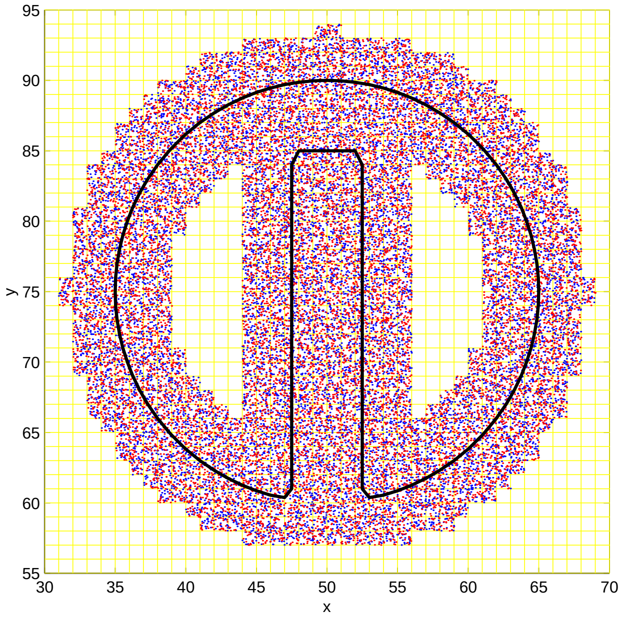
Massless Marker Particles

- Two sets of particles placed near interface
  - Positive particles with s_p = +1 in $\phi > 0$
  - Negative particles with s_p = -1 in $\phi > 0$
- Particles move with $dx_p/dt = u(x_p)$
- Each particle has a variable radius $r_p \in [\Delta x/10, \Delta x/2]$
- Particles used to enhance interface resolution



Initialization of Particles

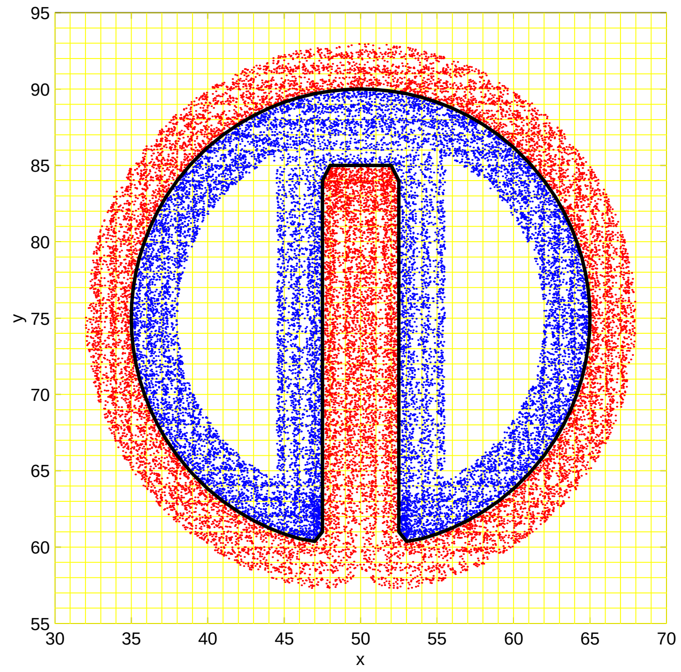
- Particles of both signs are initially randomly placed within a user defined band of the interface $\{ x \mid \phi(x) < 3\,\Delta x \}$
- Number of particles per cell used is dimension dependent, we use 4 per dimension (4 in 1D, 16 in 2D, 64 in 3D, etc)



Particle Attraction

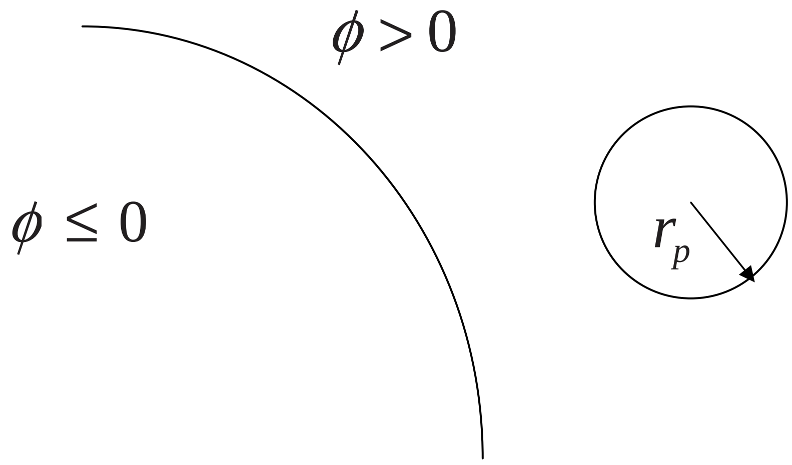
- Need to move particles to the appropriate side of interface
- Direction of shortest path to interface given by $N(x_p)$

$$x'_p = x_p + \lambda(\phi_{goal} - \phi_{current})N(x_p)$$



Coupling I: Identifying Errors

- Error detected if particle is on wrong side of interface
- First order errors in level sets, so only need first order movement of and fix by particles

