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Incentive Auction Design Alternatives: A Simulation Study

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This paper revisits the descending clock "reverse" auction design used in the FCC's 2016–17 "incentive auction". We use extensive computational simulations to investigate the quantitative significance of various aspects of the design, leveraging a reverse auction simulator and realistic models of bidder values.

Key words: incentive auction, deferred acceptance auction, reverse clock auction, spectrum auction, simulation, market design, auctions, artificial intelligence, applied game theory
History:

1. Introduction

Over 13 months in 2016–17 the US Federal Communications Commission (FCC) conducted an "incentive auction" to repurpose radio spectrum from broadcast television to wireless internet. The result of the auction was to remove 14 UHF-TV channels from broadcast use, to sell 70 MHz of wireless internet licenses for $19.8 billion, and to make 14 MHz of spectrum available for unlicensed uses. With fewer remaining channels for TV stations, the TV spectrum needed to be reorganized; stations interfere with each other, so not all of them could be reassigned channels in the compressed TV band. Each station was either "repacked" in the leftover channels or voluntarily sold its broadcast rights, either going off the air or switching to a lower-quality band. These volunteers received a total of $10.05 billion to make repacking possible by yielding or exchanging their rights.

This paper investigates the quantitative significance of various aspects of the descending clock "reverse" auction used to procure broadcast rights in the incentive auction. We take a computational perspective, running extensive simulations that leverage a reverse auction simulator and realistic models of bidder values. Why bother taking another look at the incentive auction? Because the design was both novel and extremely complex (Leyton-Brown et al. 2017), it was not possible to thoroughly consider every potential design variation before the auction was run. Our goal in this paper is to gain new insights into how well the auction design performed, particularly asking which elements of the design were most important and which variations of the design might have led to even better outcomes. Such insights can inform other important resource allocation problems that may leverage parts of the incentive auction's design. We speculate on one example: Just as historical US television broadcast rights varied by location and interference protections and were difficult to adapt to valuable new uses, historical rights to surface water in Western states have similar characteristics. Spectrum uses in a geographic area often leak interfering radiation into adjacent areas (a negative externality), and farm irrigation often leaks usable water back into river systems and acquifers (a positive externality). To develop a voluntary auction system that accounts for externalities and reorganizes water rights to unlock value, market designers will benefit from a deep understanding of which design details of the incentive auction contributed most to its successes and which could be simplified or improved. More broadly, we hope this work will serve as an example for how simulations can be employed to understand and evaluate alternative market designs in complex settings.

Roughly, the reverse auction began with a *clearing target* or number of TV channels to decommission. Stations were approached one at a time with a series of decreasing price offers for their broadcast rights. When a station refused an offer, it exited the auction irrevocably and was guaranteed a spot in the leftover channels. As prices fell and more stations declined offers, the leftover channels became more and more congested. Before any station's bid was processed, a "feasibility checker" ensured that the station could still fit in the leftover channels alongside the exited stations without causing undue interference; if it could not, that station was "frozen", meaning that its price stopped falling and it was no longer eligible to exit the auction. The reverse auction further included a provision that allowed stations to exchange their broadcast rights for both a channel in a less desirable (VHF) band and monetary compensation. After the reverse auction concluded, a "forward" ascending clock auction was used to sell licenses in the cleared spectrum to mobile carriers. An outer-loop procedure iterated between reverse and forward auctions to identify the largest possible clearing target for which forward auction revenues exceeded costs. We give a fuller and more formal summary of the auction mechanics in Section 3.

1.1. Evaluating Complex Auction Designs

The design space for combinatorial auctions is large (Pekec and Rothkopf 2003, Anandalingam et al. 2005), including choices such as eligibility rules (Engelbrecht-Wiggans and Kahn 2005), payment rules (Day and Raghavan 2007), and bidding languages (Bichler et al. 2022).

To date, the design and analysis of auctions has relied primarily on theoretical tools. Such analysis becomes increasingly difficult as auctions become more complex[1], e.g., as the number of goods at auction increases; as the number of parameters needed to describe valuation functions grows; as heterogeneity across bidders increases; etc. Nevertheless, the reverse auction design was strongly informed by theoretical analysis. Such analysis made critical contributions to our understanding of the mechanism's incentive properties, its susceptibility to collusion, and the likely revenue effects of making asymmetric offers to different stations. We survey these and other results in Section 2. In exchange for the universality of its findings, theoretical analysis necessarily relies on simplifications, such as modeling the sets of stations with interference-free repackings as a uniform matroid. Such simplifications can raise questions about the practical applicability of results. For example, Weiss et al. (2017) warn that: "Deriving analytic results for [combinatorial auctions] is very challenging... insights from small, stylized models often do not translate to practical real-world problems".

A second approach for evaluating auction designs is data driven. There is a vast literature on laboratory and field experiments for auctions (Ferejohn et al. 1979, Rassenti et al. 1982, Kwasnica et al. 2005, Cason et al. 2011, Adomavicius et al. 2012). To our knowledge no such experimental data exists evaluating the incentive auction; indeed, it would be very challenging to conduct realistic experiments, given the real auction's long length and extremely large number of participants. Data can also be obtained from an auction's practical deployment. There is an extensive structural estimation literature in auctions and matching markets.

Such analysis has lead to significant insights in data-rich domains such as school choice (Agarwal and Somaini 2018, Calsamiglia et al. 2020), highway procurement auctions (Krasnokutskaya and Seim 2011, Somaini 2020), timber auctions (Athey et al. 2011), and ad auctions (Athey and Nekipelov 2010). Unfortunately, only a single auction has been held to date using the incentive auction design, leaving us with only a single sample of past bids. We nevertheless leveraged this single datapoint to inform a valuation model, described below.

This paper focuses on a third approach, computational simulations. This approach can verify whether theoretical findings from simplified models carry over to more complex, real-world settings and does not rely on the availability of rich historical data. The FCC used simulations in the auction design process to verify the robustness of many design elements; some of these simulations were publicly described (FCC 2015b). Simulations are particularly well suited to studying market settings with complex clearing mechanisms (because such complexity tends to preclude "cleaner" analytical results) and uncontroversial models of agent behavior (because this reduces the risk that the model upon which the simulation relies will produce unrealistic behavior).

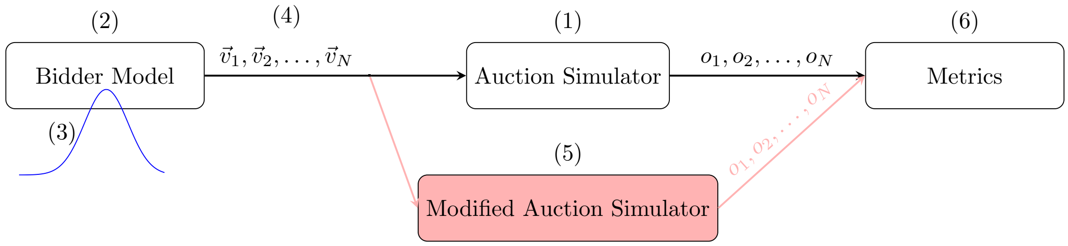

Figure 1 An overview of our simulation approach. N samples from a bidder model are fed into both a modified and unmodified simulator. The two outcome sets are converted into metrics and compared on a per-sample basis.

1.2. Our Simulation Methodology

More specifically, we advocate the following simulation methodology (see Figure 1): (1) Build an auction simulator (choosing an appropriate level of abstraction, as auction rules are often incredibly complex). (2) Create a bidder model, exposing parameters that control both valuations and behavior. (3) Establish a probability distribution over parameters of the bidder model. (4) Identify many plausible auction scenarios by sampling repeatedly from this probability distribution. (5) Run paired simulations by holding this population of scenarios fixed and varying one or more elements of the auction design. (6) Compare outcomes across paired samples using predetermined metrics.

For the study described in this paper we instantiated these steps as follows. First, we custom-built the simulator used in this paper. Other incentive auction simulators have been used in previous work (surveyed below) but ours is the most comprehensive of which we are aware, both in terms of its scale (national) and its coverage of the auction rules (e.g., multi-stage auctions including VHF bands). Our simulator's code is freely available online at `https://github.com/newmanne/SATFC`.

Next, perhaps the most critical step in our methodology is the specification of a bidder model, describing both valuations and behavior for each bidder. Since we focus on the reverse auction, each of our bidders corresponds to a television station. We considered two different valuation models: (1) the only fully specified model from the literature of which we are aware; and (2) a model we created based on publicly released bid data. We ran simulations with both models and contrast the results to investigate the robustness of our findings. Realistically modeling agent behavior is perhaps easiest in settings giving rise to *obviously dominant* strategies (Li 2017). A strategy is obviously dominant if any time an agent might deviate from the strategy, the best it can do under the deviation is no better than the worst it can do by following the strategy. Descending clock auctions make truthful bidding obviously dominant for bidders who own a single broadcast station (but not for those who own multiple stations).[2] Unfortunately, the real reverse auction lacked obviously dominant strategies because stations could do more than just accepting and rejecting an offer: they could also accept a lower price and downgrade to a VHF channel. We decided to study the problem by considering the assumptions both that VHF bidding could be abstracted away and that it could not.

For the latter case, no equilibrium bidding strategy is known, let alone any model of how bidders would behave if they believed that some or all of their opponents were bidding out of equilibrium. Given the widespread belief that many bidders in this auction were unsophisticated, we modeled bidders as bidding truthfully and myopically (see Section 4.2).

Finally, we restricted most of our comparisons to simulations that cleared the same amount of spectrum, considering two key metrics: the sum of values of stations removed from the airwaves (value loss) and the aggregate amount paid to stations (cost). When assessing design elements that did affect the amount of spectrum cleared, we assumed that it was preferable to clear more spectrum, based on statements made by the FCC about the auction's intended goals (FCC 2012).

1.3. Questions Considered in Our Analysis

We ask three categories of questions about economic design and one about algorithmic design.

1. *How important was it to expand the set of products included in the incentive auction?* In the incentive auction, it would have been straightforward only to buy back licenses from UHF stations, since only UHF spectrum needed to be cleared. Instead—considerably increasing complexity—the FCC offered to purchase licenses from both VHF and UHF stations and offered UHF stations the option of moving into the VHF band rather than going off-air. This increased the pool of stations eligible to participate in the auction by roughly 20%, with the potential both to increase efficiency (a lower-value VHF station could go off-air to make room for a UHF station) and to lower costs (c.f. Bulow and Klemperer's (1996) result that increasing competition can be more important to revenue than setting an optimal reserve price).

Was the extra complexity worth it? We found that it was (Section 6.1): under straightforward bidding, not repacking the VHF band could have increased the auction's cost by 20–30% and decreased efficiency by 5–10% (with variation depending both on random sampling and the choice of value model). Similar issues could arise in designing an auction for water rights: for example, one might ask whether the process should include ground water as well as surface water.

2. *What was the impact of the FCC's method of determining supply?* The literature has studied various auction designs in which the seller can adjust supply after observing demand (Back and Zender 2001). In the incentive auction, the FCC separately ran forward and reverse auctions at a given clearing target; then, if forward-auction demand was insufficient to cover reverse-auction costs, the clearing target was lowered and a new stage of the auction began. This design was novel and received little advance comment from stakeholders. We thus investigated how it performed, comparing it to a hypothetical auction having oracle access to the final clearing target. We found that the clearing procedure led to significantly higher costs and less efficient outcomes (Section 6.2): across all experiments, the clearing procedure increased average value loss by 5–26% and average

cost by 4–50% (with variation depending on random sampling, the choice of value model used, and whether the VHF band was repacked). We then introduce a new simple clearing procedure that performs nearly as well without knowing the final clearing target.

3. *Was price discrimination effective at reducing payments to stations and increasing the number of channels cleared?* A canonical insight from revenue maximization (Myerson 1981) is that bidders with weaker valuation distributions should be boosted to increase competition with stronger bidders. The incentive auction attempted to do something similar, reducing initial price offers for stations that reached smaller populations of viewers. The reduced price offers, called "pops scoring", were politically contentious. In Section 6.3, we show that the effects of scoring were not robust across value models. Under our new value model, pops scoring led to average costs 5% (2%) higher than head-to-head pricing when the VHF band was (was not) repacked. Under the value model from the literature, head-to-head pricing led to average costs that were 39% (5%) higher relative to pops scoring when the VHF was (was not) repacked.

4. *How important was it to optimize a solver for the computationally hard problems embedded in the auction design?* Auctions can include problems that (in general) may not be solvable in a reasonable amount of time; a well-studied example is the winner determination problem in combinatorial auctions (Rothkopf et al. 1998). Exact solutions can be replaced with approximations, but this may degrade outcomes and incentives. We investigated station repacking, a hard problem embedded in the incentive auction. Was auction performance significantly improved by the FCC's use of a customized feasibility checker to determine whether a station could be repacked alongside the set of stations continuing over-the-air broadcasting? How large might that effect have been? The auction was robust from an incentive perspective to not solving every repacking problem, but the impact on cost and efficiency was harder to reason about. This question is important because the design of customized feasibility checkers required a nontrivial effort; such efforts should only be made in the future if they yield gains. We answer this question affirmatively in Section 6.4. We show that substituting the custom feasibility checker with the best off-the-shelf alternative could have increased both average costs and value loss by more than 20%.

The rest of the paper proceeds as follows. Section 2 surveys related work. Section 3 explains the reverse auction in detail. Section 4 defines our valuation model and bidding model. Section 5 details our experimental setup and Section 6 reports our experimental findings. Endnotes appear in Appendix A and 14 subsequent appendices contain additional technical material.

2. Related Work

The incentive auction's design was strongly informed by theoretical analysis, which has shown that deferred acceptance (DA) auctions have many good properties.[3] DA auctions exhibit "unconditional winner privacy" (Milgrom and Segal 2020), meaning that winners are only required to reveal as much about their valuation as is necessary to prove that they are winners. When stations' bids are binary responses to a series of descending offers and when stations are all independently owned, DA auctions are obviously strategy-proof (Li 2017) and weakly group strategy-proof. When the sets of stations that can be jointly repacked form a uniform matroid, DA auctions repack the efficient set of stations (Bikhchandani et al. 2011); if furthermore bidder values are drawn independently from known distributions, scoring offers according to virtual values can implement the Myerson "optimal auction" (Milgrom and Segal 2020). Both Ausubel (2004) and Bikhchandani et al. (2011) study settings in which some DA auction achieves efficiency and characterize that auction. Our contribution, similar to that of Milgrom and Segal (2020), is to study properties of DA auctions in more general settings in which efficiency is not achievable.

Other questions about the incentive auction's design were addressed via simulations. The FCC conducted its own (mostly unpublished) internal simulations, leveraging a variety of techniques from operations research (Kiddoo et al. 2019). The broader research community conducted simulation studies of the reverse auction at various points in the design process and differing in the fidelity with which they modeled the auction mechanism; the degree to which they made simplifications for computational reasons; and the valuation models they employed. Before the auction mechanism was finalized, Kearns and Dworkin (2014) used simulations to characterize the space of feasible repackings based only on the interference constraints, e.g., estimating the relationship between the number of broadcasters relinquishing licenses and the feasibility of different clearing targets. This work used a bidder model that consisted entirely of determining which stations would participate, which they studied both using independent coin flips for every station and more complex models in which stations affiliated with the same broadcast networks made correlated decisions. Feasibility testing was performed using off-the-shelf SAT solvers. Later in the design process, Cramton et al. (2015) used simulations to lobby for design changes such as changing the scoring rule and removing Dynamic Reserve Pricing (DRP). They leveraged a (non-public) valuation model developed through "discussions with many broadcasters, taking into account revenue data, historical station sales prices, station affiliation information, total market revenue, and other factors". DRP was ultimately scrapped but remains a topic of interest (Bazelon 2022). Three studies were published after the auction concluded. First, Doraszelski et al. (2017) used simulations to estimate how profitable and how risky bidder collusion strategies might have been, restricting experiments to regional scale (considering subsets of at most 8% of UHF stations) and furthermore skipping certain feasibility checks ("limited repacking") to speed up computation. They concluded that the auction could have mitigated the harm imposed by owners of multiple stations artificially reducing supply by restricting the sets of stations such owners could have used to participate in the auction. This work leveraged a novel value model for reverse-auction participants, which we discuss further in Section 4.1.1 because we used it in our own simulations. Second, Newman et al. (2017) designed a simulator to evaluate the quality of the auction's deployed feasibility checker. This simulator leveraged the value model from Doraszelski et al. (2017) and operated at a national scale but was restricted to a single stage. Newman et al. (2017) concluded that the deployed feasibility checker outperformed off-the-shelf alternatives and also showed in smaller, regional simulations that the reverse auction's allocations were nearly efficient when using this feasibility checker. These results were further discussed in Leyton-Brown et al. (2017), which provided context for the unique challenges faced in the incentive auction (e.g., defining property rights for television stations), and in Milgrom and Segal (2020), a theoretical paper studying the properties of deferred acceptance auctions.[4] The simulator used in this paper substantially extends the one used in Newman et al. (2017) and thereby more accurately simulates the actual reverse auction design than any other simulator of which we are aware.[5] Our most significant extensions are integrating a new value model and supporting auctions with multiple stages. We revisit the feasibility checker experiments of Newman et al. (2017) (Section 6.4), performing a far more exhaustive assessment based on 1300 simulations instead of the original experiment's 60. The bulk of our paper, however, focuses on three other broad questions about the auction design, none of which was considered experimentally in any previous work. Finally, Ausubel et al. (2017) performed a post-mortem analysis of the incentive auction in a similar vein to this paper, but with a primary focus on the forward auction; we discuss one of their proposed amendments to the clearing algorithm in Section 6.2.2.

