Intelligent Systems (AI-2)

Computer Science cpsc422, Lecture 3

Jan, 15 2019

Lecture Overview

Markov Decision Processes

- Formal Specification and example
- Policies and Optimal Policy
- Intro to Value Iteration

Decision Processes

Often an agent needs to go beyond a fixed set of decisions – Examples?

Infinite horizon problems: process does not stop

Robot surviving on planet, Monitoring Nuc. Plant, . . .

Indefinite horizon problem: the agent does not know when the process may stop

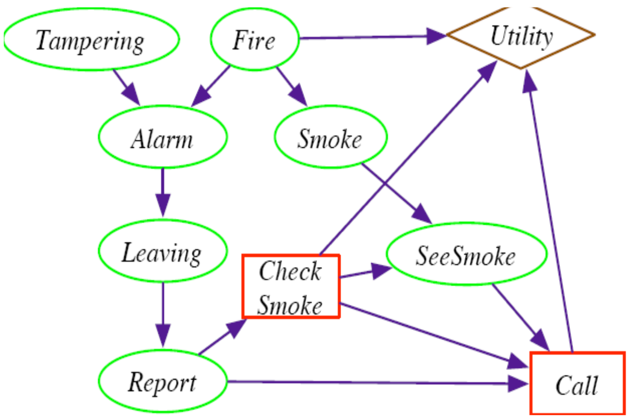
reaching location

Finite horizon: the process must end at a give time N

in N steps

Goal: represent (and optimize) an indefinite sequence of decisions

- Decision networks



Represent (and optimize) a fixed number of decisions

- Markov models

The network can extend indefinitely

How can we deal with indefinite/infinite Decision processes?

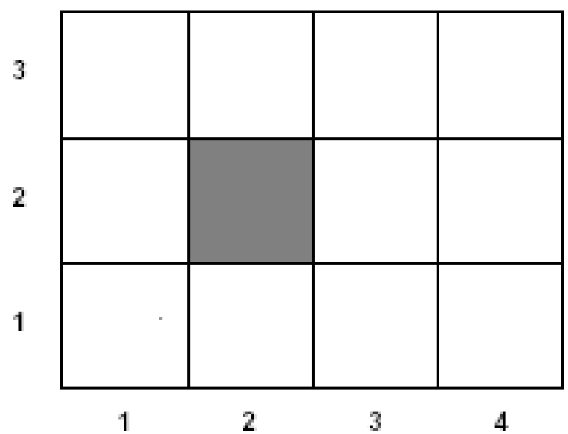
Like in a Markov Chain one random variable S_t for each time slice represents the state at time t.

And A_t be the action/decision at time t

S_0 S_1 S_2 . . . .

A_0 A_1 A_2 . . . . .

Example



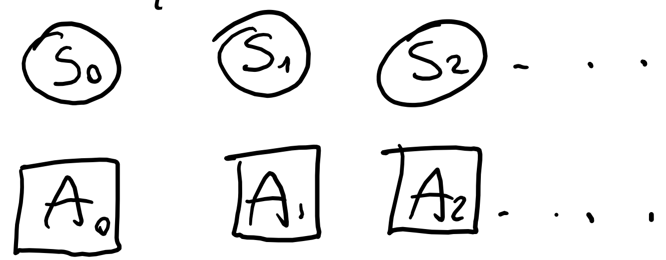
Agent moves in the above grid :

- States are the cells of the grid
- Actions *Up, Down, Left, Right*

How can we deal with indefinite/infinite Decision processes?

Like in a Markov Chain one random variable S_t for each time slice represents the state at time t.

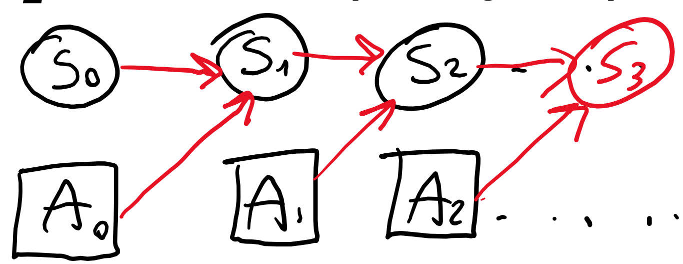
And A_t be the action/decision at time t