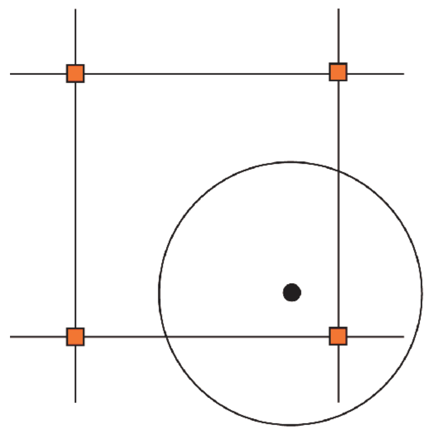


Coupling II: Quantifying Errors

- Spheres associated with particles are generators of locally defined level set functions, $\phi_p(x)$

$$\phi_p(x) = s_p\left(r_p - \|x - x_p\|\right)$$

- $\phi_p(x)$ is defined only on the corners of the cell which contains particle p



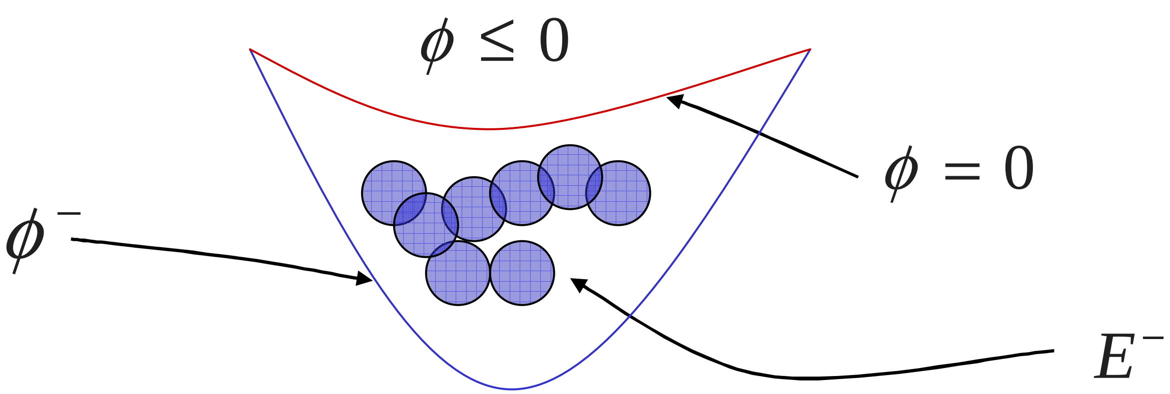
Coupling III: Reducing the Errors

- Use escaped particles to form reduced-error representation of ϕ

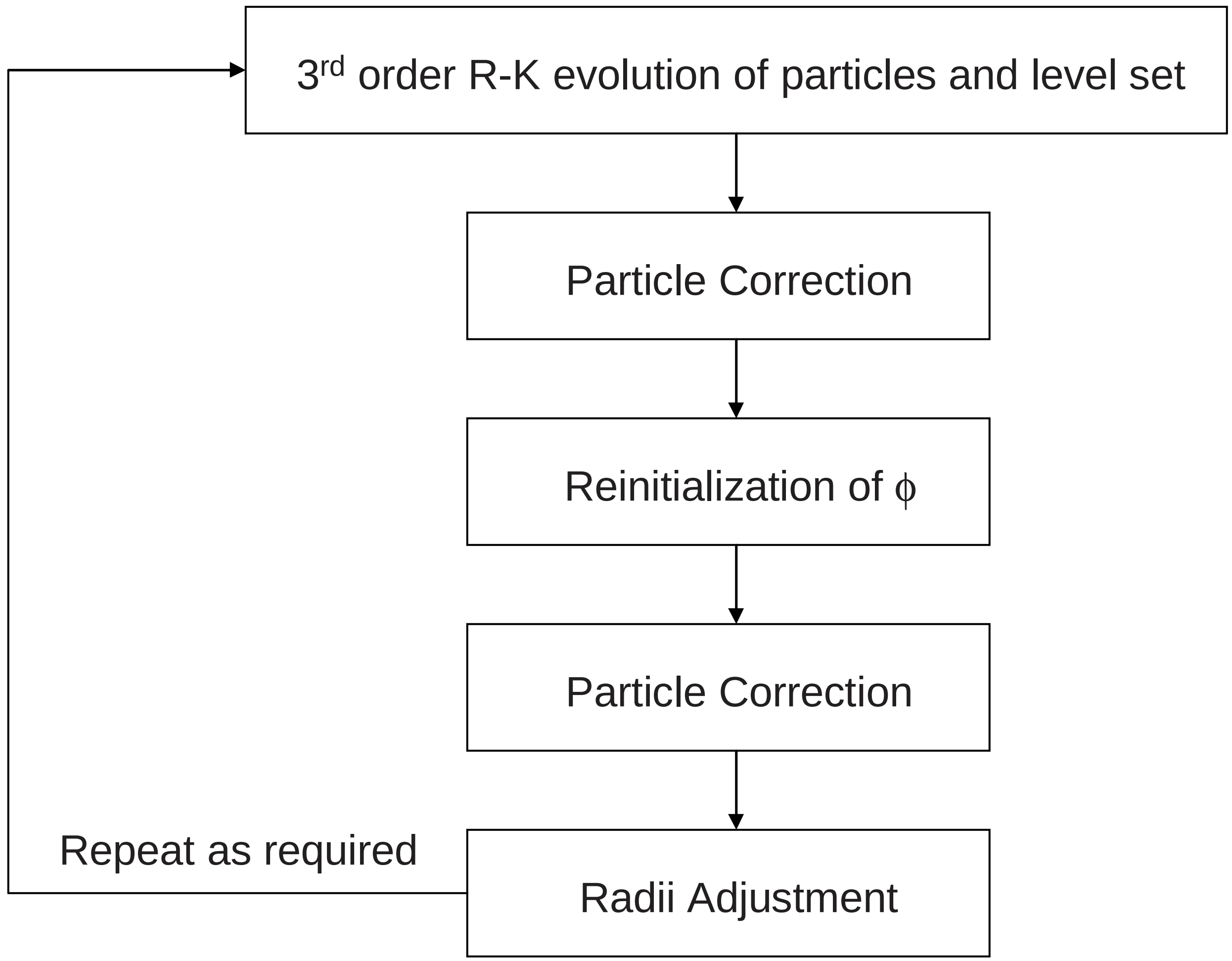
$$\phi^+(x) = \max_{p \in E^+} (\phi(x), \phi_p(x)),$$