Before proceeding, we survey some related work further afield from the incentive auction. Two papers studied how prices should be set in a clock auction. First, Nguyen and Sandholm (2014) considered how to set prices to minimize expected cost, using the reverse auction as a test setting. Their methods reduce prices until feasibility is violated and then use a final adjustment round to regain feasibility. Their results are not therefore directly comparable to the FCC's design, which maintains feasibility throughout and never increases stations' prices. Second, Bichler et al. (2020) investigated the allocative efficiency of DA auctions under different scoring rules in the problem of Steiner tree construction. They found that the choice of scoring rule affected the efficiency of the auction tremendously and that, under some scoring rules, DA auctions were competitive with other mechanisms based on well-studied approximation algorithms.

Even more generally, there exists an extensive literature on data-driven analysis and optimization of auctions. For example, one line of work considers using past bids to learn an approximately revenue-maximizing auction (Morgenstern and Roughgarden 2015) and another considers approaches for setting personalized reserve prices (Paes Leme et al. 2016, Golrezaei et al. 2017, Roughgarden and Wang 2019, Derakhshan et al. 2019, 2021). Simulating the behavior of candidate market designs in complex settings is a technique widely applicable beyond the incentive auction; a few examples from diverse domains, each with their own literature, include matching mechanisms for refugee resettlement (Delacrétaz et al. 2019); school assignment policies Allman et al. (2022); dock allocation and rebalancing in bike sharing networks (Freund et al. 2018); patrolling strategies for wildlife protection (Yang et al. 2014); reallocating fishing licenses (Bichler et al. 2019); and matching in organ exchanges (Santos et al. 2017).

3. The Reverse Auction

In this section we describe the reverse auction and show where it fits within the context of the incentive auction. We begin by introducing the station repacking problem. Solving repacking problems is a key subroutine within the reverse auction loop. We then proceed to the auction rules.

3.1. Station Repacking

Prior to the auction, each television station $s \in S$ in the US and Canada[6] was assigned to a channel $c_s \in \mathcal{C} \subseteq \mathbb{N}$. The set of channels $\mathcal{C}$ can be partitioned into three equivalence classes, referred to as *bands*. Listed in decreasing order of desirability, these bands are: UHF (channels 14–51), high VHF ("HVHF", channels 7–13) and low VHF ("LVHF", channels 1–6). We use $\text{pre}(s)$ to refer to a station's pre-auction band, sometimes called a station's home band.

Each station was only eligible to be assigned a channel on a subset of $\mathcal{C}$, given by a *domain* function $\mathcal{D}: \mathcal{S} \to 2^{\mathcal{C}}$ that maps from stations to these sets. The FCC determined pairs of channel assignments that would cause harmful interference based on a complex, grid-based physical simulation ("OET-69" FCC (2013)); this pairwise constraint data is publicly available (FCC 2015c). Let $\mathcal{I} \subseteq (\mathcal{S} \times \mathcal{C})^2$ denote a set of *forbidden station–channel pairs* $\{(s,c),(s',c')\}$, each representing the proposition that stations s and s' could not concurrently be assigned to channels c and c', respectively.

The goal of the incentive auction was to remove some broadcasters from the airwaves and assign the remaining stations new channels from a reduced set $\overline{\mathcal{C}} = \{c \in \mathcal{C} \mid c < \overline{c}\}$. This reduced set is defined by $\overline{c} \in \mathcal{C}$; each choice of $\overline{c}$ corresponds to some clearing target. The actual incentive auction ended with $\overline{c} = 37$, allowing the higher numbered channels to be used for other purposes.

A *feasible assignment* is a mapping $\gamma: \mathcal{S} \to \overline{\mathcal{C}}$ that assigns each station a channel from its domain that satisfies the interference constraints: i.e., for which $\gamma(s) \in \mathcal{D}(s)$ for all $s \in S$, and $\gamma(s) = c$ implies that $\gamma(s') \neq c'$ for all $\{(s,c),(s',c')\} \in \mathcal{I}$. As it turns out, interference constraints come in two kinds. *Co-channel constraints* specify that two stations may not be assigned to the same channel; *adjacent-channel constraints* specify that two stations may not be assigned to some pair of nearby channels. Thus, forbidden station–channel pairs are always of the form $\{(s,c),(s',c+i)\}$ for some stations $s, s' \in \mathcal{S}$, channel $c \in \mathcal{C}$, and $i \in \{0,1,2\}$.

Lastly, we define the *interference graph* as an undirected graph in which there is one vertex per station and an edge exists between two vertices s and s' if the corresponding stations participate together in any interference constraint: i.e., if there exist $c, c' \in C$ such that $\{(s,c),(s',c')\} \in I$. Figure 3 shows the interference graph for the US and Canada.

3.2. Reverse Auction

We now describe a simplified version of the reverse auction in which only the UHF band is repacked: only UHF stations participate in the auction and the only possible outcomes for each station are going off-air or continuing to broadcast in UHF. We provide pseudocode for the reverse auction in Appendix C as Algorithm 1. The real auction also repacked two VHF bands, but the inclusion of these bands complicates things significantly; we briefly sketch some details in Section 3.3.1 and further elaborate on pricing VHF options in Appendix D. The complete set of auction rules was published by the FCC in a 230-page document (FCC 2015a).

We begin by describing the reverse auction at a high level before giving more details about various key elements. First, stations respond to opening prices and decide whether to participate in the auction. Next, a solver finds an initial feasible channel assignment for all non-participating stations to minimize the number of channels required for those broadcasters, setting an initial clearing target $\bar{c}$. The auction then attempts to buy broadcast rights as necessary so that all stations remaining on air can fit into the available channels. It proceeds over a series of rounds, which consist of: (1) decrementing the clock and determining new prices, (2) collecting bids and (3) processing bids.

A forward auction to sell the cleared spectrum follows the reverse auction. If sufficient revenue is raised in the forward auction, the incentive auction terminates; otherwise, the reverse auction resumes with a lower clearing target. We elaborate on each step of the reverse auction below.

3.2.1. Prices Prior to the auction, the FCC used a *scoring rule* to assign a *score* (also sometimes referred to as a *volume*) to each station, which we denote by $\text{score}(s)$. The score was used to determine individualized opening prices and was a function of both the station's interference constraints and the population of viewers that a station reached before the auction, which we denote by $\text{Population}(s)$. We will have more to say about scoring rules in Section 6.3.

The reverse auction is a descending clock auction. The initial base clock price was $p_0 = \$900$ both in the incentive auction and in our simulations. At the start of each round, the base clock price is reduced to $p_t = p_{t-1} - d_t$, where $d_t = \max(\frac{p_{t-1}}{20}, \frac{p_0}{100})$. Scores transform base clock prices to individualized station prices; this is, prices in each round $P_{s;t}$ are computed as $P_{s;t} = \text{score}(s) \cdot p_t$. We use $P_{s;\text{Open}}$ to refer to the auction's opening prices.

A "winning" station is one that ultimately goes off-air or moves to a different band. More specifically, if the final channel assignment is γ, s is winning if $\text{post}(\gamma, s) \neq \text{pre}(s)$, where $\text{post}(\gamma, s)$

returns either the band to which s is assigned under γ or OFF if s is not assigned to a band under γ. We refer to the set of winning stations as S_{winners}. Throughout the auction, we track each station's "winning price" $\mathcal{P}:\mathcal{S}\rightarrow\mathbb{R}^+$. $\mathcal{P}(s)$ is the price that would have to be paid to s if the auction were to end immediately and s was a winning station. Initially $\mathcal{P}(s)=P_{s;\text{Open}}$.

3.2.2. Bidding When only the UHF band is repacked, a bid in a given round corresponds to a binary decision. A station can accept $P_{s;t}$, indicating that it prefers to relinquish its broadcast rights and receive $P_{s;t}$. Alternatively, a station can reject $P_{s;t}$, indicating that it prefers to continue to broadcast. If a station's bid to reject an offer is ever processed (as will be explained in the following section), it is said to have "exited" the auction. Such a station is never asked to bid again. An exited station receives no compensation and will continue to broadcast in its pre-auction band after the auction (albeit on a possibly different channel). We refer to the set of exited stations by S_{exited}.

3.2.3. Bid Processing In the bid processing step, stations are considered one after another, in descending order of $\frac{\mathcal{P}(s)-P_{s;t}}{\text{score}(s)}$. When a station s is considered, first the feasibility checker is invoked to determine whether it is possible to repack s along with the exited stations: i.e., given a time limit, it tries to find a feasible assignment for $\{s\}\cup S_{\text{exited}}$. If the feasibility checker cannot repack s, its bid is not examined, and s is said to be "frozen". A station that is guaranteed to be frozen for the remainder of the stage is called a *provisional winner*; in a UHF-only auction, every frozen station is provisionally winning. In this case, $\mathcal{P}(s)$ is not reduced and s will no longer be asked to bid. If the feasibility checker can repack s, then $\mathcal{P}(s)$ is reduced and its bid is examined. If s bid to accept $P_{s;t}$, $\mathcal{P}(s)$ is lowered to $P_{s;t}$ and s remains "active", meaning it will be asked to bid again next round. If s bids to reject $P_{s;t}$, s permanently exits the auction.

3.2.4. Transitioning Between Auction Stages A reverse auction stage ends when all stations are either frozen or have exited. Following each reverse auction stage is a forward auction stage where mobile carriers bid on licenses in the cleared spectrum. If the forward auction generates enough revenue to cover the costs of the reverse auction (the payouts to the winning stations $\sum_{s\in S_{\text{winners}}}\mathcal{P}(s)$),[7] the incentive auction terminates and each frozen station is paid $\mathcal{P}(s)$. An unsuccessful forward auction (one that does not raise sufficient revenue) triggers another stage of the reverse auction with a smaller clearing target.

The incentive auction thus determines the amount of spectrum to clear endogenously by iterating through stages of reverse and forward auctions that clear progressively less spectrum until a stage occurs in which the forward auction covers the costs of the reverse auction. When a new reverse auction stage begins, $\bar{c}$ is increased, expanding the set $\overline{\mathcal{C}}$. Given additional channels, some frozen stations may now be repackable; such stations are said to be in "catch-up" mode. At the beginning of a new reverse auction stage, the base clock p_t resets. A station in catch-up mode "unfreezes" if it

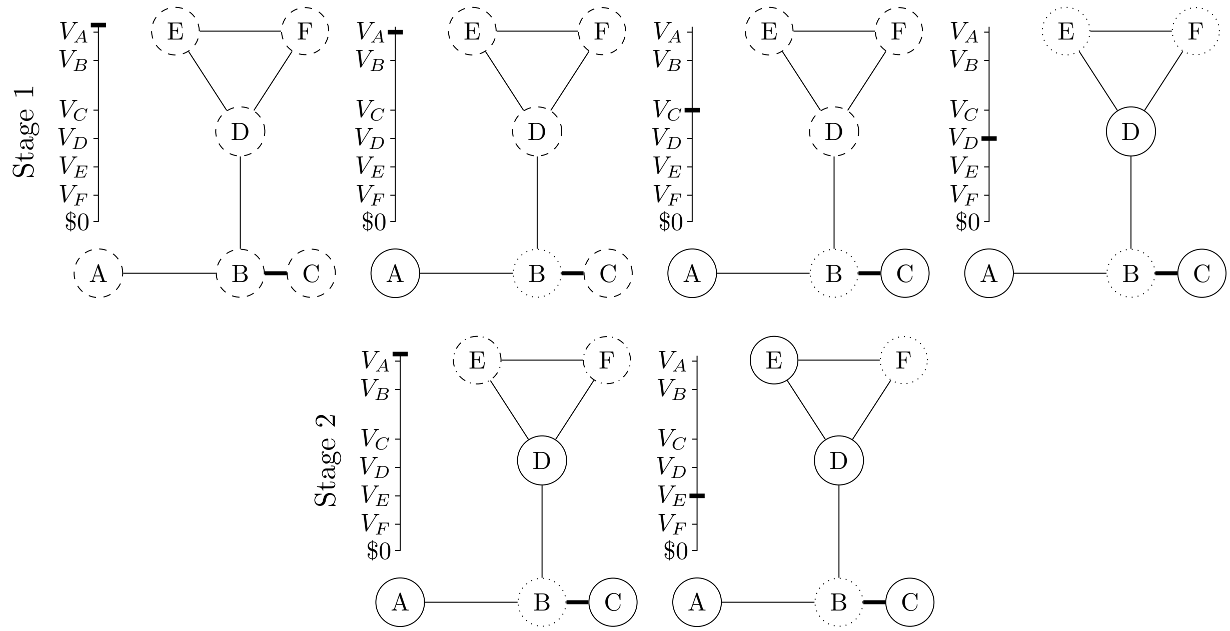

Figure 2 An illustration of the reverse auction example described in Section 3.3. Each row corresponds to a stage; a new image is drawn each time a station exits. Active stations are dashed, exited stations are solid, frozen stations are dotted, and stations with catch-up status are dashdotted. Connected stations cannot jointly broadcast on the same channel. B and C additionally cannot jointly broadcast on adjacent channels (shown by the thick bold edge between them). To the left of each image is the current clock price.

can be repacked in the first round in which it would face a weakly lower price than the price at which it froze, $\mathcal{P}(s)$. Subsequent stages otherwise proceed like the initial stage.

3.3. Worked Example

Figure 2 illustrates the reverse auction through a worked example. Consider an auction setting with six stations A, B, C, D, E, F having valuations $V_A > V_B > V_C > V_D > V_E > V_F$. Assume that stations bid straightforwardly: they accept offers above their values and reject offers below their values. Let each station be identically scored (so starting prices are the same for all stations); thus, we can drop the station subscript when discussing prices, writing P_t instead of $P_{s;t}$. Assume that the feasibility checker is perfect, always finding a feasible assignment when one exists and always determining infeasibility otherwise. For convenience, let clock decrements be so small that we can model the clock as falling continuously. Let $\mathcal{C} = \{1, 2, 3\}$ and let $\mathcal{I}$ be structured so that all stations have co-channel constraints on every channel with each neighboring station according to the interference graph in Figure 2. Let stations B and C additionally have adjacency constraints with each other due to their close proximity, prohibiting them from jointly broadcasting on adjacent channels.

Let $\overline{c} = 2$ initially, so that the auction begins trying to repack the stations into a single channel. Near the high opening prices, stations bid to remain off-air and their bids are processed because each station can exit. Nothing changes until $P_t < V_A$, at which point station A exits the auction.

This movement freezes station B at price $\mathcal{P}(B) = V_A$, since A and B cannot both broadcast on the single channel. Other stations remain bidding and prices continue to fall until $P_t < V_C$ and C exits the auction. C's exit does not impact the other stations, so they continue to bid until $P_t < V_D$. At this point, D exits the auction, freezing E and F each $\mathcal{P}(E) = \mathcal{P}(F) = V_D$. Every station is now either frozen or has exited, so the stage ends. Stations B, E, and F are frozen at the end of the stage. If the incentive auction does not proceed to another stage, the total value removed from the airwaves will be $V_B + V_E + V_F$ and the total cost of freeing up two channels will be $V_A + 2V_D$.

Let us assume that in the forward auction, wireless carriers are unwilling to pay the repacking cost of $V_A + 2V_D$ to repurpose two channels worth of spectrum. The incentive auction then proceeds to a second stage where $\bar{c} = 3$: only one channel is cleared and two channels remain for repacking stations. Stations E and F can now be repacked alongside the exited stations and so enter catch-up mode. B cannot be repacked and remains frozen. P_t resets to a high value and then descends until $P_t < V_D$. At this point, E and F both transition from catch-up mode to bidding, since V_D was the price at which they froze. The auction continues until $P_t < V_E$, at which point E exits and freezes F at a price $\mathcal{P}(F) = V_E$. Again, all stations are either frozen or exited, so the second stage completes. Assuming the ensuing forward auction raises sufficient revenue, stations B and F will removed from the airwaves for a value loss of $V_B + V_F$ and a cost of $V_A + V_E$.