Make the same two assumptions we made for Markov Chains

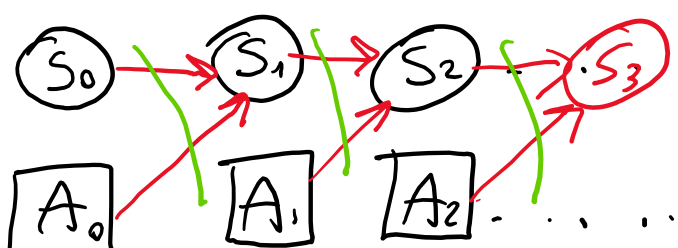
(a) The action outcome (the state S_{t+1} at time $t+1$) only depends on S_t and A_t

$$P(S_{t+1} \mid S_t, A_t, S_{t-1}, A_{t-1}, \dots) = P(S_{t+1} \mid S_t, A_t)$$



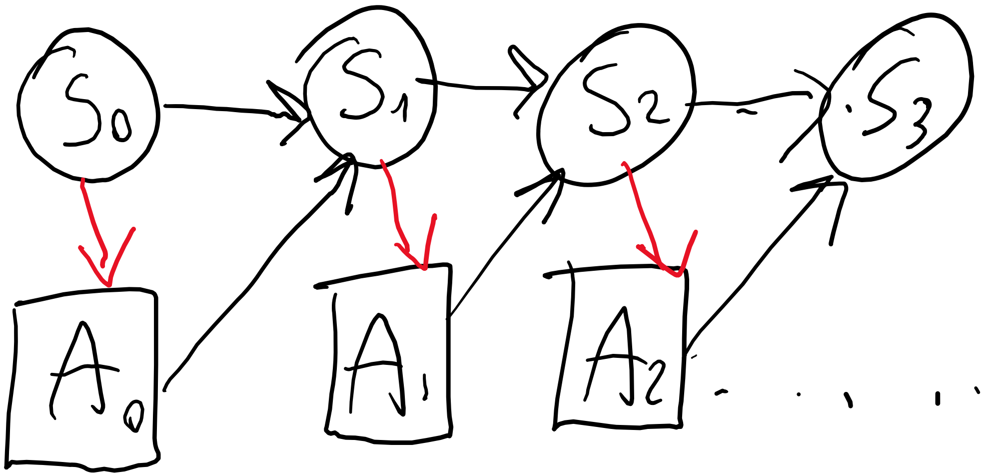
(b) The process is *stationary…*

$P(S_{t+1} \mid S_t, A_t)$ the same for all t

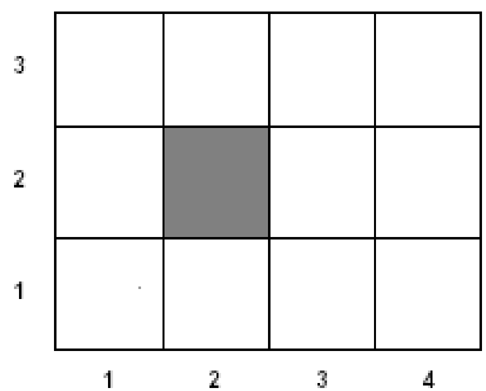


same conditional probabilities

Agent knows in which state it is at time **t** when it selects the action at time **t**