$$\phi^-(x) = \min_{p \in E^-} (\phi(x), \phi_p(x)),$$

$$\phi'(x) = \begin{cases} \phi^+(x), & \text{if } |\phi^+(x)| \leq |\phi^-(x)|, \\ \phi^-(x), & \text{otherwise.} \end{cases}$$



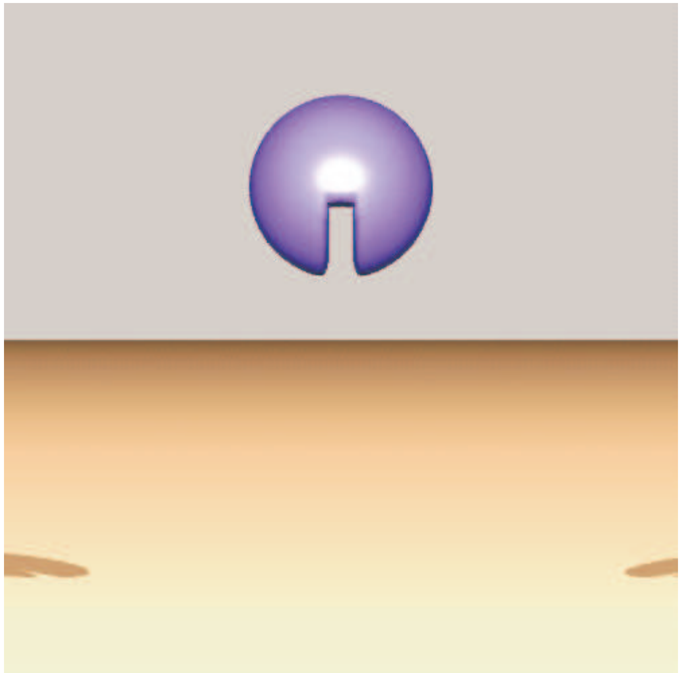
Sample Computational Cycle



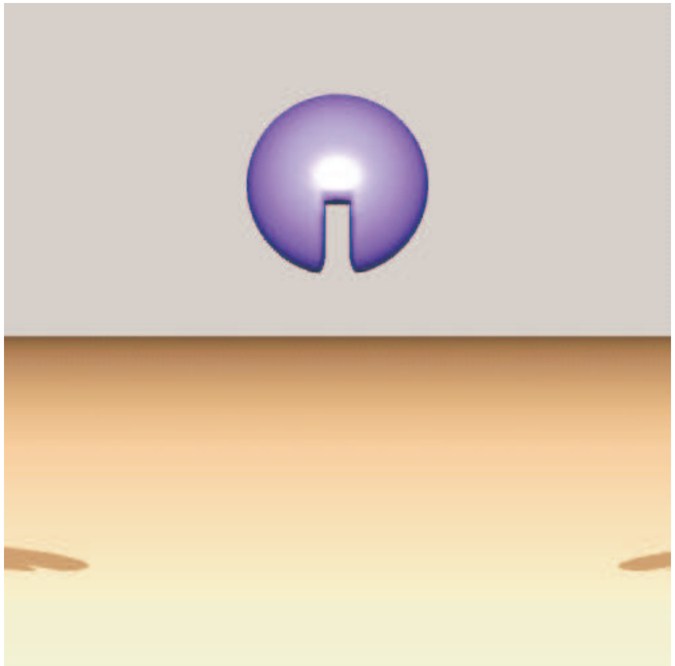
Notched Sphere

- 3D version of Zalesak's disk
- 100^3 grid, notch width 5, roughly 64 particles per cell