3.3.1. Repacking VHF We conclude by noting some of the reverse auction modifications when the VHF band is also repacked and stations can additionally bid to move between bands. Let $b_{s,t} \in B_{s,t}$ represent station s's bid in round t, where $B_{s,t} \subseteq \{\text{OFF, LVHF, HVHF, UHF}\}$ denotes the options available to s in round t. To reduce strategic options available to bidders, bidding is a restricted by a "ladder constraint": Stations can never move "down" the ladder of bands (ordered as in the list above, with UHF at the top and OFF at the bottom) and can never go higher than their home band. For example, a station with a home band of HVHF currently assigned to OFF could bid to remain in OFF, move to LVHF, or exit to HVHF, but would not be allowed to move to UHF since UHF is above its home band. If the station were instead assigned to LVHF, it could bid to remain in LVHF or exit to HVHF; it would not be able to bid for OFF because OFF is below LVHF, and it would still not be able to bid for UHF because UHF is above its home band. In each round, each bidding station is offered a separate price for each of its legal movements. We use $P_{s;b;t}$ to represent the price offered to station s in round t for selecting band b. We discuss how prices are computed in Appendix D. A final difference we will mention is the introduction of fallback bids. While any number of stations can go off-air, the VHF bands have limited capacity. As a result, the auction may not be able to accommodate certain bids for moving into VHF bands. For example, consider a UHF station s that bid for going off-air in the previous round and in the current round bids to move to HVHF. If s is not frozen when its bid is examined, the feasibility checker will try to determine if it is possible to fit s alongside the set of exited stations whose home band is HVHF. If the feasibility checker cannot find a feasible repacking, s's fallback bid is examined. This fallback bid can be either to exit or duplicate the previous bid (i.e., in the example, it would be as if s had again bid to go off-air). We denote such bids as $\text{fallback}_{s,t}$.

4. Value and Bidding Models

This section begins by describing two value models. The first follows Doraszelski et al. (2017); we created the second for this study, based on bid data released after the auction by the FCC. An advantage of considering two different value models is that we were able to compare simulation results under both settings to assess the robustness of our findings. We conclude the section by describing a model of how stations bid as a function of these values.

4.1. Value Models

Each station s has a value $v_{s,b}$ for broadcasting in each permissible band b. We normalize so that a station has no value for being off-air, i.e., $v_{s,\text{OFF}} = 0$. Both models only provide $v_{s,\text{UHF}}$, that is, home band values for UHF stations. For the two VHF bands in the auction, lower and higher VHF, we model a UHF station's value for switching to the HVHF band as $\frac{2}{3} \cdot v_{s,\text{UHF}} \cdot \mathcal{N}(1, 0.05)$ and similarly for the LVHF band as: $\frac{1}{3} \cdot v_{s,\text{UHF}} \cdot \mathcal{N}(1, 0.05)$—i.e., roughly two thirds and one third of the station's UHF value with some multiplicative Gaussian noise. We describe robustness experiments on alternate parameter choices in Appendix E; our qualitative findings remain the same. We generated values for VHF stations by computing a hypothetical UHF value and then applying the fractional reductions for VHF bands just described.

4.1.1. The MCS Value Model. Doraszelski et al.'s (2017) valuation model, which we dub the MCS (Max of Cash flow and Stick value) model,[8] treats a station's value as the maximum of its cash flow value as a business and its stick value. The stick value represents the value of the broadcast license and tower, independent of the business; it can be more appropriate than cash flow when valuing non-commercial stations. Both of these were estimated from various sources including transaction data of station sales, advertising revenue, and station features.

4.1.2. A Novel Value Model based on Bid Data. Two years after the incentive auction concluded, the FCC released the auction bids. We used this data to construct a "realistic" model for station valuations. While the bids are not sufficient to reveal station values, they do allow us to infer bounds on values. In some cases these bounds are relatively tight, with upper and lower bounds separated by a single clock interval. Most of the time they are looser, in some cases, to an extent that does not allow us to improve on the trivial bound.

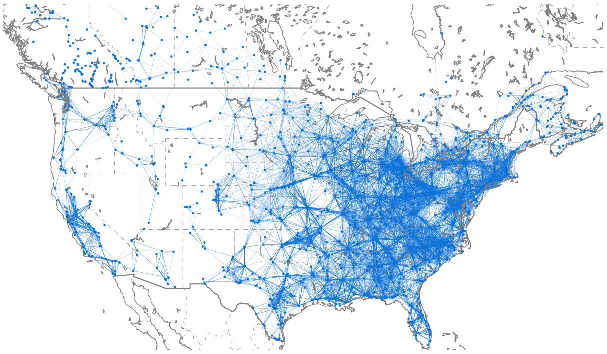

Figure 3 **Interference graph derived from the FCC's constraint data.**

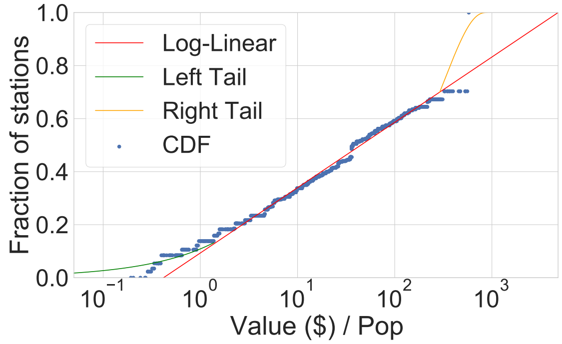

Figure 4 **CDF for our maximum likelihood estimate of N and its log-uniform distribution fit, plus generalized Pareto left and right tails.**

We inferred bounds on each UHF station's home band value, $v_{s,\mathrm{UHF}}$. For details, see Appendix F.1. We then used these bounds to fit a model. We assumed that value is proportional (in expectation) to population,[9] $v_{s,\mathrm{UHF}} = \mathrm{Population}\,(s) \cdot n_s$. Here n_s is some number in units of $/pop, sampled from an unknown cumulative distribution function (CDF) N. Our upper and lower bounds on a given station's home band value can be translated into upper and lower bounds on a station's $/pop by dividing by s's population. We computed a non-parametric maximum likelihood estimate of the distribution function N. Note that by definition $N(y_s) - N(x_s) = \Pr(x_s \le n_s \le y_s)$. Our goal was to maximize the product of these terms subject to constraints ensuring that N is a valid CDF. The results (Figure 4) suggest that N is a log-linear function. Data was sparsest in the tails of the distribution, especially in the right tail, so we replaced the log-linear segments in both tails with Generalized Pareto Distributions (GPDs). For more details, see Appendix F.2.

To generate UHF values, we multiply a station's population by a sample from the modeled distribution of N. In what follows, we refer to this model as the BD (bid data) model.

4.2. Bidding Model

We now describe our model of how stations bid. A station participated in our simulations if its opening price for going off-air exceeded its value for continuing to broadcast in its home band, i.e., if $P_{s;\mathrm{OFF};\mathrm{Open}} \ge v_{s,\mathrm{pre}(s)}$. After excluding 64 non-mainland stations (see Appendix G), we considered 1813 stations eligible to participate in our simulations: 1407 UHF, 367 HVHF, and 39 LVHF.

Our two value models differ substantially in the participation rates that they predict. In the MCS model, station values tend to be low relative to opening prices, leading to very high participation rates. For example, considering 10 000 sampled value profiles of UHF stations, the mean number of participants was 1416. 1196 of these stations participated in every sample, and only 46 had less than an 80% chance of participating. In contrast, when running the same experiment with the BD model, no station participated in every sample and average participation rates were 60%

(877 stations), closer to the 64% participation rate of UHF stations (930 stations) in the incentive auction. For reference, a total of 1030 stations (including VHF stations) actually did participate in the incentive auction (FCC 2019).

In auctions with UHF options only, in which bidders are able only to remain off-air or to exit the auction, a single station faces a strategic situation in which its utility is "obviously" maximized by remaining off-air if the price exceeds its value and by exiting otherwise—that is, by bidding myopically (Li 2017). The situation when VHF options are included is no longer obvious in this sense, but we continue to assume for simplicity that when bidding in round t, a station selects the offer that myopically maximizes its net profit, $\arg\max_{b \in B_{s,t}} P_{s;b;t} + v_{s;b}$. When fallback bids are required, we again assume that stations select the option that maximizes their net profit. We note that in the released bid data, only 52% (13%) of bids to move into LVHF (HVHF) were successful. Given that stations seeking to move to a VHF band faced a meaningful risk that their bid might fail to execute, some bidders might have benefited by bidding on VHF bands before it was straightforwardly optimal to do so. However, despite the potential drawbacks of straightforward VHF bidding, we are not aware of any behavioral rule that can be applied to all bidders and that is arguably more realistic.

5. Simulator Design and Experimental Considerations

As the reverse auction is simplest to reason about when only the UHF band is repacked, we ran both simulations that only repacked the UHF band and simulations that also repacked the VHF band. This allowed us to investigate which of our results generalize across settings. We ran simulations on both value models described in Section 4. Unless otherwise stated, in every one of our experiments we took 50 samples per treatment. We gave feasibility checks 60 seconds to complete (as in the real auction, though of course we were unable to use exactly the same hardware) unless otherwise stated.[10] We now explain how we compare simulated outcomes and discuss some simplifications our simulations make relative to the real auction process. We report additional elements of our experimental setup details in Appendix G.

5.1. Metrics

One goal for the auction is efficiency: for any given clearing target, to maximize the total value of the stations that remain on the air, or equivalently, to minimize the total value of the stations removed from the airwaves. We focus on the latter definition—*value lost* instead of *value preserved*—because it is unaffected by value estimates for large, highly valuable stations that do not participate in the auction. That is, *value preserved* includes the values of easy-to-repack stations, even those that do not participate in any interference constraints, and leads to efficiency estimates near 100% when few stations go off-air relative to the number that remain on air, even when the number of stations going off-air is large relative to the number required by an optimal solution.

We define the *value loss* of an auction outcome as $\sum_{s \in \mathcal{S}} v_{s,\text{pre}(s)} - v_{s,\text{post}(\gamma,s)}$. Ideally, our metric for allocative efficiency would be the ratio of the value loss of a simulation's final assignment, γ, relative to an assignment from an efficient repacking γ^* that minimizes the value loss for a given value profile, i.e., $\frac{\sum_{s \in \mathcal{S}} v_{s,\text{pre}(s)} - v_{s,\text{post}(\gamma,s)}}{\sum_{s \in \mathcal{S}} v_{s,\text{pre}(s)} - v_{s,\text{post}(\gamma^*,s)}}$. In general, however, γ^* is too difficult to compute, so we convert our absolute metric into a relative one by comparing the value loss between two simulations' final assignments (i.e., the ratio of value loss ratios, noticing that the denominators which depend on γ^* cancel out in this case).

Our second metric is the cost of reaching the specified clearing target: the prices paid to all winning stations, $\sum_{s \in S_{\text{winners}}} \mathcal{P}(s)$.

We use the terms "efficiency" and "cost" below as abbreviations that refer to value loss and the total payments made to broadcasters that go off-air or change bands. Outcomes with high efficiency (low value loss) and low cost are preferable. It is straightforward to compare two outcomes if they both clear the same amount of spectrum and one is both more efficient and cheaper; otherwise, any comparison requires a judgement call about how the two metrics should be traded off.

5.2. Impairments

The incentive auction's design requires it to begin from a feasible channel assignment for the non-participating stations. However, it may not always be possible to find a feasible repacking for the non-participating stations in the set of remaining channels $\overline{\mathcal{C}}$ induced by the initial clearing target $\overline{c}$. The FCC's rules therefore allowed a small number of stations to be assigned to channels *within* the spectrum that was otherwise resold (i.e., channels in $\mathcal{C} \setminus \overline{\mathcal{C}}$), even though doing so degrades the desirability of the mobile broadband licenses sold in the forward auction. Such stations are referred to as "impairing". There are two types of impairments: those caused by non-US stations that would be present even if every US station participated in the auction ("essential impairments") and those caused by non-participating US stations.

Despite our best efforts to make our simulations realistic, we could not replicate the optimization procedure the FCC used to determine the initial clearing target $\overline{c}$ and the set of impairing stations. This optimization relied on data that is not publicly available: the Inter-Service Interference ("ISIX") constraints that determine which geographic areas are impaired when a station is placed on channels to be resold. Without the ISIX data, we also could not replicate the analogous optimizations that the FCC conducted between auction stages.

Instead of determining the initial clearing target via an optimization, except when otherwise noted, we started our simulations at the 84 MHz clearing target (the clearing target at which the real incentive auction concluded) and ran auctions for only a single stage. We did this both because running multiple stages of the reverse auction is computationally expensive and because multi-stage simulations depend on additional assumptions about the forward auction. A channel uses 6 MHz of spectrum, so a clearing target of 84 MHz corresponds to clearing 14 channels; see Figure B.11 for more details on possible clearing targets. We do explore multi-stage auctions that begin from other clearing targets (including 126 MHz, the clearing target on which the incentive auction began) in Section 6.2. Our methodology for selecting which stations to impair is described in Appendix H. In our simulations, stations marked as impairing do not interfere with each other or with stations assigned to channels in $\overline{\mathcal{C}}$.

6. Experiments

Our experiments are divided into four categories, based on which element of the auction design they investigate: (1) repacking the VHF band in addition to UHF; (2) choosing the order in which stations are processed via a scoring function; (3) determining a clearing target by iterating between reverse and forward auction stages; and (4) determining which stations to freeze by checking the feasibility of station repackings.

6.1. Repacking the VHF Band

The incentive auction reduced only the number of UHF channels, but repacked stations in three bands: UHF, HVHF, and LVHF. Repacking the two VHF bands offered the potential for cost savings and efficiency gains, as UHF stations might have been willing to accept a smaller payment to move to a VHF channel instead of going off the air and this could have constituted a net gain, even taking into account the need to compensate VHF stations for going off-air to make space. An optimal repacking for a given value profile can only become weakly more efficient when the VHF bands are included as more configurations of stations become available.

However, adding extra bands to the reverse auction complicated an otherwise elegant design (see Section 3.3.1 and Appendix D). Stations no longer possessed obviously dominant strategies and might have benefited from reasoning about when to move up the ladder. Price calculations became more involved as each option had to be priced appropriately. The bidding language had to be augmented with fallback bids. Also, unlike in UHF-only settings where freezing is permanent, VHF stations can freeze and later unfreeze within the same stage if other stations move out of their home bands, complicating bid processing. All of this extra complexity made the auction more difficult to explain to station owners, which mattered since some of the participants were relatively unsophisticated and encouraging them to participate was a first-order concern. It is thus sensible to ask whether the additional complexity was worthwhile. While repacking VHF raised the possibility of more efficient allocations, such gains might have been small, arising only under exotic bidding

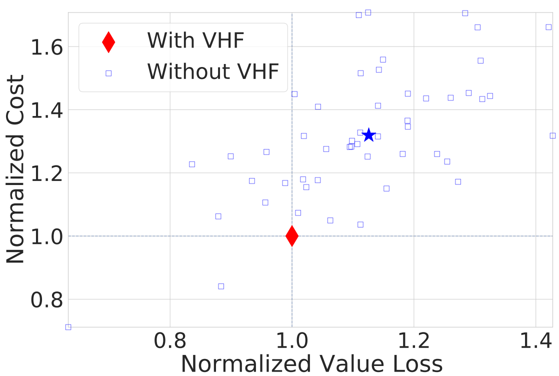

Figure 5 **Comparing auctions that only repack the UHF band against auctions that also repack VHF bands.**

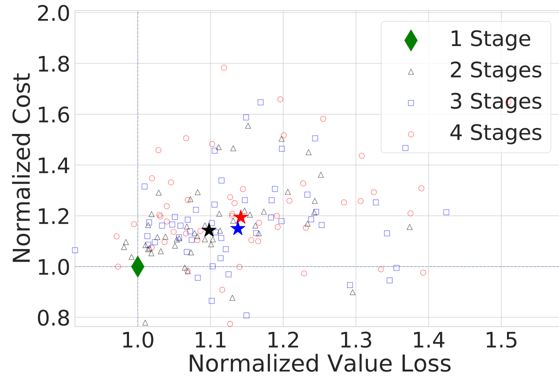

Figure 6 **Comparing auctions running through 1-4 stages, ultimately ending on the same clearing target, with no impairing stations.**

behavior, or have come at a high cost. We thus asked: What changes to efficiency and cost arise when VHF options are included and bidders bid straightforwardly?

