Williams et.al. – Approximate Dynamic Programming for Communication-Constrained Sensor Network Management

An Experimental Review and Visual Tool

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Project Outline – CPSC532M
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Mar 17, 2008

Outline

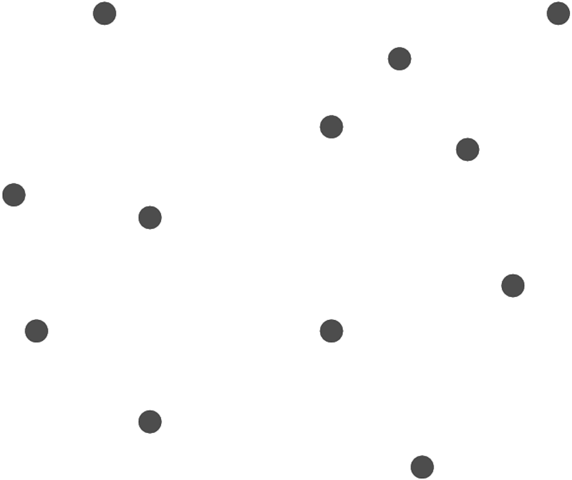
1. Problem Introduction
2. Dynamic Programming Formulation
3. Project

Based on: J. L. Williams, J. W. Fisher III, and A. S. Willsky. Approximate dynamic programming for communication-constrained sensor network management. *IEEE Transactions on Signal Processing*, 55(8):4300–4311, August 2007.

Sensor Networks

- Spatially distributed autonomous devices
- Sensor to monitor:
  - temperature, sound, pressure
  - pollutants
  - external agents
  - battlefield surveillance
- Wireless communication
- Limited energy

Example



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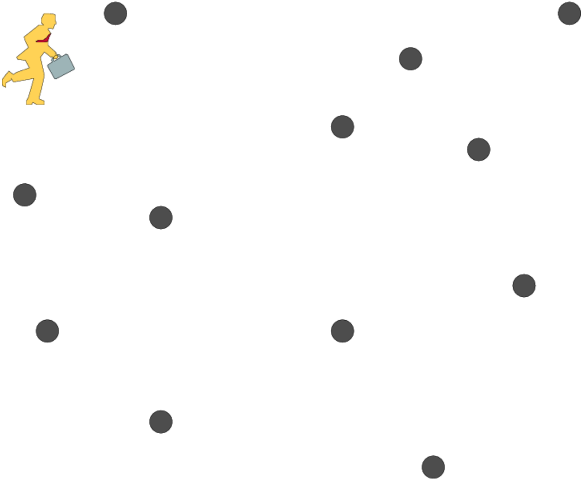

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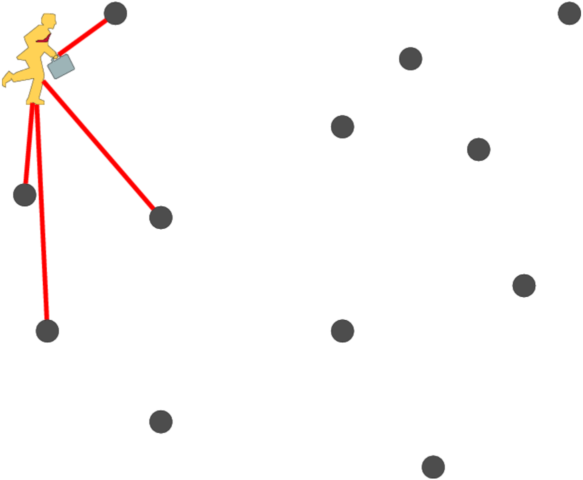

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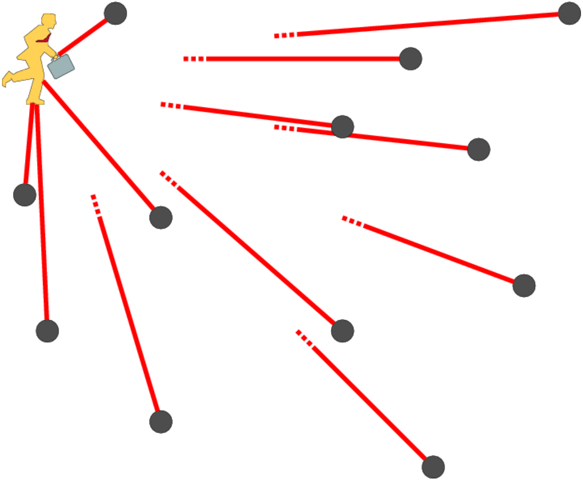