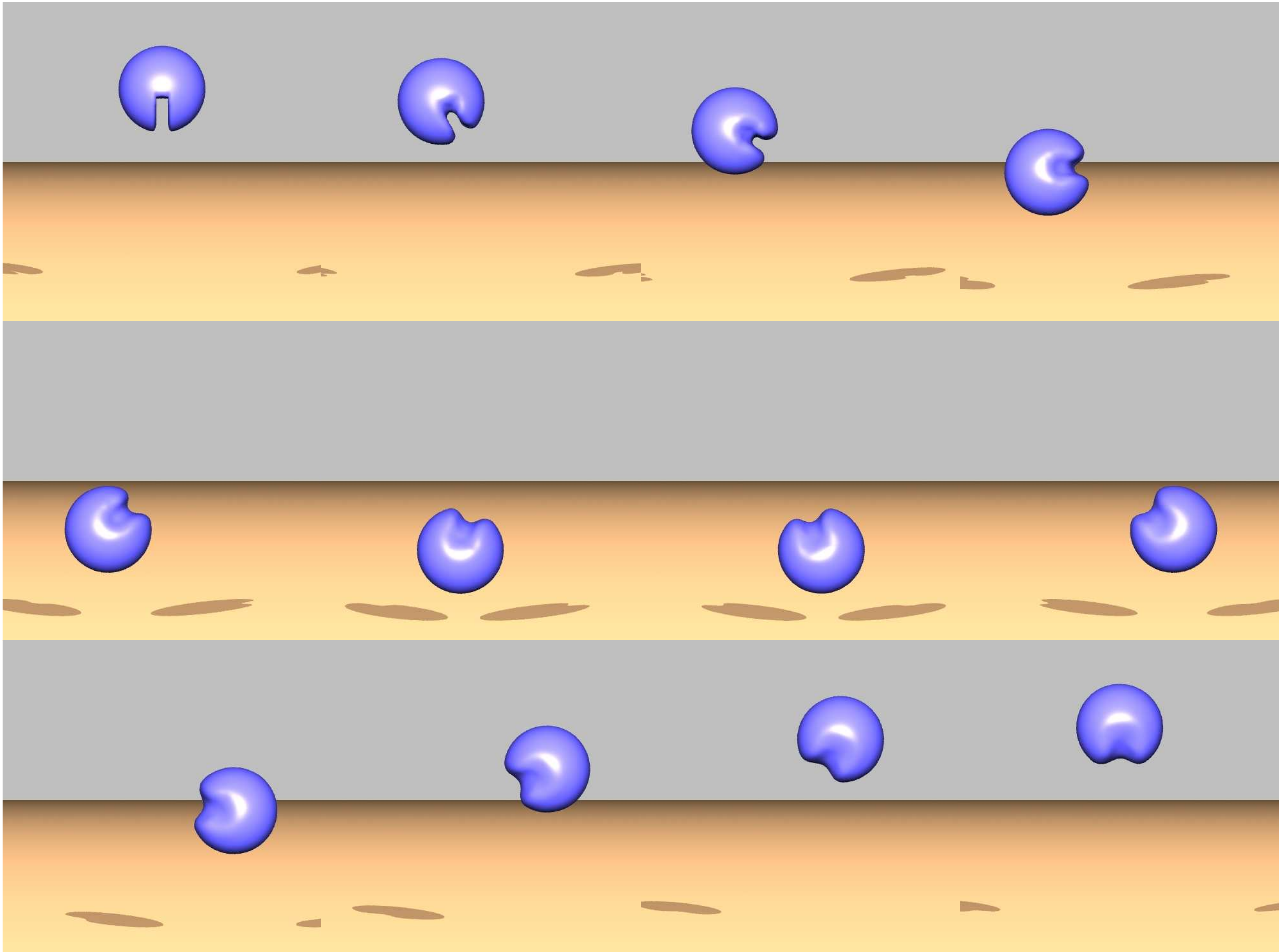
Level Set Only

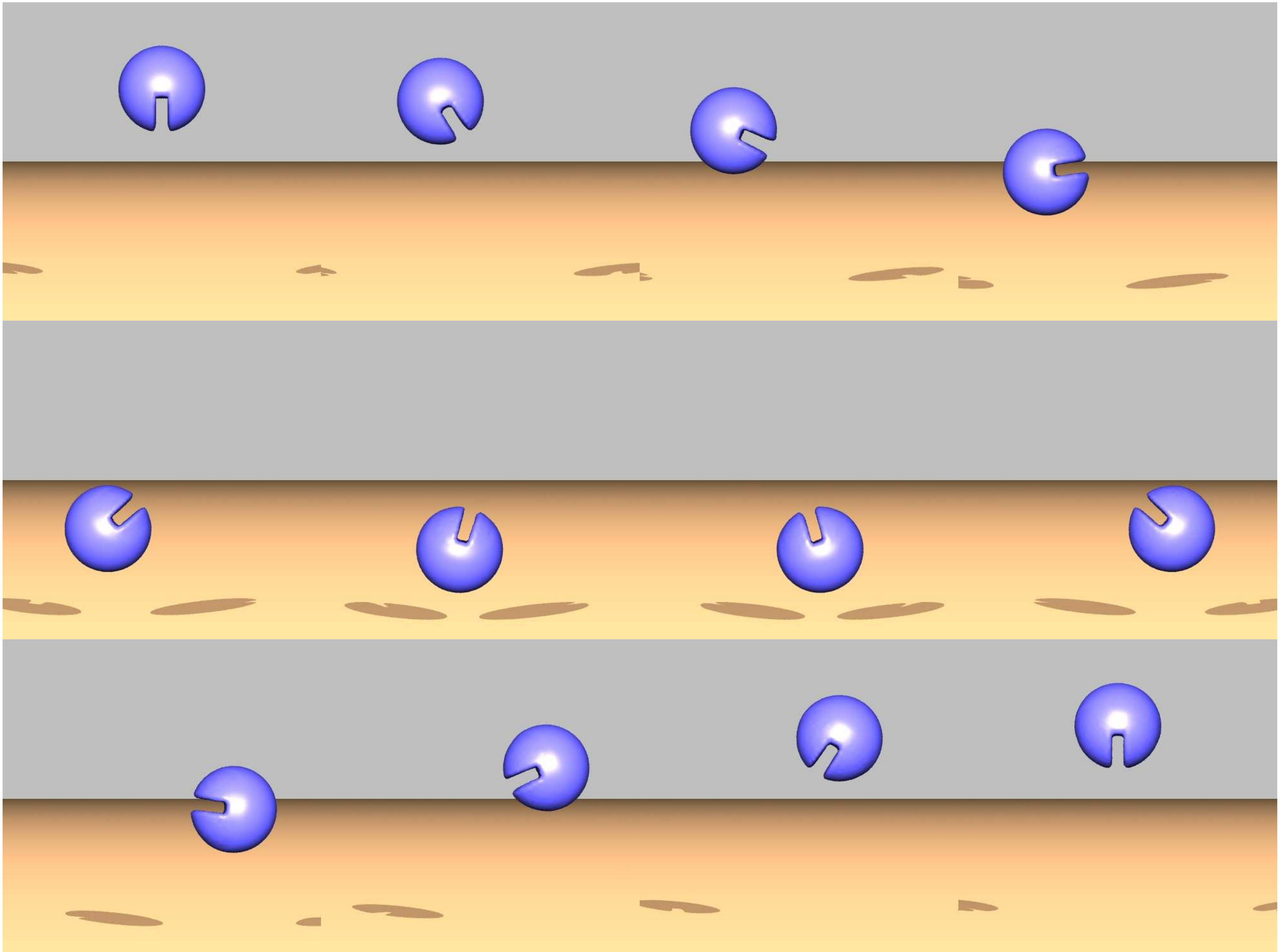


Particle Level Set



Rendering by Sou Cheng Choi

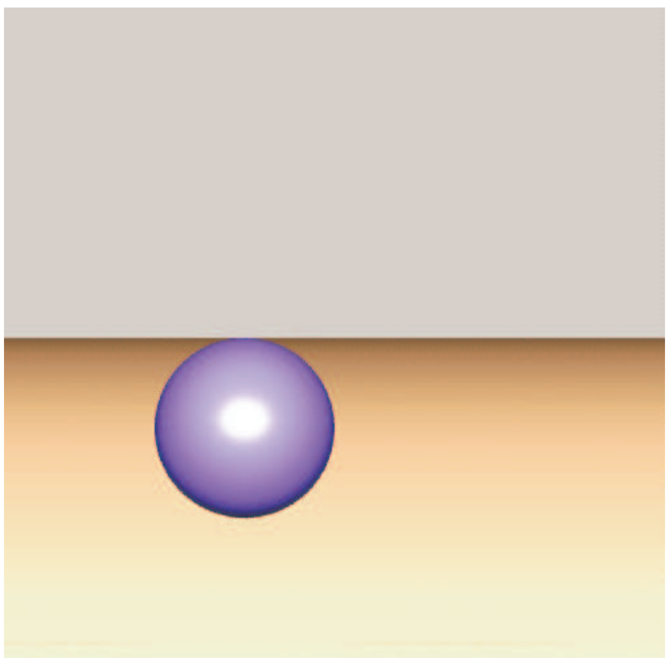