To answer this question, we ran two sets of auctions: the first repacking both the VHF and UHF bands, the other repacking only the UHF band. Our results are shown in Figure 5. (In what follows, we typically present figures for repacking both UHF and VHF using the BD value model, but discuss our findings across all experimental settings. The remaining graphs are presented in Appendix I.) Each point in the figure represents the outcome of one simulation, with its x-axis position denoting its efficiency and its y-axis position denoting its cost. Since it is difficult to show graphically which auctions use the same paired value profiles for even a modest number of samples, rather than plotting raw efficiency and cost on each axis we instead plot normalized efficiency and cost. That is, we select one setting (in this case, auctions that repack VHF) as the reference treatment, and then plot the ratio of the efficiency (cost) of each simulation from additional treatments relative to the corresponding simulation using the same value profile in the reference treatment. With this choice, the reference treatment always corresponds to the point $(1,1)$, represented in our figures by a diamond. For each treatment we also plot a star to indicate each metric's mean value.

The experiments just described took a little over one CPU year to run. Using both value models, we observed that repacking the VHF band led to a significant reduction in payments—on average, sampled UHF-only auctions cost 1.23 and 1.32 times as much as their VHF-repacking counterparts using the MCS and BD value models, respectively. The impact on efficiency was more modest and less uniformly positive. Under the MCS (BD) value model, sampled UHF-only auctions experienced 1.05 (1.12) times higher value loss on average. On the whole, our simulations suggest that if bidders continued to bid straightforwardly despite the complex design, repacking the VHF band was an important design choice that likely led to lower costs and also somewhat more efficient outcomes.

6.2. Multi-Stage Clearing

The incentive auction allowed for multiple stages of reverse followed by forward auctions in order to let market forces determine the appropriate amount of spectrum to clear. Each successive stage began with a reverse auction clearing a smaller amount of spectrum to achieve a lower overall cost than the previous stage. The actual incentive auction went through four stages.

One obvious practical drawback of a multi-stage approach is its impact on auction length: running multiple stages takes time. As noted by Ausubel et al. (2017) "One potential criticism of the incentive auction is that it lasted too long. If, after the initial commitment, the FCC had selected a clearing target of 84 MHz (instead of 126 MHz), it is very likely that the auction would have concluded after a single stage with significantly fewer rounds."

A less obvious concern is how the multi-stage approach impacted the final outcome. In general, if the clock auction perfectly optimizes for each clearing target and if the stations packed for the higher target are not a subset of those packed for the lower target, then one should expect a multi-stage auction to perform worse than an auction that starts by setting the correct target. We investigate these dynamics in Appendix L, considering elaborations of our previous worked example. First, we show how stations packed in late rounds of an early stage can cause problems in subsequent stages. Then, we show the clock auction's greedy optimization procedure can also lead to the reverse: an auctioneer *benefiting* from running a multi-stage rather than a single-stage auction.

Having observed that multi-stage clearing can yield both economic benefits and harms, we turn to simulations, asking two questions: (1) What, if any, economic costs arose due to multi-stage clearing? (2) Would an "early-stopping" alternative to the reverse auction have performed better?

6.2.1. The Economic Impact of Multi-Stage Clearing. To assess the economic impact of multi-stage clearing, we ran experiments that began trying to clear 126, 114, 108, and 84 MHz of spectrum, each proceeding to follow the ordering of clearing targets selected by the FCC (see Figure B.11), and each terminating at 84 MHz, leading to four-, three-, two- and single-stage auctions, respectively. These experiments took 31 years of CPU time.

A complicating factor in this comparison is that for any given value profile, the final set of impairing stations may differ based on the starting stage, which muddies the interpretation of cleared spectrum as a measure of performance. Our impairment mechanism (described in Appendix H) attempts to remove impairments in between stages. The cleared spectrum is only comparable if exactly the same set of impairing stations remain at the auction's termination. To improve the comparison, we analyze the results of these experiments in two ways. The first is to consider a world without impairments, corresponding to the case where the FCC is willing in principle to pay any amount to stations to eliminate impairments and therefore sets high opening prices. Under this assumption we observed that running the auction through multiple stages degraded both cost and efficiency, especially when the VHF band was repacked (Figure 6). In VHF-repacking simulations using the MCS (BD) model, on average four-stage auctions cost 1.50 (1.19) times as much as their single-stage counterparts and had 1.26 (1.14) times the value loss. The results were similar but less dramatic in magnitude for UHF-only auctions. In UHF-only simulations using the MCS (BD) model, four-stage auctions cost roughly 5% (10%) more and had roughly 5% (10%) additional value loss compared to perfectly forecasting the clearing target. In all cases, even if the exact stage was not perfectly selected, starting the auction closer to the final stage would also have yielded significant improvements to both metrics on average.

A second way of analyzing the results is to account for impairing stations. Of course, this leads to "apples-to-oranges" comparisons in the sense that the quality of cleared spectrum varies across simulations. We do not have a reliable way of assessing how much a given impairment devalues spectrum. The answer surely depends on the location and power of the exact stations in question, but a blunt proxy number for how bad a set of impairing stations might be is the sum of their population (less is better). In our experiments, there were never any impairing stations remaining at the final stage when using the MCS value model, so results remained unchanged for this model. We visualize results for the BD model in Figure I.14. We observe much higher variance than in the previous analysis. With VHF repacking, we again observe that multi-stage auctions performed much worse than single-stage auctions, with four-stage simulations costing 1.25 times more and experiencing 1.10 times more value loss than single-stage auctions. In the UHF-only case we no longer observe the trend that more stages led to worse outcomes—on average, four-stage simulations cost and had value loss about 1% higher than single-stage simulations, and two- and three-stage simulations performed a little better (1–2%) on average on each metric. However, single-stage auctions resulted on average in the least impaired spectrum (according to our population metric): four-stage, three-stage, and two-stage simulations had mean impairing populations of 206, 205, and 206 million respectively compared to 192 million for single-stage simulations. Our results for repacking the VHF band look similar, with 205, 203, and 207 million mean impairing population for four-, three-, and two-stage auctions respectively compared to 192 million for single-stage auctions.

6.2.2. Early Stopping The results just presented led us to ask: is there any other way the FCC could have determined a clearing target endogenously with less impact on cost and efficiency? We propose a simple answer, which we call *early stopping*: for each candidate clearing target, conduct the forward auction before the corresponding reverse auction, and stop each reverse auction as soon as its cost exceeds the forward auction revenue. To see why this would help, consider again our example in Figure 2, but now imagine that the auctioneer knows going into the first reverse

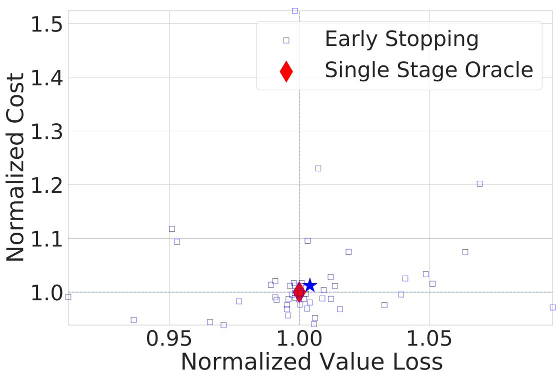

Figure 7 Comparing early stopping auctions against single-stage auctions.

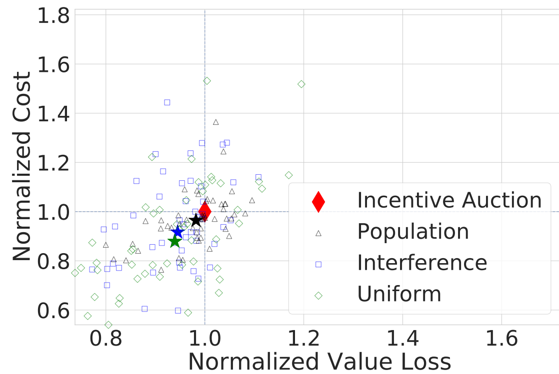

Figure 8 Comparing auctions using four different scoring rules.

auction stage that the forward auction revenue is some number less than V_A. Once station B freezes at a price of V_A, the provisional cost exceeds the forward auction revenue. Following our proposal, the first reverse auction stage would immediately stop and so station C would not exit. Then, in the second stage, station B would exit next and the outcome would exactly match that of an auction in which the clearing target had initially been set to the second stage target. In general, continuing the reverse auction instead of aborting can only result in more stations exiting, limiting flexibility in later stages.[11] The original design proposed that the forward and reverse auctions be run in parallel, but due to finite staffing resources, the auctions had to be sequenced. We note that early stopping represents a potential algorithmic improvement even over the original parallel design.

Early stopping does not always outperform the original design; it is possible to construct examples (see Appendix M) where the commitments made in earlier stages are better than those made in later stages. Nevertheless, we believed that early stopping would typically help in practice. To test this, we ran two sets of experiments. The first set compared early stopping auctions against single-stage stations that "knew" the correct clearing target. The second set compared the amount of spectrum cleared by early stopping auctions to auctions using the original design. Both of these experiments required forward auction revenues as inputs. We had little data to use for modeling these revenues—one observation for each of the four clearing targets that were reached in practice—so we adopted a convenient model described in Appendix N. Unlike previous experiments where we assumed a predetermined number of stages, we now determined the auction's end by comparing forward auction revenues to clearing costs. This meant that auctions could end at any stage, so the amount of spectrum sold could differ across paired simulations. We assume that clearing more spectrum is preferable to clearing less, noting the FCC's stated goals. We ran early stopping auctions with an initial clearing target of 126 MHz (corresponding to the first stage of the real auction) and following the 600 MHz band plan (see Figure B.11) from that point on.[12]

Once these experiments completed, we determined the final stage of each early stopping simulation and ran a corresponding single-stage auction in order to assess the "penalty" due to early stopping vs. perfect forecasting of the clearing target. The results are shown in Figure 7. For computational reasons, we only ran UHF-only simulations; even so, the experiments took more than 15 CPU years overall. To avoid making comparisons between different qualities of spectrum, we disabled our impairment mechanism and let prices start as high as required for full participation. We observed that in both value models, early stopping performed well relative to single-stage auctions, with increases to average cost and value loss of no more than about 1%. These results are particularly encouraging when compared to the multi-stage experiments described earlier, in which we observed significant gaps between multi-stage clearing and perfect forecasting.

We next ran simulations that compared early stopping to the original clearing procedure to determine which cleared more spectrum. Under both value models, on average more mobile licenses were created when by early stopping than the original algorithm—0.50 and 0.22 extra licenses under the BD and MCS models, respectively. Early stopping also led to shorter auctions, averaging 30% and 76% of the rounds required by the original algorithm under BD and MCS, respectively.

We conclude by reflecting on some concrete numbers from the real auction. At the end of the first round of the first stage, payments to frozen stations already exceeded $50 billion; when the stage finished, these payments were $86 billion. In the subsequent forward auction, revenues were only $23 billion. Early stopping would have terminated the reverse auction during the very first round of bidding. The first reverse auction stage took one full month to resolve. This month, and possibly more time in future stages, would have been saved if early stopping had been implemented.[13]

6.3. Scoring Rules

Stations' starting prices in the incentive auction were not all the same: they were set proportionally to an assigned *score*, determined by a *scoring rule*. Under truthful bidding in a UHF-only auction, an unfrozen station will remain off-air for exactly as many rounds as it takes for its price to fall below its value, at which point it will exit. Using scoring to raise or lower a station's initial price relative to others is one control knob an auctioneer has over the order in which stations exit.

Theoretically, scoring performs two distinct functions. First, because every descending clock auction is equivalent to a greedy algorithm for packing stations into the broadcast spectrum, it may be possible to pack a larger and more valuable set of stations if the algorithm prioritizes stations that interfere with fewer neighbours. In the FCC's design, this was achieved by offering higher prices to stations with more "interference links"[14] so that those stations would be less likely to exit. Second, scoring reduces an auction's expected cost by offering lower prices to stations that would be likelier to accept them, following Myerson (1981). In the FCC's design, this was implemented by reducing prices offered to stations serving smaller populations. (This design element was controversial: it was vigorously opposed by a coalition of owners of lower powered stations serving smaller populations, the "Equal Opportunity for Broadcasters Coalition", whose starting prices in the clock auction were reduced.) Overall, the two elements were combined by setting opening prices in proportion to the square root of the product of a station's population and its interference links.

In our experiments, we compared the following four scoring rules: (1) "Incentive Auction", the scoring rule used in the actual auction; (2) "Interference", the square root of a station's interference links; (3) "Population", the square root of a station's population; and (4) "Uniform", scoring each station identically. As in the actual auction, we normalized all scores so that the highest scoring UHF station had a score of 1 million.

We note a subtlety in these experiments: scoring rules impact prices, so simulations differing only in their scoring rules will not necessarily select the same set of impairing stations given the same value profile. To prevent "apples-to-oranges" comparisons across auctions clearing spectrum of varying quality, we disabled our impairment mechanism and ran simulations with high enough base clock prices to elicit full UHF-station participation (thereby avoiding all "unessential" impairments).[15][16] Running with no impairment mechanism corresponds to considering a world in which the FCC is unwilling to accept any degradation to the cleared spectrum.

Results for our simulations using the various scoring rules are shown in Figure 8. Under the MCS value model, when the VHF band was repacked, we observed that simulations using only interference scoring cost 1.24 times as much and experienced 1.11 times as much value loss as simulations that combined population and interference scoring (the FCC's scoring rule). Uniformly scoring stations was not effective in this setting, with simulations averaging nearly 50% higher costs and 25% higher value loss. When only the UHF band was repacked, the scoring rule appeared to matter much less. Here, the interference scoring rule outperformed other scoring rules on average, with mean costs and value losses of 0.93 and 0.99 times respectively compared to corresponding simulations using the FCC's scoring rule. Under the BD value model, we observed much higher variance across simulations. If we nevertheless consider average performance, the FCC's scoring rule was outperformed in both metrics by every other scoring rule considered regardless of whether the VHF band was repacked. When the VHF band was repacked, the lowest value losses and costs were achieved, surprisingly, by the uniform scoring rule (94% mean value loss and 88% cost relative to the FCC's scoring rule). When only the UHF band was repacked, the lowest average value losses and costs were achieved by interference scoring (97% and 95% of the FCC scoring rule). On the whole, beyond a single setting (MCSMCS values and UHF + VHF repacking) we did not find robust evidence for population-based scoring. These experiments required 3.5 years of CPU time to run.

All of the scoring rules that we considered are static: they assign each station a single, fixed score. The theory of descending clock auctions also considers scores that respond to auction history in a dynamic fashion (provided that they never increase a station's price). We leave the investigation of dynamic scoring rules for this setting as future work, but note that our simulations' failure to crisply recommend any of the static scoring rules we considered may motivate such an investigation.

6.4. Feasibility Checking

The feasibility checker determines if a given station can be repacked alongside the stations that have previously exited the auction. Feasibility checking was a large concern in the incentive auction because the station repacking problem is hard both theoretically—it is NP-complete—and in practice. When the feasibility checker cannot find a way to repack a station, either by proving that no repacking exists or by running out of time, a station freezes: it stops bidding and its compensation stops falling. The feasibility checker's quality therefore has a direct effect on both the cost and efficiency of the auction: if the feasibility checker fails to find an assignment that repacks a station when such an assignment exists, this station will freeze at an unnecessarily high price. If the station would otherwise never freeze at all, the value loss of the allocation is changed.

Do auctions achieve better efficiency and/or lower costs as the feasibility checker improves? Our intuition is that better feasibility checking should lead to better results. However, this intuition is not always correct; see Appendix O for a counterexample.

6.4.1. Effect of the Feasibility Checker on Auction Outcomes. SATFC 2.3.1, the feasibility checker that was used in the incentive auction, was designed over several years (Fréchette et al. 2016, Newman et al. 2017).[17] The solver combines complete and local-search SAT-encoded feasibility checking with a wide range of domain-specific techniques, such as constraint graph decomposition. Automatic algorithm configuration techniques were applied to construct a portfolio of eight complementary algorithms from these various components. We now investigate whether the effort invested in making SATFC 2.3.1 was helpful, or whether a more off-the-shelf solution would have sufficed. To do so, we ran simulations in which we exchanged SATFC 2.3.1 with alternative solvers.