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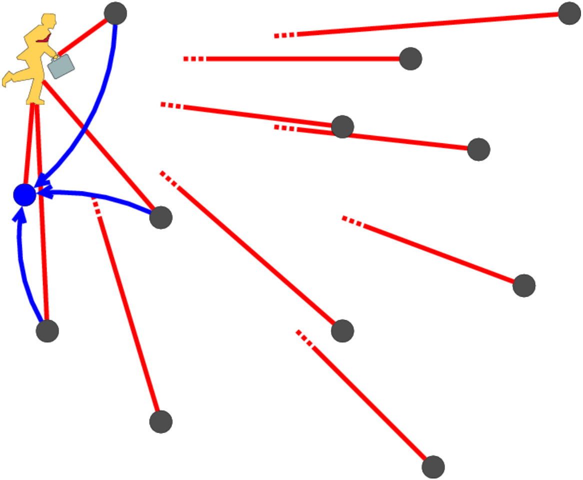

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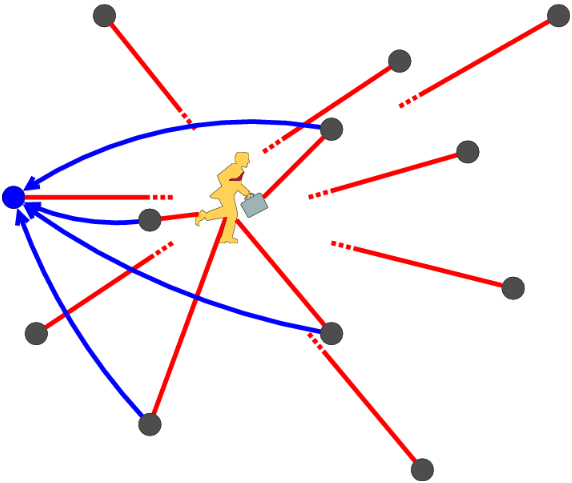

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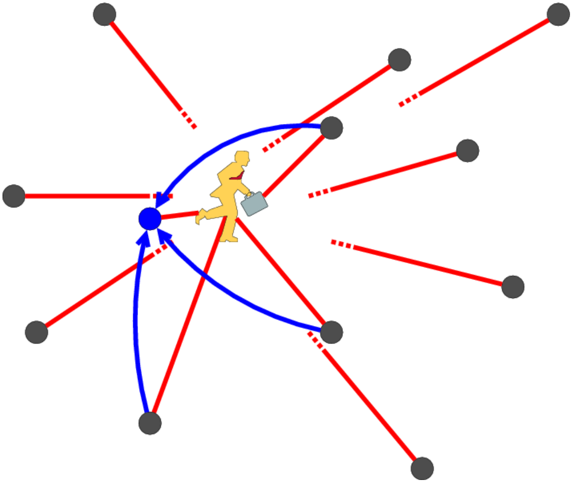

The Problem

- Identify the state (position, velocity) of the object
  - Probability Distribution Function (pdf)
  - Estimate the object's next state
- Subset of sensors and a leader sensor

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- Identify the state (position, velocity) of the object
  - Probability Distribution Function (pdf)
  - Estimate the object's next state
- Subset of sensors and a leader sensor
- Objectives:
  - Maximize the information estimation performance
  - Minimize the communication cost

Dynamic Programming Approach

- Objective function: estimation performance or communication cost
  - Lagrangian relaxation
- State: object actual position
- Policy: leader and sensor subset selection
- Rolling discrete horizon
  - In each step, N steps are calculated, and one is taken

Object Dynamics

- Object state: $x_k = \begin{bmatrix} p_x \\ v_x \\ p_y \\ v_y \end{bmatrix}$
- Object dynamics: $x_{k+1} = Fx_k + w_k$
- $F = \begin{bmatrix} 1 & T & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & T \\ 0 & 0 & 0 & 1 \end{bmatrix}$
- w_k is a white Gaussian noise process