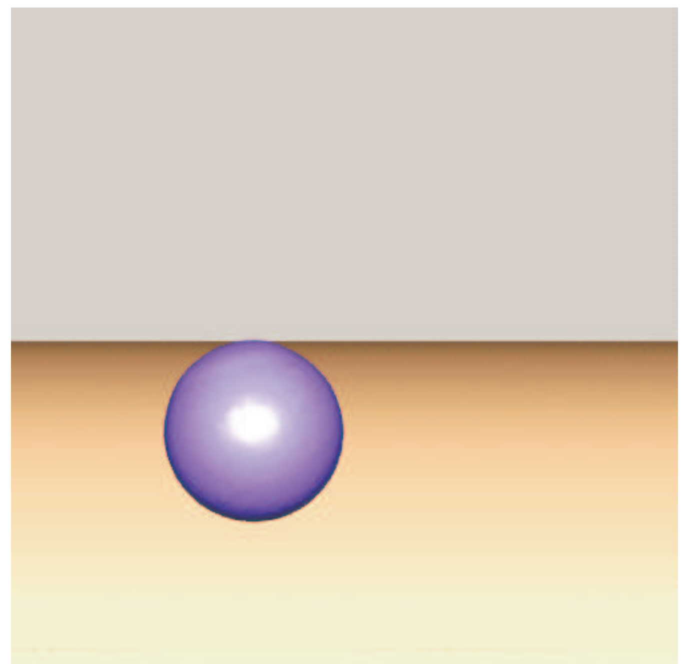
Pushing the Limits

- Fully 3D vortex stretch of sphere (vortex in x-y and x-z planes)
  - 100^3 grid, error is evaluated by time reversing the flow
  - [LeVeque, 1996]