Newman et al. (2017) ran 22 solvers from ACLib (Hutter et al. 2014) (a library of solvers that support algorithm configuration) on a benchmark set of sampled station repacking problems from their reverse auction simulations.[18] The results are shown in Figure K.20. We selected certain solvers from this figure and ran simulations, using each as the feasibility checker. Specifically, we compared the following solvers: *SATFC 2.3.1*: the feasibility checker used in the incentive auction; *Greedy*: a solver that simply checks whether a previous assignment can be augmented directly, without changing the assignments of any other stations (this algorithm is the simplest reasonable feasibility

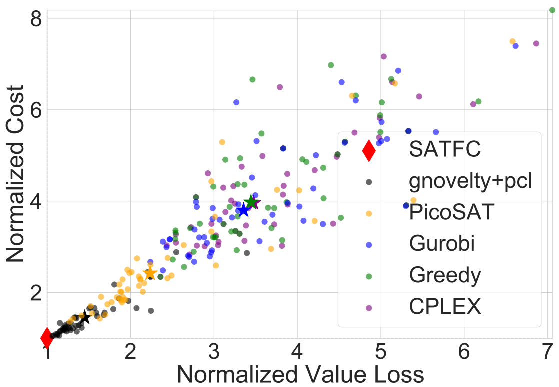

Figure 9 **Comparing auctions using different feasibility checkers.**

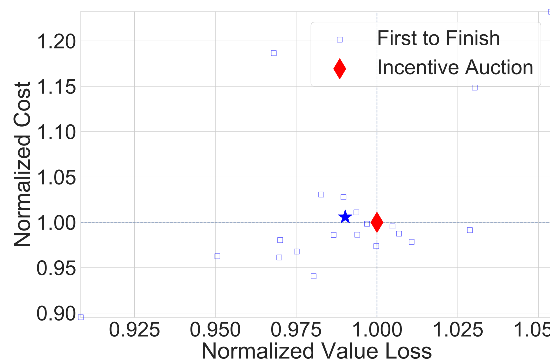

Figure 10 **Comparing the first to finish algorithm against the standard bid processing algorithm for single-stage 126 MHz auctions.**

checker and thus serves as a baseline); *PicoSAT*: To our knowledge, alongside MIP approaches, the only other solver that has been used in publications on the incentive auction, probably because it was shown to be the best among a set of alternatives in an early talk at the FCC on the subject (Leyton-Brown 2013); *Gurobi* and *CPLEX*: MIP solvers initially considered by the FCC; and *Gnovelty+PCL*: the best performing of the 22 ACLib solvers on the benchmark data described above. We note that similar experiments were performed in Newman et al. (2017). The experiments presented here go beyond those both by being larger in scale and by the inclusion of a new feasibility checker (Gnovelty+PCL), UHF-only experiments, and consideration of a new value model.

Our experiments took just over two CPU years. Results are shown in Figure 9.[19] We observed that stronger feasibility checkers led to better outcomes according to both of our metrics: the relative rankings of the solvers in the benchmark study translated exactly into the relative rankings across both of our metrics, regardless of whether the VHF band was repacked and regardless of our choice of value model. In particular, we observed that SATFC 2.3.1 dominated all other solvers on both metrics, not only on average but in each individual simulation and across both value models. Reverse auctions run using the best off-the-shelf solver, *gnovelty+pcl*, cost between 1.22 and 1.45 times more on average (depending on bands being repacked and the value model used) and lost between 1.22 and 1.45 times as much broadcaster value as those based on SATFC 2.3.1.[20]

6.4.2. Alternative Bid Processing Algorithms. Given our findings about the impact of a strong feasibility checker, we asked: Could allowing more time for solving individual repacking problems have led to fewer stations freezing unnecessarily and thereby have improved economic outcomes? Figure K.20 makes a compelling case that problems that are not solved quickly are overwhelmingly infeasible, yet it is difficult to say without experimentation what the impact of solving the remaining truly feasible problems might be. Significantly increasing the cutoff time given to each problem would not have been practical. The first round of the reverse auction had 1030 bidding stations. A worst-case scenario of each sequential repacking problem taking one hour, say, would have required more than 42 days. This would have slowed the pace of the auction to a halt!

Going beyond the feasibility checker comparisons in Newman et al. (2017), we explored two variants to the bid processing algorithm that increase the computation time available to processing stations before declaring them frozen while also respecting the time constraints of the auction. The first optimistically does not freeze a station with an indeterminate check, allowing future rounds to revisit whether the station is indeed frozen; the second leverages parallel computation to make better use of the full time window allotted to a given round.

Revisiting Indeterminate Feasibility Checks. The incentive auction froze a station whenever the feasibility checker could not prove that the station could be repacked. We note that indeterminate cases did not have to be treated this way: incentive constraints require only that a station's winning price does not decrease if it cannot exit. While calling these stations infeasible saves computation time as there is no need to recheck them in later rounds, it is possible that as other stations exit, a given station's repacking problem becomes more constrained and thus easier to solve. An alternative is for the bid processing algorithm, upon encountering an indeterminate feasibility check for a station, simply not to process that station's bid. Such a station is not frozen and will be asked to bid again in the next round. The prices offered to that station decreases as normal, but its winning payment $\mathcal{P}(s)$ will not decrease unless the solver finds a feasible repacking for the station. The auction designers rejected this alternative on the grounds that the actual rules would be simpler for bidders, but until now the alternative has never been quantitatively investigated.

Figure I.18 compares the results of the standard bid processing algorithm against one that does not freeze stations with indeterminate results. These experiments took 3 CPU years to run. We observed on average, in UHF-only simulations, small (about 0.5–1%) improvements to both metrics when we revisited indeterminate results.

The First to Finish Algorithm. Bid processing for each round was required to complete within a fixed time window. Since stations were checked sequentially, with a fixed cutoff time allotted to each check, and since empirically most checks finished very quickly (see Figure K.20), many rounds terminated earlier than they were required to. Recognizing that stations' incentives are unaffected by the order in which their feasibility checks are performed, we propose an alternate bid processing algorithm (dubbed "first to finish") that aims to order feasibility checks in ascending order of their runtimes, ensuring that a round never terminates early unless all feasibility problems have been solved.[21] This can be achieved by running all feasibility checks in parallel, each with a cutoff equal to the entire amount of time remaining in the round. As soon as any feasibility check completes, we process the corresponding station's bid. If the station remains in its current band, we leave all other feasibility checks running unchanged; otherwise, we update them to include the changes implied by the just-processed station, and restart all of them in parallel with a new cutoff corresponding to all of the remaining time. We provide pseudocode in Appendix J as Algorithm 2.

The first-to-finish algorithm consumes dramatically more compute power than the standard bid processing algorithm: checking the status of all 1030 participating stations in parallel would require on the order of 10 000 CPUs, since each run of SATFC 2.3.1 uses 8 parallel threads. At the scale of the incentive auction and given modern cloud computing resources, such computational requirements would not have been prohibitive. However, given that most feasibility checks are extremely fast (see Figure K.20), a more practical variant might be to run each check sequentially with a tiny cutoff before switching to fully parallel. In our experiments (described below), in all rounds at most 27 stations had checks that took longer than 1 second, meaning this variant would have required dramatically fewer CPUs than suggested by the worst-case bound.

Our experiments compare the first-to-finish algorithm[22] with a one-hour-per-round cutoff against the traditional bid processing algorithm with the usual one-minute-per-problem cutoff. Due to the computational requirements of these experiments, we considered only 20 samples of UHF-only simulations; still, they took more than 6 CPU years to run. The results are shown in Figure 10. We observed moderate noise in both metrics and a small average effect: about a 1% improvement in both metrics under the MCS value model, and a 1% improvement to value loss and a 0.5% increase in cost under the BD value model. We conclude that increased cutoffs would not have made a significant difference to the auction outcome. We do expect that the first-to-finish algorithm would have yielded larger gains if paired with a weaker feasibility checker or a larger incentive auction that gave rise to even harder feasibility checking problems.

7. Conclusions

We outlined a six-step simulation methodology and instantiated it in the context of the incentive auction to investigate previously unanswerable questions about the cost and efficiency of certain alternative designs. To validate the robustness of our results, we used two quite different value models: one from the empirical economics literature and another that we constructed to rationalize public bid data. Our main findings were that: repacking VHF led to significantly lower costs and more efficient outcomes; the multiple stage clearing rule substantially both increased costs and reduced efficiency; a simple amendment to the clearing algorithm could nearly eliminate multi-round inefficiency; the performance of pops scoring relative to other scoring rules varied widely based on the value model and whether the VHF band was repacked; the specialized feasibility checker developed for the auction significantly improved both cost and efficiency; and alternative bid processing algorithms, while helpful, would not have made a significant difference to auction outcomes. We hope these specific insights can help to inform future auction designs.[23] More broadly, our analysis demonstrates the practicality[24] and promise of large-scale computational analysis of the simulated behavior of candidate market designs in highly complex settings.

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Appendix A: Endnotes

1. For example, Kelly and Steinberg (2000) note of the Progressive Adaptive User Selection Environment (PAUSE) auction format that their design is "probably too complex to admit much theoretical analysis".

2. An agent can deviate from truthful bidding in this setting either by (1) accepting an offer below its value, or (2) by rejecting an offer above its value. Since prices only descend, these deviations either lead to selling at a loss or not selling at all, thus having a best-case utility of zero. Since truthful bidding has a worst-case utility of zero, it constitutes an obviously dominant strategy in this setting.

3. Theory on DA auctions has continued to evolve since the auction: for example, Gkatzelis et al. (2017) generalize deferred acceptance auctions to non-binary settings.

4. Neither of these papers included novel experiments; unlike our current study, their focus was not evaluating the empirical performance of counterfactual designs.

5. We make no comment about the fidelity of the FCC's own simulator, since details about it are not publicly available.

6. Canadian stations did not bid in the auction but could be reassigned new channels.

7. Technically, the forward auction had to generate about $2 billion more: it also had to cover FCC expenses and the estimated costs of station retuning. In our examples and experiments, we ignore these additional requirements and only focus on the payments to winning stations.

8. Our implementation follows the value model described in a 2016 draft of the paper; the authors have subsequently made revisions (e.g., to the way that population is counted for low-power non-commercial stations). We used parameters obtained directly from the authors.

9. A television station's value obviously depends on more than just the population it can reach. It is likely that other important features include the station's location (e.g., high-income versus low-income areas), and whether or not it is a commercial operation, for example. We opted for a simple model that only uses population, which the FCC used as a proxy for station value, acknowledging it will not capture all of the heterogeneity in station value.

10. We note a subtlety regarding runtime measurements. Since the feasibility checker uses a walltime cutoff, there will be some degree of unavoidable noise (i.e., problems that require time very similar to the cutoff threshold). As a result, different auction trajectories starting from the same value profile can occasionally be caused by such measurement noise rather than a given design change being tested. This is difficult to control for, but the effect is random and averages out across samples. One alternative (which we did not employ) to enhance repeatability would have been to cache results (e.g., as described in Section 4.4 of Fréchette et al. (2016)) so that a given station repacking problem always produced a deterministic outcome.

11. A suggestion along similar lines was given by Ausubel et al. (2017). Rather than swapping the order of the reverse and forward auctions, they proposed forecasting reasonable bounds on forward auction revenue (i.e., asserting that it would be unlikely for telecoms to pay more than $X for a given amount of spectrum) for each stage. The reverse auction would then terminate when the provisional cost reached $X, the following forward auction would be skipped, and the next stage of the reverse auction would be begin. They argue that first, this would have reduced the auction duration, and second, that bidders in the forward auction had a sense in early stages that the prices were too high for the auction to terminate, so they did not bid sincerely.

12. We note an implementation detail: The reverse auction detects "unconstrained" stations and forces them to exit at the end of each round. Unconstrained stations are those that can be provably feasibly repacked for the remainder of the stage, and hence would otherwise simply bid until their clocks wound down to zero or they exited of their own accord. In an early stopping auction, forcing these stations to exit no longer makes sense, as such stations may not be unconstrained in the next stage. Therefore, we disabled these checks in our early stopping simulations.

13. One advantage of the ultimately implemented reverse-first ordering is that it allowed the forward auction to terminate early when it was clear from looking at the high-value products (Category 1 products in High Demand Partial Economic Areas) that the auction would require another stage, skipping the rounds that would have otherwise been required to let every product's demand settle. The speedups from terminating the reverse auction early would need to be traded off against terminating the forward auction early; we did not perform such an analysis.

14. More precisely: "an index of the number and significance of co- and adjacent channel interference constraints that station would impose on repacking." (FCC 2015a)

15. We continued to lock the VHF band until the base clock price reached the FCC's starting base clock price as described in Appendix H, because the heuristics that set VHF prices are fragile and we were concerned that altering them might cause unintended effects on the VHF allocation.

16. Another reason not to run our impairment mechanism in this case is that the starting base clock price would surely change under a shift in the distribution of station scores; by not running our impairment procedure our results are not sensitive to this parameter.

17. While the prototype of SATFC was developed quickly, various unrelated legal and logistical delays to the auction's launch provided SATFC's authors with time to refine it.

18. This benchmark can be downloaded at `https://www.cs.ubc.ca/labs/beta/www-projects/SATFC/cacm_cnfs.tar.gz`

19. Our simulator always attempted to solve each feasibility check using the greedy algorithm before calling another solver, so every other feasibility checker can be understood as a sequential portfolio of the greedy solver and itself. We used this heuristic for good reason: the vast majority of feasibility checking problems encountered in a typical simulation can be solved greedily. Lastly, we note that for consistency we always used SATFC 2.3.1 as the feasibility checker in all experiments during the impairment phase (see Appendix H).

20. We emphasize that these experiments considered only each solver's default parameter setting, and do not investigate the performance each solver could have achieved if it had been tuned or otherwise customized.

21. In fact, we first raised this idea as the last details of the incentive auction were being finalized. Although the design team received the idea positively, it was not pursued because altering the bid processing algorithm would have contradicted the published auction rules, which was no longer possible by that time.

22. Since we lacked access to the required number of parallel computers, we ran a sequential version of the first to finish algorithm. The sequential version runs all of the unprocessed checks for a small cutoff and then processes them in order of completion time until encountering a movement or exit bid, at which point checks are restarted. If at the end of the cutoff, only indeterminate problems remain, the cutoff is set to the minimum of double the current cutoff or the remaining time left in the round.

23. Kiddoo et al. (2019) remark: "Numerous government regulators around the world have sought FCC input on how they could apply market-based mechanisms such as the Incentive Auction to clear spectrum".

24. Altogether, our final experiments took more than 60 CPU years to run. Writing this paper consumed perhaps 200–300 CPU years of compute in total, since we often found ourselves needing to rerun experiments as we uncovered bugs, changed parameters, or refined our experimental questions. We do not provide these CPU time numbers with the goal of impressing the reader with how much compute power we used. Instead, we hope (a) to give other researchers a sense of the scale of experiments that we consider necessary for answering questions like the ones we tackled here, and (b) to reassure the reader that we ran more or less the largest experiments that were practically feasible.

Appendix B: FCC Band Plan

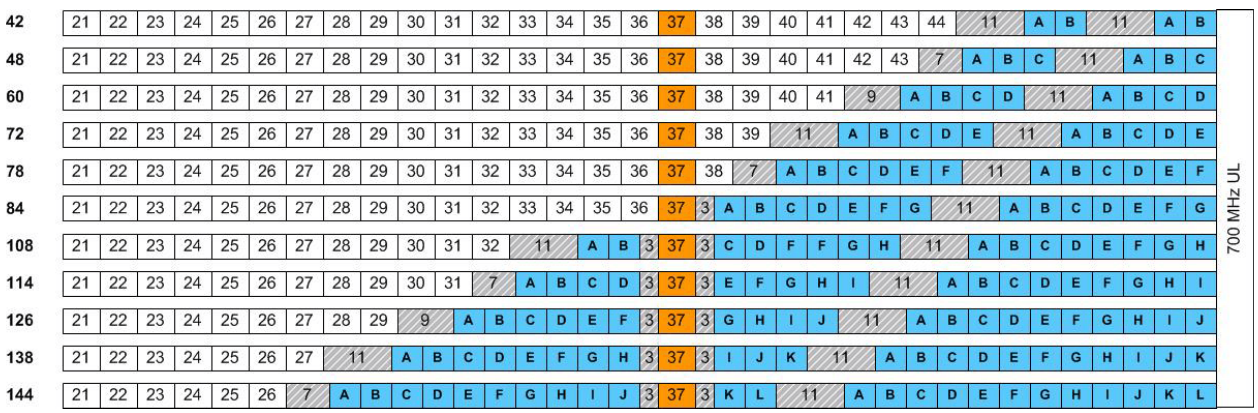


Figure B.11 **The FCC's band plan for 600 MHz. Each row corresponds to a different clearing target and shows which channels would be repurposed and which would remain for TV broadcasting.**

Appendix C: Reverse Auction Pseudocode

Algorithm 1 Multi-Stage UHF-Only Reverse Auction Without Impairment

Input: S_{exited} contains initial set of non-participating stations; we assume no impairments. S_{bidding} contains the set of participating stations. $\bar{c}$ is the initial clearing target. $\mathcal{C}$, $\mathcal{I}$, $\mathcal{D}$ are the set of channels, the interference constraints, and the station domains respectively. FC is the feasibility checker. cutoff is the amount of time given to FC. p_0 is the starting clock price.

