Level Set Methods
CPSC 542D, Term 2, Winter 2006–2007
Assigned Monday, January 15. Due Monday January 22.

Homework #1

1. **Classifying sets.** For each of the sets S_i listed below, very briefly describe:

   - The qualitative shape of the set.
   - The convexity, connectedness and/or boundedness of the set.
   - The dimension and codimension of the set (**not** its boundary).
   - The boundary of the set (if any).
   - The interior of the set (if any).
   - Whether the set could be represented by an implicit surface function $\phi(x) : \mathbb{R}^n \to \mathbb{R}$ such that either $S_i = \{x \in \mathbb{R}^n \mid \phi(x) \leq 0\}$ or $S_i = \{x \in \mathbb{R}^n \mid \phi(x) = 0\}$. If yes, give such a function and specify whether S_i is the zero sublevel set (the interior, codimension 0) or the zero level set (the surface, codimension 1). If no, explain why not.

   In what follows, x_j is the j^{th} component of the n dimensional vector $x \in \mathbb{R}^n$.

   (a) $S_1 = \{x \in \mathbb{R}^3 \mid x_1 = 0\}$.

   (b) $S_2 = \{x \in \mathbb{R}^3 \mid x_2 \leq \sin(\pi x_1)\}$.

   (c) $S_3 = \left\{ x \in \mathbb{R}^3 \,\middle|\, \sqrt{\sum_{i=1}^3 x_i^2} \leq 5 \right\}$.

   (d) $S_4 = \{x \in \mathbb{R}^2 \mid 0 \leq x_2 \leq \cos(\pi x_1) \wedge -2 \leq x_1 \leq +2\}$.

   (e) $S_5 = \{x \in \mathbb{R}^2 \mid \sqrt{x_1^2 + x_2^2} = 1\}$.

   (f) $S_6 = \{x \in [0, 2\pi) \times \mathbb{R} \mid \sin(x_1) \leq x_2 \leq \cos(x_1)\}$. Let x_1 have periodic conditions on the edges of its range.

   (g) $S_7 = \{x \in \mathbb{R}^3 \mid \sqrt{(r-2)^2 + x_3^2} \leq 1\}$, where $r = \sqrt{x_1^2 + x_2^2}$.

   (h) $S_8 = \{x \in \mathbb{R}^4 \mid a^T x = 1\}$, where $a = \begin{bmatrix} 1 & 1 & 1 & 1 \end{bmatrix}^T$.

   (i) $S_9 = \{x \in \mathbb{R}^3 \mid x_2 = \sin(x_1) \wedge x_3 = \cos(x_1)\}$.

   (j) $S_{10} = \{x \in \mathbb{R}^2 \mid \mathbf{A}x \leq b\}$, where

   $$\mathbf{A} = \begin{bmatrix} 1 & 1 \\ -1 & 0 \\ 0 & -1 \end{bmatrix}, \qquad b = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}.$$

   (k) $S_{11} = \{x \in \mathbb{R}^2 \mid x_2 = x_1 - 1 \wedge x_1 = x_2^2\}$.

   (l) $S_{12} = \{x \in [-1, +1] \mid x \text{ is rational}\}$.

2. **A dynamic surface.** Consider a square with sides of full length $l = 0.2$ centered at $c = (0.5, 0.0)$ in $\mathbb{R}^2$. We wish to move this square under the velocity field

$$\frac{dx}{dt} = \frac{d}{dt}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = v(x) = 2\pi \begin{bmatrix} -x_2 \\ x_1 \end{bmatrix}, \qquad x \in \mathbb{R}^2,$$

which is equivalent to a rigid body rotation about the origin.

Using a point cloud representation and MATLAB's initial value ODE solvers (probably `ode23` or `ode45`), simulate this motion for $t \in [0, 1]$ (one full rotation). There is no need to maintain connectivity information between the points.

Use `subplot` and `plot` to display the motion at $t_i = i/8$ for $i = \{0, 1, 2, \ldots, 8\}$ as a 3×3 grid of subplots. Use variants of the `axis` command to ensure that the scaling of the axes is equal (so that the square looks like a square) and each of the subplots uses the sames bounds. Use the `title`, `xlabel` and `ylabel` commands for labelling. Submit both your (labelled) plot and your (documented) code listing.

On the course web site you will find `example.m`, which shows the type of code comments and figure labels I expect. If necessary, you can also add figure labeling by hand after printing. Don't forget to include your name in both the code and on the figures.