Level Set Only



Particle Level Set



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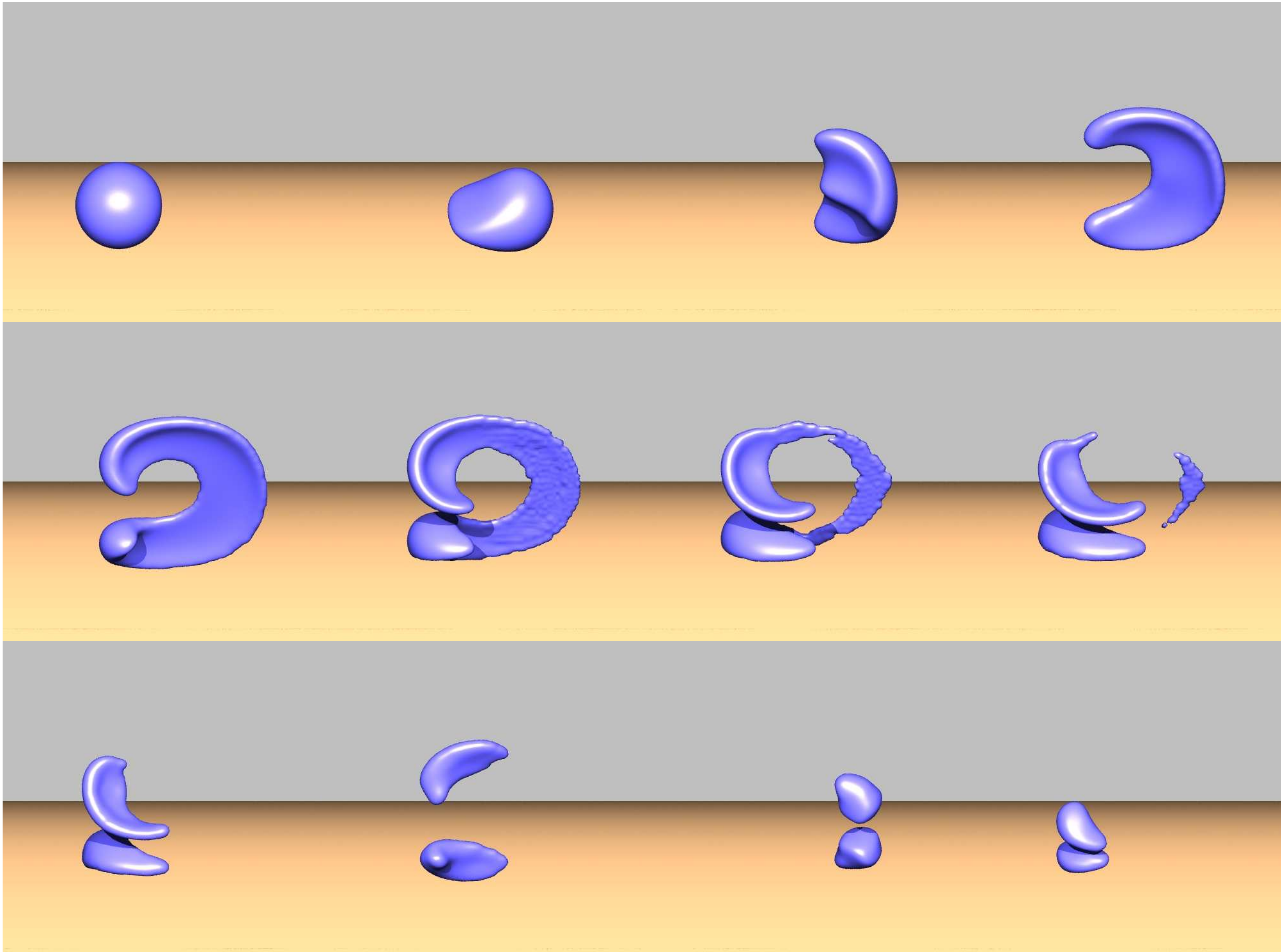
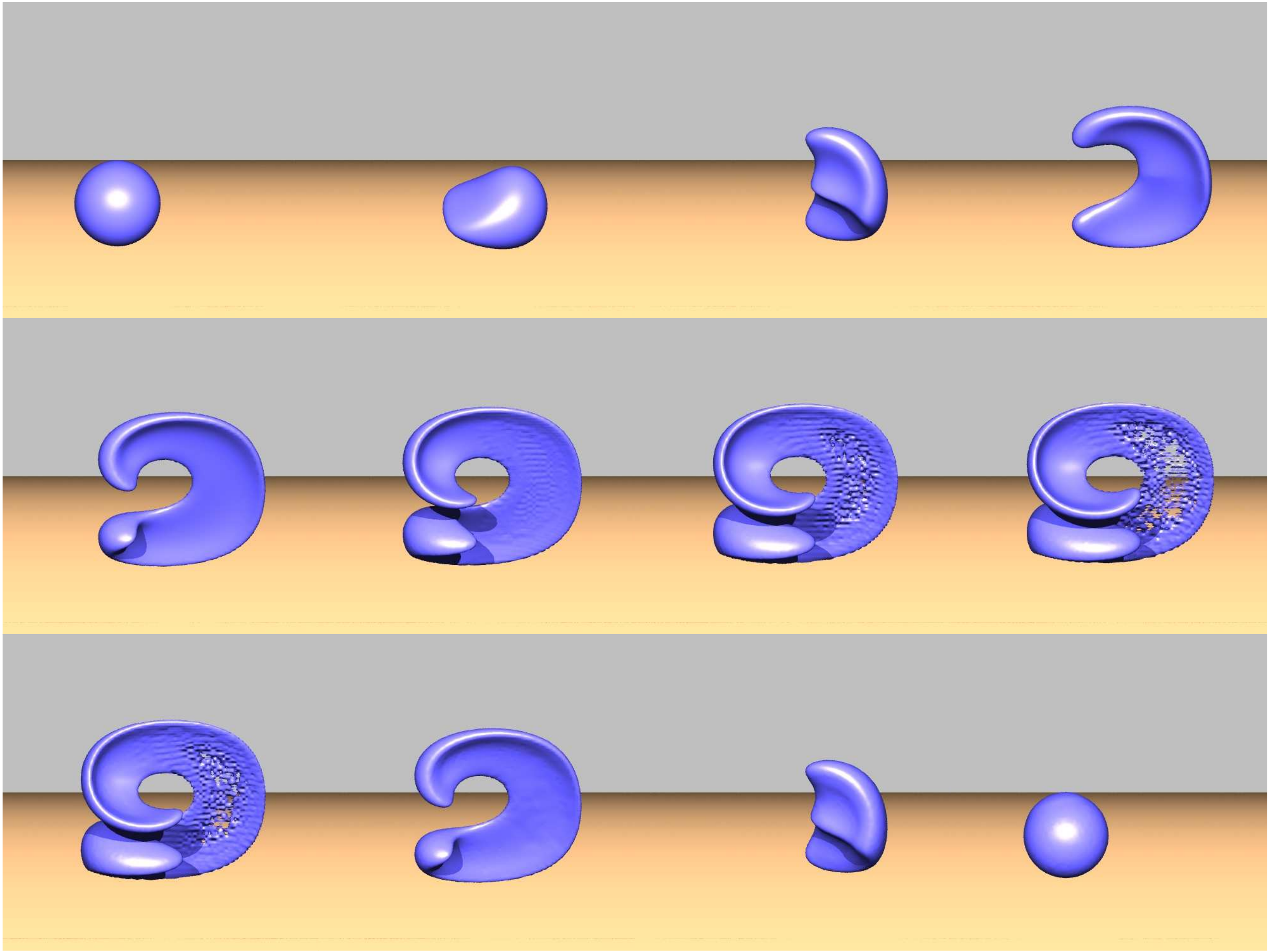
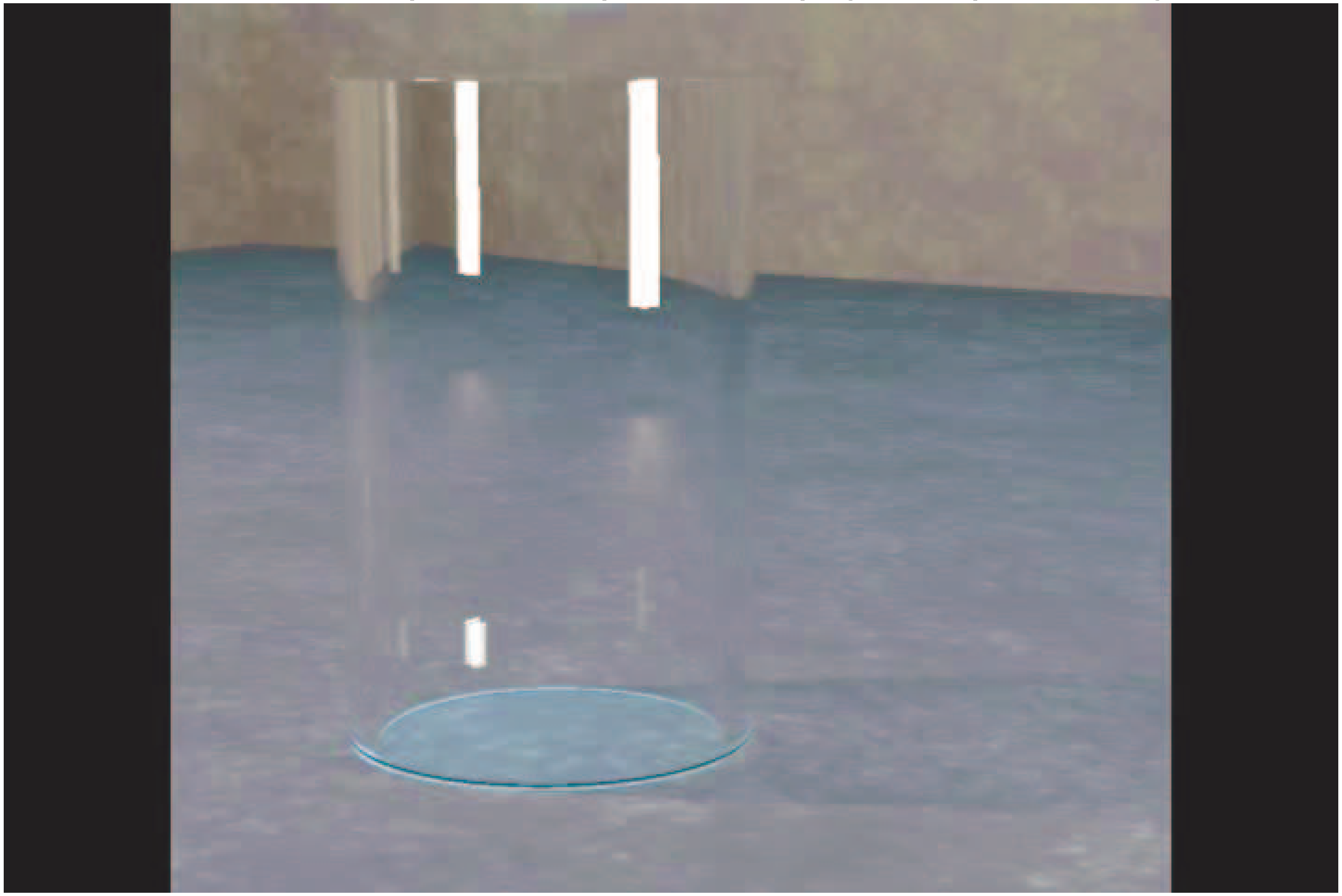
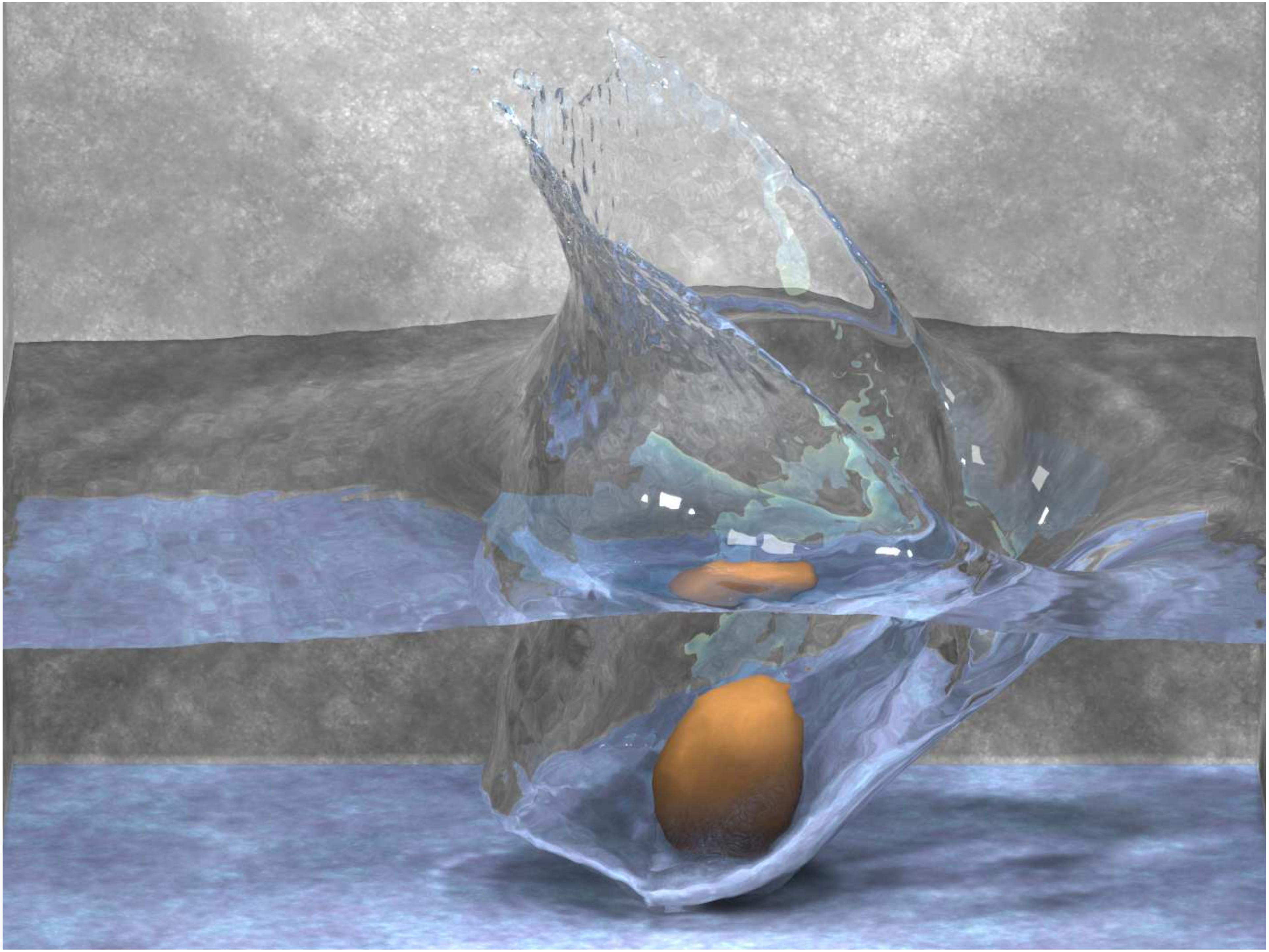




Application: Animating Fluids

- [Enright, Marschner, & Fedkiw, Siggraph 2002]
  - Coupled particle level set with ghost fluid method to track complex free surface evolution
  - Animation demands computation speed and physical plausibility





What About Shocks?

- Level set regularizes regions of high curvature
  - An error for rigid body rotations and other pure advections
  - Correct behavior around shocks
- Particle level set assumes that particles are correct
- Example: square moving inward with unit normal speed