$S_{\text{winners}} \leftarrow \{\}$ ▷ Set of provisionally winning stations, initially empty
$\mathcal{P}(s) \leftarrow 0\ \forall s \in S_{\text{bidding}}$ ▷ Provisionally winning prices; initially all 0
$\overline{\mathcal{C}} \leftarrow \{c \in \mathcal{C} \mid c < \bar{c}\}$ ▷ Channels available for repacking
$t \leftarrow 1$

while The incentive auction has not terminated **do**
  // Start a new stage
  $p_t \leftarrow p_0$ ▷ Reset clock prices
  while $|S_{\text{bidding}}| > 0$ **do**
    $p_t \leftarrow p_t - \max(0.05 \cdot p_t, 0.01 \cdot p_0)$ ▷ Decrease clock price

    // Check if any stations frozen in earlier stages have "caught up". Can be skipped in the first stage.
    for $\{s | s \in S_{\text{catchup}} \wedge p_t \cdot \text{SCORE}(s) \leq \mathcal{P}(s)\}$ **do**
      $S_{\text{catchup}} \leftarrow S_{\text{catchup}} \setminus \{s\}$
      if REPACKABLE($\{s\} \cup S_{\text{exited}}$) **then**
        $S_{\text{bidding}} \leftarrow S_{\text{bidding}} \cup \{s\}$
      else
        $S_{\text{winners}} \leftarrow S_{\text{winners}} \cup \{s\}$

    // Update prices
    for $s \in S_{\text{bidding}}$ **do**
      $P_{s;t} \leftarrow p_t \cdot \text{SCORE}(s)$

    // Bid processing
    Collect bid $b_{s;t}$ from each station in S_{bidding}
    for $s \in S_{\text{bidding}}$ **do**
      if Not REPACKABLE($\{s\} \cup S_{\text{exited}}$) **then** ▷ s freezes
        $S_{\text{winners}} \leftarrow S_{\text{winners}} \cup \{s\}$
        $S_{\text{bidding}} \leftarrow S_{\text{bidding}} \setminus \{s\}$
        $\mathcal{P}(s) \leftarrow P_{s;t}$
      else if $b_{s;t} = \text{Exit}$ **then** ▷ s can be repacked and bid to exit
        $S_{\text{bidding}} \leftarrow S_{\text{bidding}} \setminus \{s\}$
        $S_{\text{exited}} \leftarrow S_{\text{bidding}} \cup \{s\}$
    $t \leftarrow t + 1$

  Conduct a forward auction for the cleared spectrum $\{c \in \mathcal{C} \mid c \geq \bar{c}\}$
  Reverse auction cost $\leftarrow \sum_{s \in S_{\text{winners}}} \mathcal{P}(s)$
  if Revenue raised in forward auction $\geq$ Reverse auction cost **then**
    Terminate the incentive auction
  else
    $\bar{c} \leftarrow$ next clearing target ▷ Selected according to band plan
    $\overline{\mathcal{C}} \leftarrow \{c \in \mathcal{C} \mid c < \bar{c}\}$ ▷ Update channels available for repacking
    $S_{\text{catchup}} \leftarrow \{s \in S_{\text{winners}} \mid \text{REPACKABLE}(\{s\} \cup S_{\text{exited}}\})$ ▷ Find stations that may unfreeze
    $S_{\text{winners}} \leftarrow S_{\text{winners}} \setminus S_{\text{catchup}}$
// Incentive auction terminated
Pay $\mathcal{P}(s)$ to $s \in S_{\text{winners}}$

function REPACKABLE(S)
  Ask FC to find an assignment γ such that $\gamma(s) \in \mathcal{D}(s) \cap \overline{\mathcal{C}}\ \forall\ s \in S$, and $\gamma(s) = c \Rightarrow \gamma(s') \neq c'$ for all $\{(s,c),(s',c')\} \in \mathcal{I}$
  if A feasible assignment is found within cutoff seconds **then**
    Return True
  else
    Return False

Appendix D: VHF Option Pricing

In this appendix, we describe how VHF options are priced in the reverse auction.

D.1. An Ideal Case: Similar Stations

The easiest case to reason about is when there are three stations which serve identical populations: a UHF station, an HVHF station, and an LVHF station, with the property that none of these stations can co-exist on the same band, and that if any station moves to a lower band, its interference constraints in the new band are identical to the station it displaces. There are then four ways to clear the UHF channel, each of which create the same amount of value and hence should cost the same amount.

1. The UHF station can go to OFF for a price of $\$X$
2. The UHF station can go LVHF for a price of $\$Y$ (where $\$Y < \$X$) and the LVHF station can go to OFF at a price of $\$X - \$Y$
3. The UHF station can move to HVHF for a price of $\$Z$ (where $\$Z < \$Y$) and the HVHF station can go OFF at a price of $\$X - \$Z$
4. The UHF station can move to HVHF for a price of $\$Z$, the LVHF station can go to OFF for a price of $\$X - \$Y$, and the HVHF station can go to LVHF at a price of $\$Y - \$Z$.

D.2. Benchmark Prices

The reverse auction uses *benchmark prices* based on the above thought experiment: the auction first pretends that we are in the scenario of the above example where comparable stations exist and computes a benchmark price which it later transforms into an actual price as described below.

To compute the benchmark price for a station s in round t for the option of moving into b, the previous round's benchmark price is decremented by a **fraction** of d_t. This fraction is called a *reduction coefficient*, $r_{s;t;b}$. We will describe the computation of reduction coefficients shortly.

$$p_{s;t;b} = p_{s;t-1;b} - r_{s;t;b} \cdot d_t \tag{1}$$

The initial benchmark prices, denoted $p_{0,b}$, are chosen prior to the auction (with $p_{s;0;\mathrm{UHF}} = 0$). The benchmark price is then converted to an actual price as follows.

$$P_{s;t;b} = \mathrm{score}\,(s) \cdot \max\left\{0, \min\left\{p_{s;t;\mathrm{OFF}}, p_{s;t;b} - p_{s;t;v_{s,\mathrm{pre}(s)}}\right\}\right\} \tag{2}$$

Let us break down this formula: The prices are weighted by volume as before. The max ensures the price is non-negative and the min upper bounds the price by the price offered to a UHF station to go to OFF. Lastly, the second term in the min reflects the pricing division described in Section D.1.

D.3. Vacancy

Before describing the calculation of reduction coefficients, we need to introduce *vacancy*. Vacancy, denoted $V_{t,s,b}$, is a heuristic that estimates the competition that a station s faces for a spot in band b in round t, with higher values indicating less competition. More formally, vacancy is a volume weighted average of a function f over potential competitors for the vacant space in the band: Let $G(t,s,b)$ denote the set containing both s

and stations which have the possibility to interfere with s in b which are currently bidding on options below b. The function f is computed by taking the number of channels in b to which s can be feasibly assigned (given the current assignment of the exited stations) and normalizing by the number of channels in b. If s cannot be assigned to any channels, 0.5 is used in place of the numerator.

$$f(t,s,b) = \frac{\#\text{ of feasible channels for } s \text{ in } b \text{ in round } t}{\#\text{ of channels in } b} \tag{3}$$

$$V_{t,s,b} = \frac{\sum_{s' \in G(t,s,b)} \text{score}(s') \cdot f(t,s',b)}{\sum_{s' \in G(t,s,b)} \text{score}(s')} \tag{4}$$

D.4. Reduction Coefficients

We now provide equations for computing the reduction coefficients. The full decrement is always applied to going off-air, that is $r_{t,s,\text{OFF}} = 1$.

The reduction coefficient for moving to HVHF is:

$$r_{t,s,\text{HVHF}} = \frac{p_{0,\text{HVHF}} \cdot \sqrt{V_{t,s,\text{UHF}}}}{(p_{0,\text{OFF}} - p_{0,\text{HVHF}}) \cdot \sqrt{V_{t,s,\text{HVHF}}} + p_{0,\text{HVHF}} \cdot \sqrt{V_{t,s,\text{UHF}}}} \tag{5}$$

Finally, the reduction coefficient for moving to LVHF is:

$$r_{t,s,\text{LVHF}} = \left(\frac{(p_{0,\text{LVHF}} - p_{0,\text{HVHF}}) \cdot \sqrt{V_{t,s,\text{HVHF}}}}{(p_{0,\text{OFF}} - p_{0,\text{LVHF}}) \cdot \sqrt{V_{t,s,\text{LVHF}}} + (p_{0,\text{LVHF}} - p_{0,\text{HVHF}}) \cdot \sqrt{V_{t,s,\text{HVHF}}}} \right) \cdot (1 - r_{t,s,\text{HVHF}}) + r_{t,s,\text{HVHF}}$$

Appendix E: Robustness Experiments

Recall from Section 4.1 that we model a UHF station's value for switching to the HVHF band as $\frac{2}{3} \cdot v_{s,\text{UHF}} \cdot \mathcal{N}(1, 0.05)$ and similarly for the LVHF band with $\frac{1}{3}$ instead of $\frac{2}{3}$. These fractions were chosen for simplicity, drawing on some degree of domain knowledge about the values of VHF bidders. After performing our analysis, we wanted to understand how much our results depended on the specifics of our modelling choices. What if we had used other fractions instead? Here we describe the results of rerunning several experiments under three alternate sets of fractions. The FCC set opening off-air prices for HVHF and LVHF stations at $\frac{3}{5}$ and $\frac{1}{4}$ respectively of corresponding UHF stations. We reran experiments using $\frac{3}{5}$ and $\frac{1}{4}$ as the fractions in our value model. We refer to this parameterization as "Lower Values", as stations have lower values for the VHF bands when compared to our standard value model. For symmetry, we also reran experiments incrementing the fractions by corresponding amounts, leading to $\frac{11}{15}$ and $\frac{5}{12}$. We refer to this parameterization as "Higher Values". We also were interested in whether the Gaussian noise was consequential, so we created another parameterization where no noise was applied which we refer to as "No Gaussian Noise".

For each of our three value model parameterizations, we reran one experiment related to each question we posed in the introduction. Specifically, we compared: auctions that repacked the VHF band with those that did not ("Repacking VHF"); auctions that ran for four stages against those that simply ran for a single stage beginning at the fourth clearing target ("Clearing Procedure"); auctions that used the FCC's scoring rule against those that only used the population component ("Scoring Rules"); and lastly, auctions that used SATFC 2.3.1 as their feasibility checker against those that used the best off-the-shelf feasibility checker

Model Change	Value Loss Repacking VHF	Clearing Procedure	Scoring Rules	Feasibility Checker
None	0.92 (0.15)	0.95 (0.24)	0.94 (0.08)	1.44 (0.31)
Lower Values	0.96 (0.14)	0.95 (0.18)	0.97 (0.08)	1.51 (0.41)
Higher Values	0.87 (0.15)	0.91 (0.17)	0.93 (0.09)	1.53 (0.33)
No Gaussian Noise	0.9 (0.13)	0.94 (0.19)	0.95 (0.08)	1.48 (0.31)

Model Change	Cost Repacking VHF	Clearing Procedure	Scoring Rules	Feasibility Checker
None	0.78 (0.15)	0.85 (0.2)	0.96 (0.19)	1.44 (0.29)
Lower Values	0.86 (0.13)	0.89 (0.21)	0.98 (0.18)	1.48 (0.32)
Higher Values	0.74 (0.16)	0.81 (0.17)	0.93 (0.18)	1.55 (0.34)
No Gaussian Noise	0.79 (0.14)	0.85 (0.18)	0.95 (0.19)	1.49 (0.31)

Table 1 A summary of the results of changing the value model on four replicated experiments from the main paper. Each row corresponds to a different change to the value model, and each column corresponds to an experiment. The "None" row refers to the original experiment. Values in each cell represent the mean and standard deviation of value loss (upper table) and cost (lower table) of an altered auction design relative to the real auction design across all paired simulations.

("Feasibility Checker"). We ran 50 paired simulations for each experiment, just as we did for the corresponding main results. These experiments took roughly 20 CPU years to run.

The results are summarized in Table 1. We observed slightly different results among the three parameterizations: for example, the cost savings estimate from repacking VHF bands varied from 14–26% depending on how substitutable stations felt the VHF bands were for the UHF band. However, no change to the value model substantially altered the conclusions we drew from any experiment, giving us confidence that our results are relatively robust to the particular choices we made.

Appendix F: Additional Details of the BD Value Model

F.1. Inferring Bounds on Values from Bids

The FCC's data contains the selected band and price (and fallback band and price when applicable) for each bid that was processed. It additionally contains the offers and set of bands for which each station s accepted opening prices, which we call $\text{PermissibleStartBands}_s$.[25]

We began with the trivial bounds $0 \leq v_{s,\text{UHF}} \leq \infty$. We tightened bounds by applying the following rules[26] to the released bids:

1. $\text{OFF} \in \text{PermissibleStartBands}_s \implies v_{s,\text{UHF}} \leq P_{s;\text{OFF};\text{Open}}$
2. $\text{OFF} \notin \text{PermissibleStartBands}_s$[27] $\implies v_{s,\text{UHF}} \geq P_{s;\text{OFF};\text{Open}}$
3. $s \in S_{\text{winners}} \land \text{post}(s) = \text{OFF} \implies v_{s,\text{UHF}} \leq \mathcal{P}(s)$ [28]
4. $b_{s,t} = \text{OFF} \lor \text{fallback}_{s,t} = \text{OFF} \implies v_{s,\text{UHF}} \leq P_{s;\text{OFF};t}$
5. $(b_{s,t} = \text{Exit} \lor \text{fallback}_{s,t} = \text{Exit}) \land \gamma_t(s) = \text{OFF} \implies v_{s,\text{UHF}} \geq P_{s;\text{OFF};t}$

In words, we inferred that a station that included (did not include) starting off-air as a permissible option had a value less than (greater than) its opening price for starting off-air. A station that froze while bidding for off-air and became a winner had a home band value less than its compensation. Whenever a station bid to remain off-air, including as a fallback bid when attempting to move between bands, we inferred that the station's value was less than its price for remaining off-air. Similarly, whenever a station bid to drop out of the auction (including as a fallback bid) while off-air, we inferred that the station's value was greater than its price for remaining off-air.

F.2. Fitting our Value Distribution

Let x_s and y_s represent lower and upper bounds respectively on s's observed \$/pop sample n_s, i.e., $x_s = \frac{v_{s,\text{UHF,Lower Bound}}}{\text{Population}(s)}, y_s = \frac{v_{s,\text{UHF,Upper Bound}}}{\text{Population}(s)}$. Let Z be a list containing all of the x_s and y_s in ascending order, such that Z_1 is the smallest element and $Z_{2|S|}$ the largest. Our problem was then as follows.

$$\text{maximize} \prod_{s\in S} \big(N(y_s) - N(x_s)\big) \tag{6}$$

$$\text{subject to } N(x_s), N(y_s) \in [0,1]\, \forall s \in S \tag{7}$$

$$N(Z_1) \le N(Z_2) \le \cdots \le N(Z_{2|S|}) \tag{8}$$

Noting that taking the log of the objective function preserves its maximum, we minimized the negative log likelihood, $-\sum_{s\in S} \log\left(N(y_s) - N(x_s)\right)$. This is a constrained optimization problem with a nonlinear convex objective function and convex constraints. The constraints ensure that the values of N must fall between 0 and 1 and that N must be non-decreasing. The solution uniquely identifies the value of N at each point x_s and y_s. We translated this problem into a second-order cone program using `cvxpy` (Diamond and Boyd 2016) and solved it using the `ECOS` (Domahidi et al. 2013) solver.

We then fit a GPD to each of our tails. Specifically, we labelled the bottom 15% segment of the CDF the left tail and top 30% segment the right tail. The CDF of the GPD is $F(x) = 1 - (1 + \frac{x-\mu}{\sigma} \cdot \xi)^{\frac{-1}{\xi}}$ for $\xi \neq 0$, where μ, σ, ξ are the location, scale, and shape parameters respectively. We fit the left tail to the set of points Z falling in the region described (about 7% of our data). We did not have any data to fit the right tail of the distribution, so we fit a GPD using control points such that a \$/pop value 12.5% higher than any we observed occurred at $y = 0.975$ and one 25% higher than any we ever observed occurred at $y = 0.99$.