Measurement Model

- Measurement model: $z_k^s = h(x_k, s) + v_k^s$

$$h(x_k, s) = \frac{a}{\|Lx_k - y^s\|_2^2 + b}$$

- v_k^s is a white Gaussian noise process
- $z_{0:k}$ is the history of all measurements $\{z_0, z_1, \ldots, z_k\}$
- Estimation: in progress

Communication

- Cost (per bit) of direct communication: $\tilde{C}_{ij} \propto \left\| y^i - y^j \right\|_2^2$
- Cost considering path $\{i_0, i_1, \ldots, i_{n_{ij}}\}$:

$$C_{ij} = \sum_{k=1}^{n_{ij}} \tilde{C}_{i_{k-1} i_k}$$

- Transmission of probabilistic model: B_p bits
- Transmission of measurements: B_m bits

Constrained Dynamic Programming

- Conditional belief state: $\mathbb{X}_k \triangleq p(x_k|z_{0:k-1}) \in \mathcal{P}(\mathcal{X})$
- Control: $u_k = (l_k, \mathcal{S}_k)$, $l_k \in \mathcal{S}$ and $\mathcal{S}_k \subset \mathcal{S}$
- Control policy – time k: $\mu_k(\mathbb{X}_k, l_{k-1})$
- Control policy set: $\pi_k = \{\mu_k, \ldots, \mu_{k+N-1}\}$

Optimization Problem

$$\min_{\pi} E\left[\sum_{i=k}^{k+N-1} g\left(\mathbb{X}_i, l_{i-1}, \mu_i\left(\mathbb{X}_i, l_{i-1}\right)\right)\right]$$

$$\text{s.t.} E\left[\sum_{i=k}^{k+N-1} G\left(\mathbb{X}_i, l_{i-1}, \mu_i\left(\mathbb{X}_i, l_{i-1}\right)\right)\right] \leq M$$

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- Constraint addressed through Lagrangian relaxation:

$$L_k\left(\mathbb{X}_k, l_{k-1}, \pi_k, \lambda\right) = E\left[\sum_{i=k}^{k+N-1} g\left(\mathbb{X}_i, l_{i-1}, \mu_i\left(\mathbb{X}_i, l_{i-1}\right)\right)\right.$$

$$\left. + \lambda\left(\sum_{i=k}^{k+N-1} G\left(\mathbb{X}_i, l_{i-1}, \mu_i\left(\mathbb{X}_i, l_{i-1}\right)\right) - M\right)\right]$$

Dynamic Programming Function

$$L_k\left(\mathbb{X}_k, I_{k-1}, \pi_k, \lambda\right) = E\left[\sum_{i=k}^{k+N-1} g\left(\mathbb{X}_i, I_{i-1}, \mu_i\left(\mathbb{X}_i, I_{i-1}\right)\right)\right.$$
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$$\left. + \lambda\left(\sum_{i=k}^{k+N-1} G\left(\mathbb{X}_i, I_{i-1}, \mu_i\left(\mathbb{X}_i, I_{i-1}\right)\right) - M\right)\right]$$
$$J_k^D\left(\mathbb{X}_k, I_{k-1}, \lambda\right) = \min_{\pi_k} L_k\left(\mathbb{X}_k, I_{k-1}, \pi_k, \lambda\right)$$
$$J_k^L\left(\mathbb{X}_k, I_{k-1}\right) = \max_{\lambda \geq 0} J_k^D\left(\mathbb{X}_k, I_{k-1}, \lambda\right)$$

Complexity Evaluation

- N_s sensors
- N_p samples for each policy
- N steps (horizon)
- $N_s 2^{N_s}$ possible controls
- $N_s 2^{N_s} N_p$ new total samples
- $O(N_s^N 2^{N_s N} N_p^N)$

Linearized Gaussian Approximation

- Considering the smoothness of $z_k^{\mathcal{S}_k}$
- Function for entropy evaluation is simplified
- $O(N_s^N 2^{N_s N})$

Greedy Sensor Subnet Selection

- Each stage is divided in substages
- A new problem is defined, algebraically equivalent to the original problem
- Sensors that increase the cost in a substage are discarded
- Sensors are limited to a small neighbourhood
- Worst-case complexity: $O(NN_s^3)$

Dynamic Programming Problem Implementation

- Implementation of the problem described in this paper
- Both communication contraint (CC) and information constraint (IC) approaches
- Repeat simulation results
- Obtain extended resuls
- Tool for visualizing:
  - Agents position
  - Object state
  - Leader and subset selection

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