While we recognize that the value distribution's right tail is the most *ad hoc* part of our model, we believe that the choices we made about this part of the distribution are unlikely to have substantially impacted our results. The reason is that stations with sufficiently high values are unlikely to choose to participate even at the incentive auction's opening prices, at which point it does not matter to the auction's outcome exactly how high those values are. Indeed, we calculated that only 3.4% of UHF stations nationally would have *any* chance of participating in the auction conditional on their values having been sampled from any part of the right tail. Since each station has a 30% chance of having its value sampled from the right tail in any given simulation, in expectation any change to this section of the curve would alter the bidding behaviour of fewer than 1% of UHF stations. Furthermore, it is likely that conditional on a station's value having been drawn from the right tail and the station having chosen to participate, these stations would exit early in the auction before interference constraints start to bind tightly, and that this behaviour would not depend on the exact shape of the right-tail distribution.

Appendix G: Additional Simulation Details

G.1. Computational Environment

Our experiments were performed on two different compute clusters (the same cluster was used consistently for each experiment). Our first cluster ran on AWS, using c5.18xlarge nodes. Each node had 72 vCPUs (each a hyperthread on a 3.0 GHz Intel Xeon Platinum 8000-series processor) and 144 GB of RAM. Our second cluster consisted of nodes equipped with 32 2.10GHz Intel Xeon E5-2683 v4 CPUs with 40960 KB cache and 96 GB RAM. SATFC runs a portfolio of 8 algorithms in parallel; jobs were always scheduled such that each simulation had access to 8 CPUs and 20 GB of RAM.

G.2. Handling Missing Data in the MCS Value Model

The MCS model does not provide values for stations in Hawaii, Puerto Rico, or the Virgin Islands. We therefore excluded all of these "non-mainland" stations from our analysis completely when using both value models to preserve the same interference graph across simulations. We note that there are few such stations and they reach relatively small populations, so this exclusion is unlikely to have impacted our qualitative findings.

The MCS model also does not provide values for 25 mainland UHF stations. For these stations, we set $v_{s,\text{UHF}}$ to be proportional to population. We sampled the constant of proportionality from a log-normal distribution, where the mean and variance were calculated from samples of $\frac{v_{s,\text{UHF}}}{\text{Population}(s)}$ from other stations in that station's Designated Market Area (DMA)[29] or nationally if there were no other stations in the DMA.

G.3. Additional Value Model Details

The Gaussian noise in our value model can occasionally cause a station to violate $v_{s,\text{UHF}} > v_{s,\text{HVHF}} > v_{s,\text{LVHF}}$. In these rare cases, we resample. We rounded all values to the nearest thousand dollars. In rare cases when the value model provides an extremely low sample for $v_{s,\text{UHF}}$, we set $v_{s,\text{UHF}} = \$3000$.

G.4. Initial Band Assignment

In the real auction, stations could choose to participate conditional on being initially assigned to one of a subset of bands. One role of the initial clearing target optimization was to accommodate such stations. Since we could not replicate this optimization, we did not allow stations to select their preferred starting bands. Instead, in our simulations, all participating stations start off-air. In the real incentive auction, 85% of stations indicated off-air as their preferred starting band and only 5% of stations declined off-air as a permissible starting band.

G.5. Canadian Stations

Canadian stations were also involved in the repacking process (they could be moved to a new channel, but did not participate in the auction and could not be purchased). We included all 793 Canadian stations in our simulations.

G.6. Station Volumes

We used population and interference values from the FCC's opening prices document (FCC 2015d), noting that the links heuristic is calculated on the full interference graph. We continue to use these values in our UHF-only auctions, even though the underlying interference graph changes (the number of links goes down). We observe that the interference graph somewhat similarly changed between each stage of the incentive auction (the values correspond to the full graph, or the final stage of the band plan) but nevertheless the same values for the links heuristic were used throughout the auction.

Appendix H: Impairing Stations

In this appendix, we describe how we choose which set of stations will be impairing given that we cannot replicate the initial clearing target optimization.

Given a clearing target, our simulator determines which stations to impair as follows. First, we ensure full participation of UHF stations by increasing the FCC's base clock price in 5% increments until no UHF stations reject their initial offers. We then solve an optimization problem to find an initial assignment γ that minimizes the number of essential impairing stations, breaking ties by minimizing the aggregate population of impairing stations. Having dealt with the essential impairments, we then run the auction starting from the inflated base clock price. While the base clock price remains higher than its normal starting point, the VHF band is considered "locked"—only UHF stations are asked to bid and stations cannot bid to move into the VHF band. When the FCC's starting base clock price is reached, any station that would not have participated at the opening prices will have exited or be frozen. We consider any stations that have frozen at this point to be impairing; we do not include impairing stations in any of our metrics unless explicitly noted. At this point, VHF stations make their participation decisions, the VHF band is "unlocked", and the auction proceeds as normal.

In a multi-stage auction, impairing stations may be able to leverage the newly available spectrum to become non-impairing. At the start of each new stage, we again solve an optimization procedure to move as many essential impairments to $\mathcal{C}$ as possible. We then proceed with the between-rounds transition as described in Section 3.2.4, except that p_t can be reset back to the impairment regime, following the same rules as above until the FCC's base clock price is reached. Lastly, we note that there are no essential impairments given a clearing target of 84 MHz and that there are 9 at a clearing target of 128 MHz.

Appendix I: Full Experimental Results

In this appendix, we provide figures showing the results of our simulations in each of our experiments. Results are shown for both value models and for both auctions that repack and do not repack the VHF band as available.

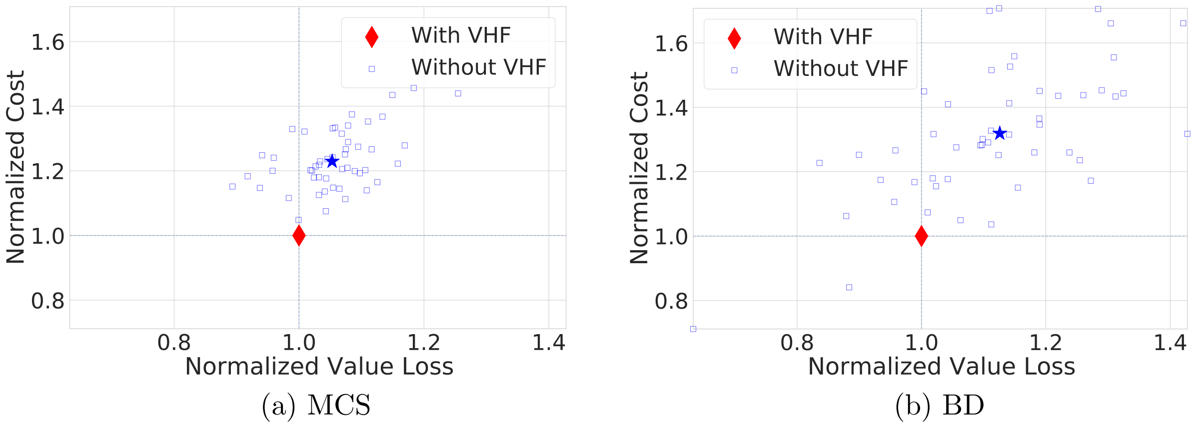


Figure I.12 Comparing auctions that only repack the UHF band against auctions that also repack VHF bands.

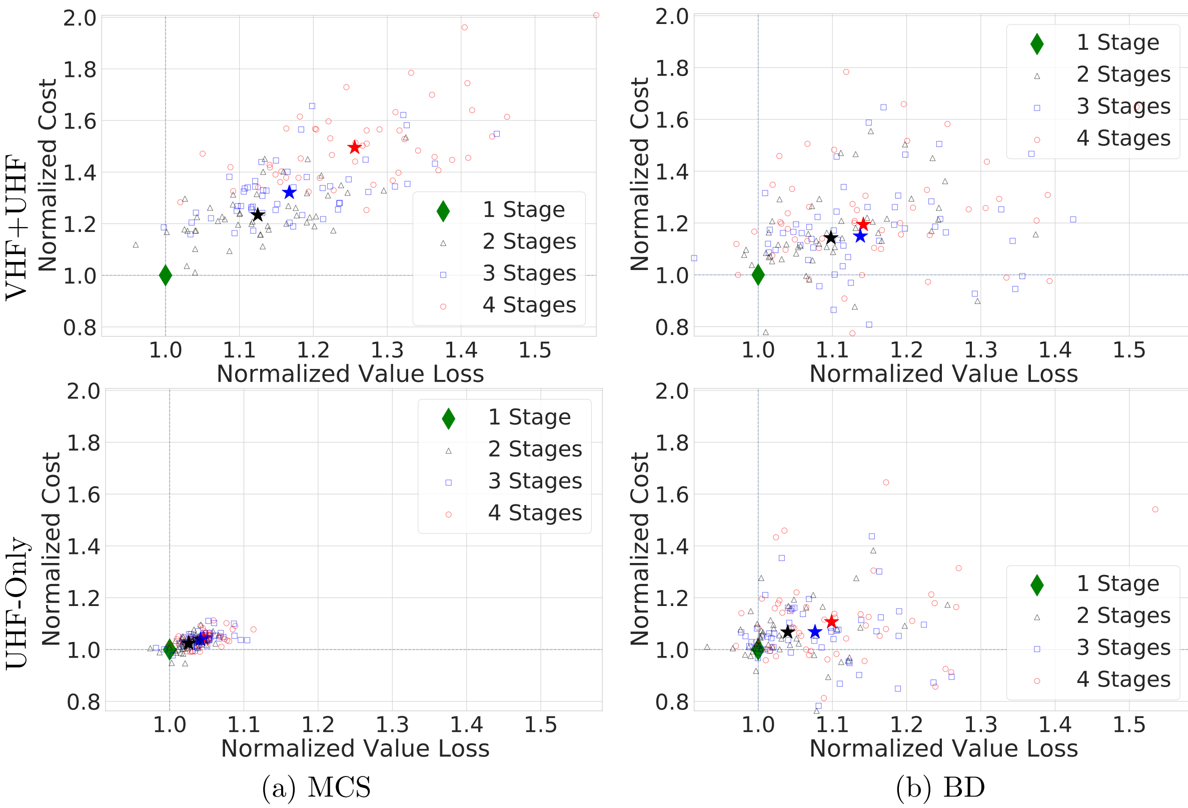
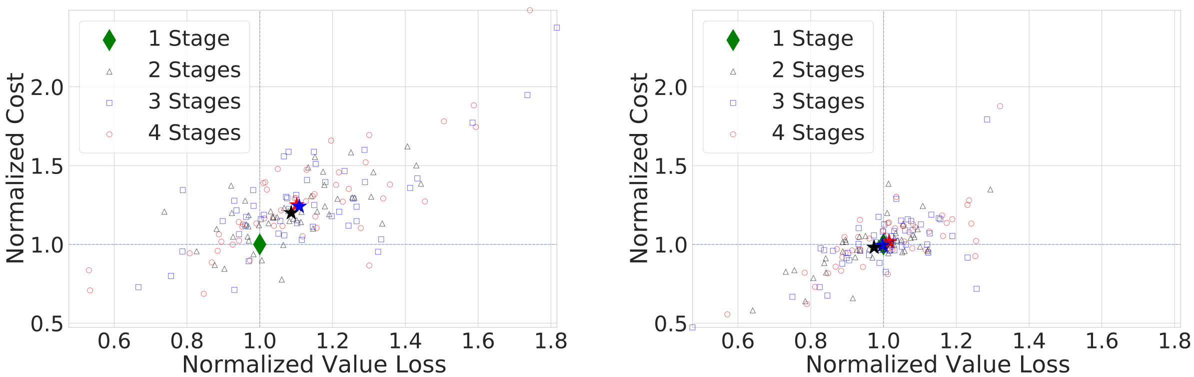
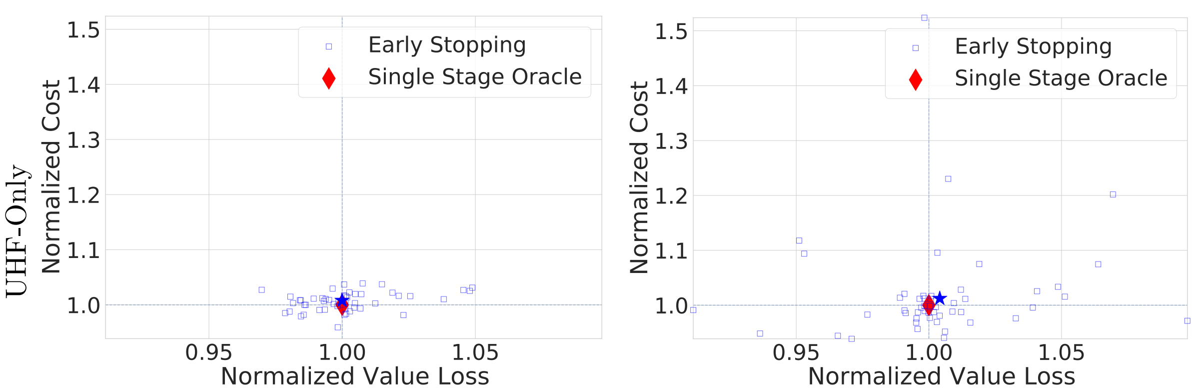


Figure I.13 Comparing auctions running through 1-4 stages, ultimately ending on the same clearing target, with no impairing stations.



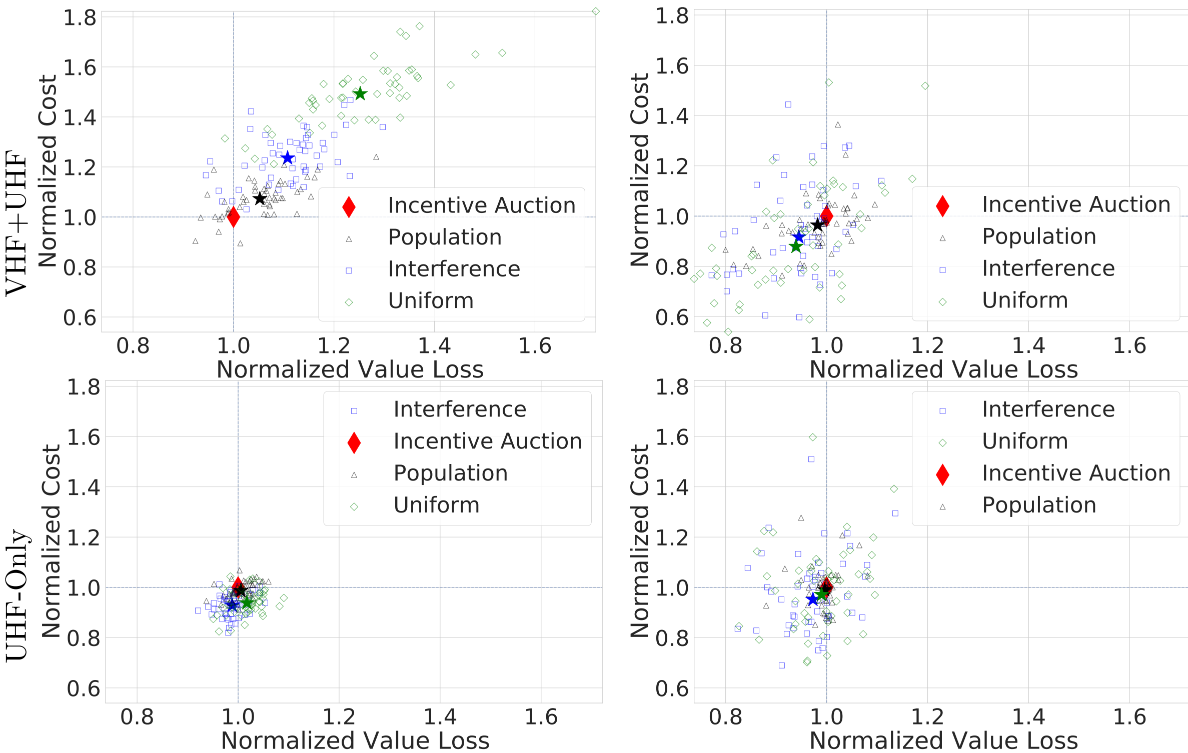
(a) VHF+UHF (b) UHF-Only

Figure I.14 Comparing auctions running through 1-4 stages, ultimately ending on the same clearing target, factoring in impairing stations. The BD value model is used for all simulations.



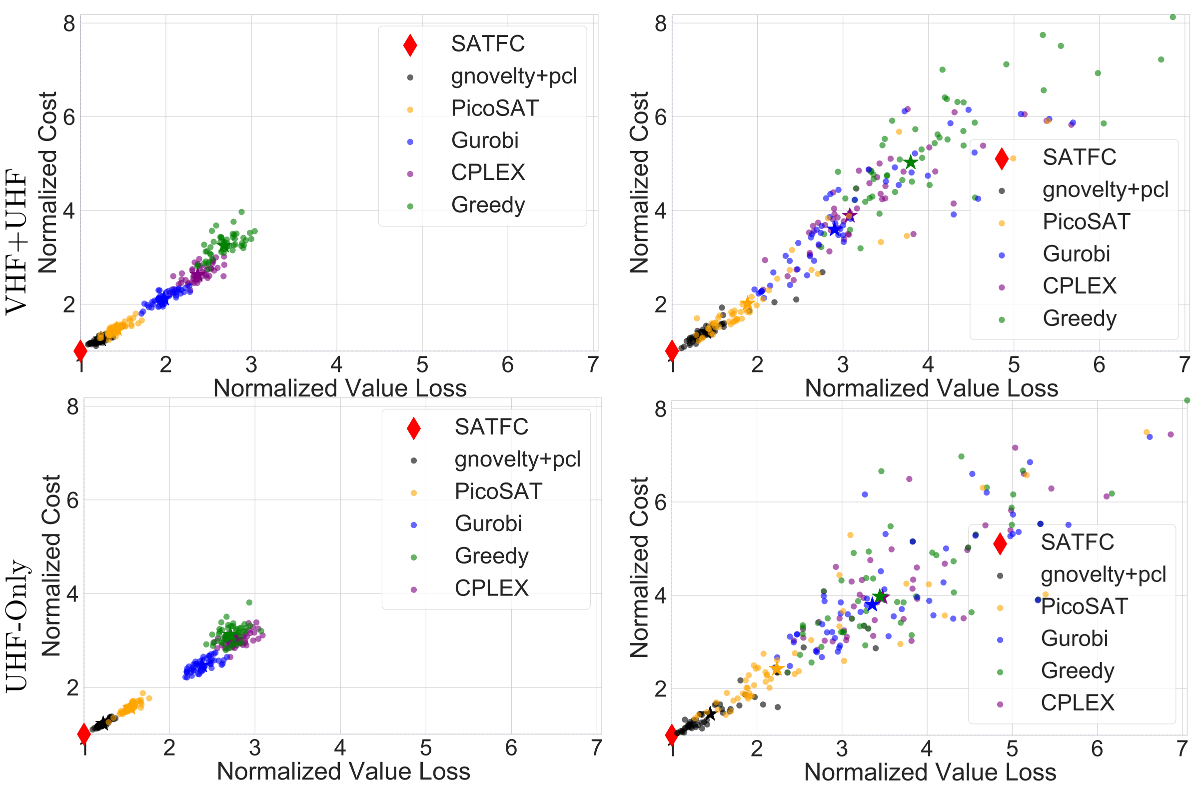
(a) MCS (b) BD

Figure I.15 Comparing auctions using the early stopping algorithm against single-stage auctions ending on the same clearing target as their corresponding early stopping auction.



(a) MCS (b) BD

Figure I.16 Comparing auctions using four different scoring rules.



(a) MCS (b) BD

Figure I.17 Comparing auctions using different feasibility checkers.

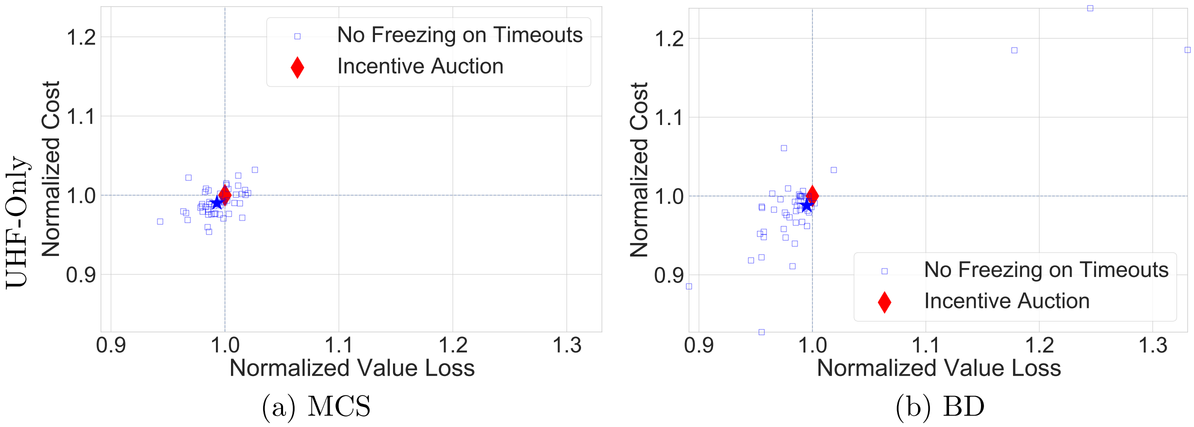


Figure I.18 Comparing the standard bid processing algorithm with one that does not freeze stations with indeterminate feasibility checks.

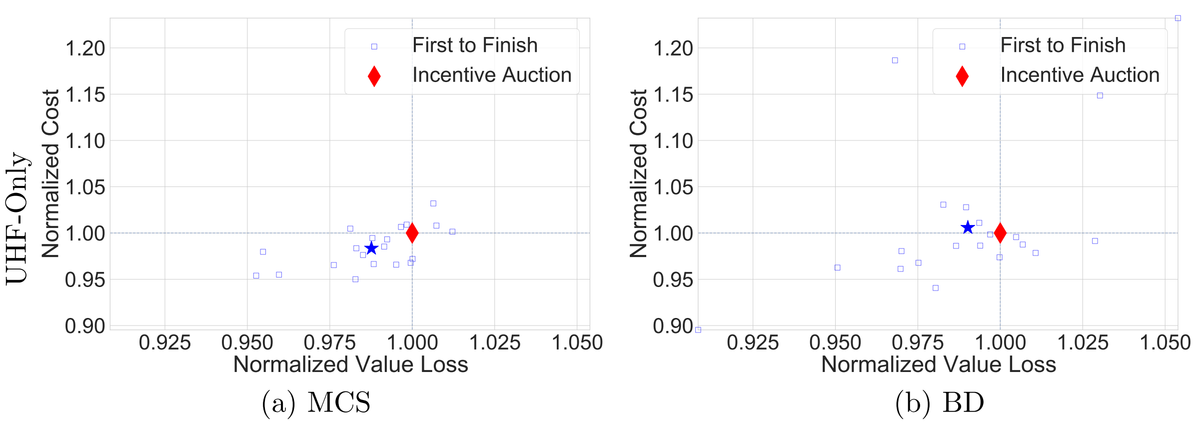


Figure I.19 Comparing the first to finish algorithm against the standard bid processing algorithm for single-stage 126 MHz auctions.

Appendix J: First-to-Finish Algorithm Pseudocode

Algorithm 2 First-to-Finish Bid Processing

```
Unprocessed ← S_bidding
while Time remains in the round and |Unprocessed| > 0 do
    Launch a parallel feasibility check for each s ∈ Unprocessed, with a cutoff equal to time remaining in the round
    repeat
        Wait for a check to complete
        if s is not frozen then
            Process s's bid and remove s from Unprocessed
    until A station exits or moves bands
    Interrupt all ongoing checks
```

Appendix K: Station Repacking Problem Runtimes

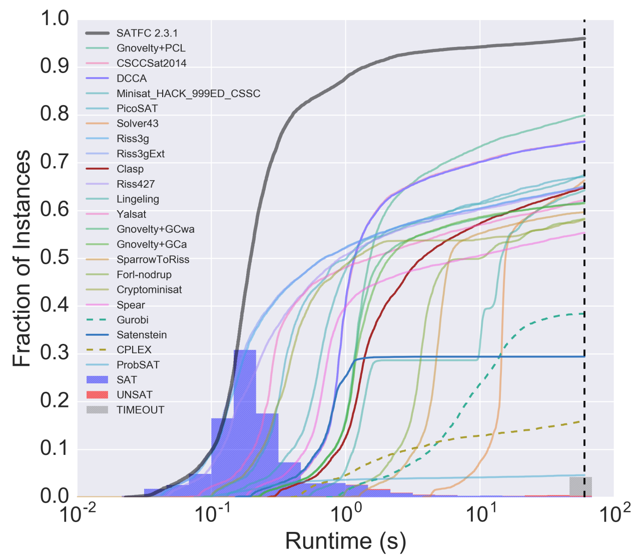


Figure K.20 Empirical Cumulative Density Function of runtimes for default configurations of MIP and SAT solvers and for SATFC 2.3.1 on a benchmark set of $10\,000$ non-trivial problems. Figure taken from (Newman et al. 2017). The curves show fraction of instances solved (y axis) within different amounts of time (x axis; note the log scale). The legend is ordered by percentage of problems solved before the cutoff. The histogram indicates density of SAT and UNSAT instances binned by their (fastest) runtimes; unsatisfiable instances constituted fewer than 1% of solved instances.

Appendix L: Single-stage vs Multi-stage Auctions Examples

In this appendix, we give an example of how running a single stage auction can lead to a better outcome than iterating through multiple stages (for a fixed final clearing target).

Example L.1 *We return to the example in Figure 2 and ask what would have happened if the auction had initially set* $\bar{c} = 3$ *and tried to repack stations into two channels from the outset. We visualize this scenario in Figure L.21. As before, A is the first to exit. However, B does not freeze this time around, because A and B are both repackable using two channels. When* $P_t < V_B$*, B will exit. At this point C freezes at* $\mathcal{P}(C) = V_B$*. D* *is the next to exit, followed by E, finally freezing F at $\mathcal{P}(F) = V_E$. Recall that the value loss and cost of the two stage auction were $V_B + V_F$ and $V_A + V_E$ respectively. In the single-stage alternative, the value loss is $V_C + V_F$ and the cost is $V_B + V_E$: the outcome is preferable according to both of our metrics!*

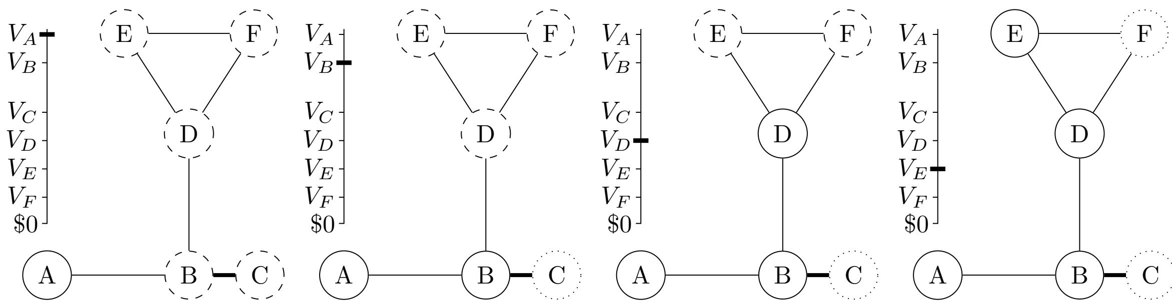


Figure L.21 **Returning to the example of Figure 2, but now repacking two channels right from the outset.**

Next, we provide an example showing that this is not always the case. Here, we show an example where a single stage auction leads to a worse outcome than a multi-stage auction.

Example L.2 *Consider a setting with four identically scored stations A, B, C, D with $V_A > V_B > V_C > V_D$, $V_B < V_C + V_D$ and $V_A < 2V_B$. In the final stage, the feasible sets are $\{A, B\}, \{A, C, D\}$ and all subsets of these sets. As prices drop, A exits first, followed by B, which freezes C and D at prices of V_B. So the single-stage approach gives a value loss of $V_C + V_D$ and a cost of $2V_B$*

In the multi-stage setting, in the first stage assume the feasible sets are $\{B\}, \{A, C, D\}$ and all subsets of these sets. A will exit first, freezing B at a price of V_A. B and C then exit, concluding the stage. In the second stage, B never unfreezes. Therefore, the total cost of the multi-stage approach is V_A and the value loss is V_B. By the inequalities assumed above, this is a cheaper, more efficient outcome than the single-stage auction that predetermined the amount of spectrum to clear.

Appendix M: Early Stopping Counterexample

In this appendix, we provide an example where early stopping leads to a worse outcome than the standard clearing procedure.

Example M.1 *Consider four identically scored stations A, B, C, D with $V_A > V_B > V_C > V_D$, $V_B < V_C + V_D$ and $V_A < 2V_B$. Let the forward auction run first and have a purchasing price of V_A. Let the feasible sets in the first stage be $\{A, C\}, \{A, D\}, \{B\}$ and all subsets of these sets. In the second stage, the feasible sets add $\{A, B\}, \{A, C, D\}$ and all subsets. In both cases, the auction begins with A exiting and B freezing at price V_A. In an early stopping auction, this will trigger the end of stage one. In stage two, B unfreezes and exits. Then C and D freeze at price V_B. This leads to a value loss of $V_C + V_D$ and a cost of $2V_B$. In an auction without early stopping, C would exit, freezing D at price V_C. This would trigger the stage to end. In the next stage, B would remain frozen and D would unfreeze and exit, leading to a value loss of V_B and a cost of V_A. Using the inequalities on the values above, the auction that does not use early stopping performs better in both metrics.*

Appendix N: Modeling Forward Auction Revenue

When using the early stopping algorithm, the reverse auction takes as input the output of the previous forward auction (the amount that mobile carriers will pay for the spectrum). Therefore, in order to simulate early stopping auctions, we need to model forward auction revenues.

While we have access to the real forward auction revenues, the incentive auction only went through four stages, and simulations could potentially go to stages beyond the fourth. To address this issue, we performed a log-log fit on the number of mobile licenses and forward auction proceeds in the first three stages (i.e., with 3 data points we fit $ln(\#\text{licenses}) = a \cdot ln(cost) + b$ for some constants a, b). We ignored the revenue in the fourth stage when performing this fit because the price per license rose significantly relative to the other stages, and we suspect that the price increase was likely due to an understanding among bidders that the auction would terminate in this stage. After the fit was established, we scaled the entire model by a constant so that the model's prediction for the fourth stage matched the observed real revenue. The results appear in Figure N.22.

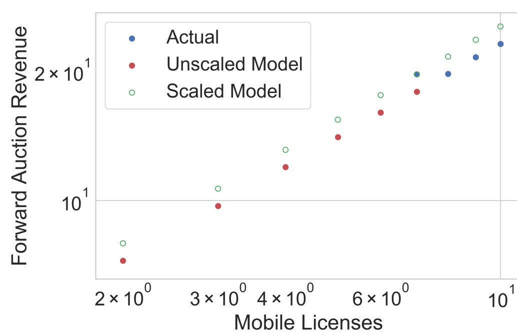


Figure N.22 Forward auction revenues observed in the incentive auction ("Actual") and our model of forward auction revenue.

We used the same forward auction revenues across all sampled station value profiles for a given value model. When using the MCS model, we observed that reverse auction simulations rarely produced high enough procurement costs to trigger early stopping in the first stage, leading to uninformative behavior in which the auction simply ends after the first stage. To get around this, we scaled our forward auction model downwards by a factor of 2 when running simulations for the MCS.

Appendix O: Feasibility Checker Counterexample

In this appendix, we provide an example showing that a better feasibility checker does not necessarily lead to better outcomes, and can even lead to worse ones.

For a fixed cutoff, a feasibility checker can be thought of as a mapping from a set of stations to $\{Feasible, Infeasible, Unknown\}$. We can define an ordering over feasibility checkers such that a feasibility checker F_1 is strictly better than a second F_2 if and only if for all possible sets of stations $s \in 2^S$, $F_2(s) = Feasible \implies F_1(s) = Feasible$ and $\exists s$ such that $F_1(s) = Feasible$ and $F_2(s) = Unknown$. Importantly note that for the purposes of this definition we don't care about the feasibility checker's ability to prove infeasibility: while this is important for saving time, it does not ultimately impact the result of the auction since infeasibility and indeterminate solutions are treated identically. We now proceed to the example.

Example O.1 *Imagine a UHF-only setting involving four bidding stations* A*,* B*,* C*, and* D*. Let* $V_A = V_B = V_C = V_D = V$ *and let all stations have the same score. The constraints are such that the repackable sets are either* $\{B,C,D\}$ *or* $\{A,D\}$ *(and all subsets). There are two feasibility checkers:* F_1 *can find all feasible repackings, but* $F_2(\{A,D\}) =$ *Unknown. Consider the first round in which each station is being offered a price* p *just below* V *and assume that the bid processing order is* D,A,B,C*.* D *exits the auction. Under* F_1*,* A *is allowed to exit the auction. This freezes* B *and* C*. The value loss for these two stations will be* $2V$ *and the payment will be just under* $2V$*.* F_2*, however, cannot pack* A*, and so* A *freezes.* B *and* C *then exit the auction. The value loss in this scenario is* V *and the payment is just under* V*, so this outcome is strictly better than the previous even though* F_1 *is strictly better than* F_2*.*