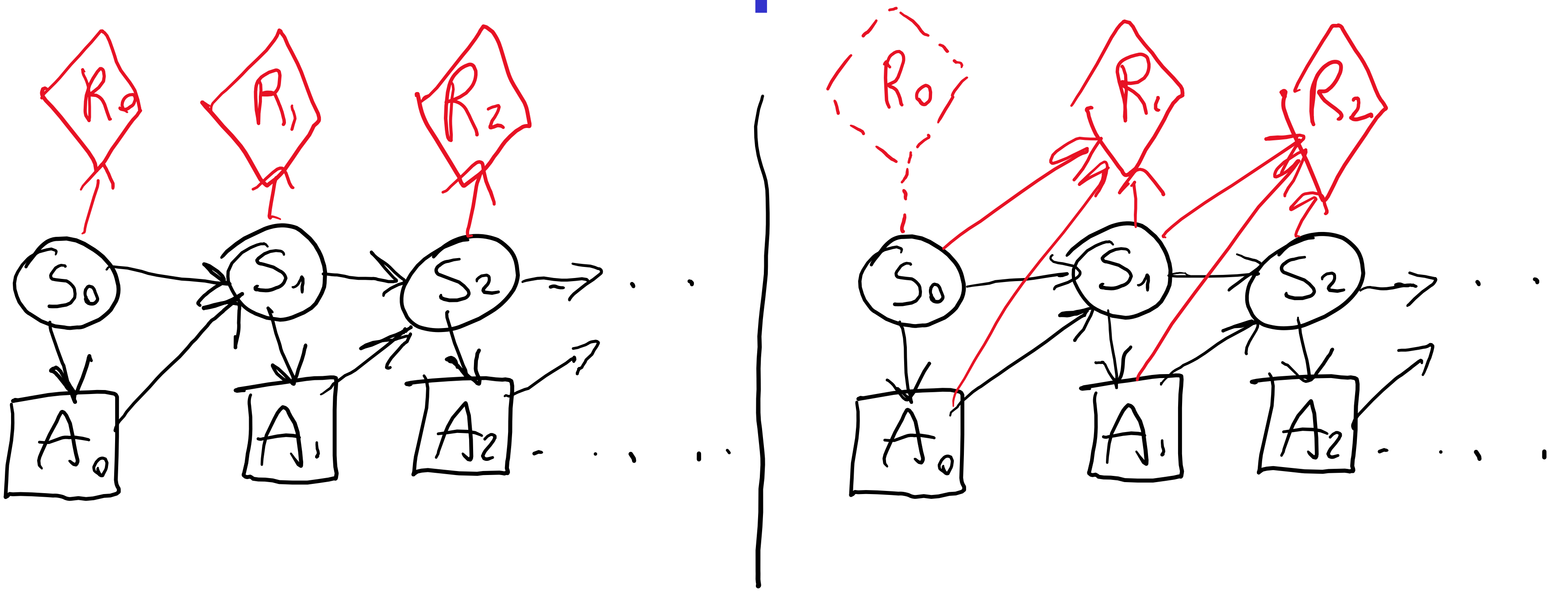


The robot always knows in which cell it is…



So what will be a policy ?

How can we deal with indefinite/infinite Decision processes?

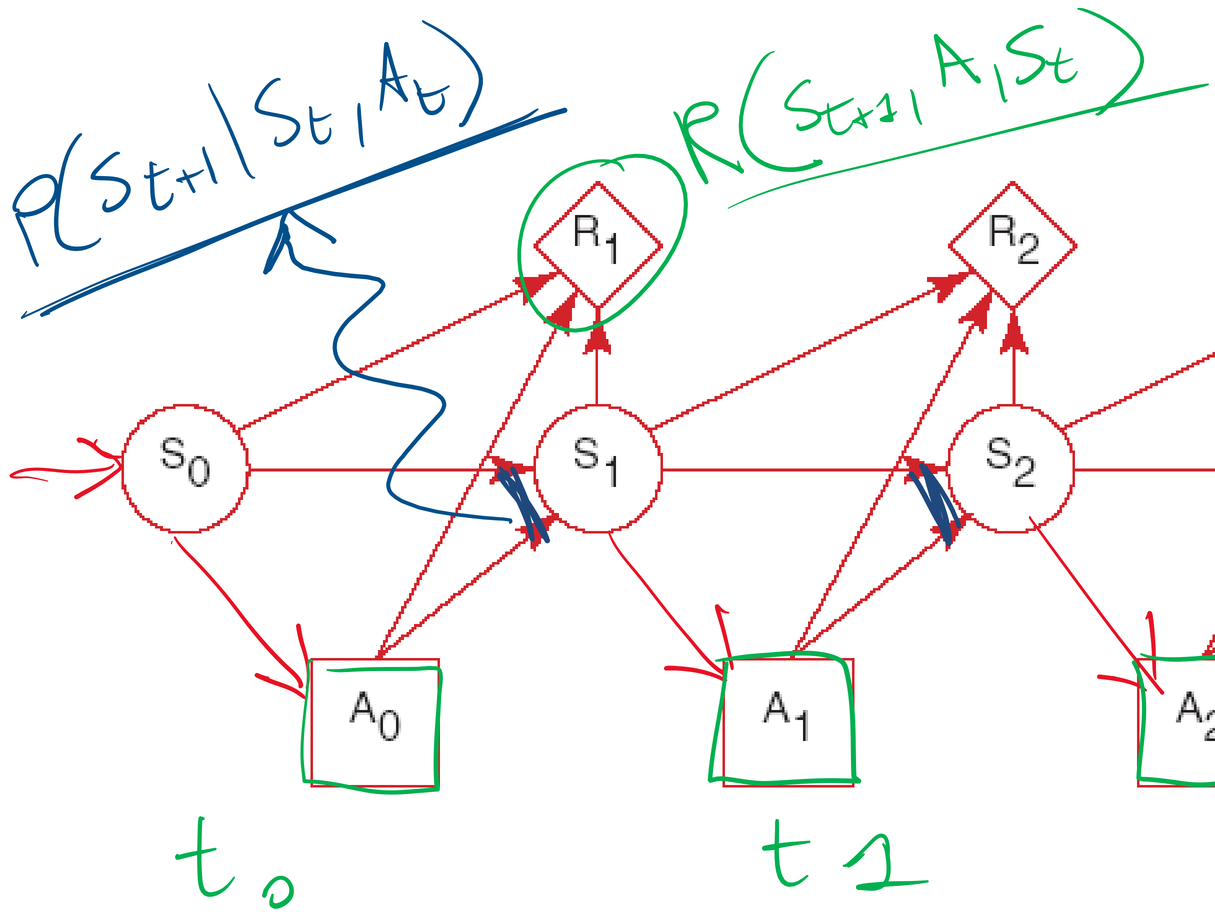


We also need a more **flexible specification for the utility**. How?

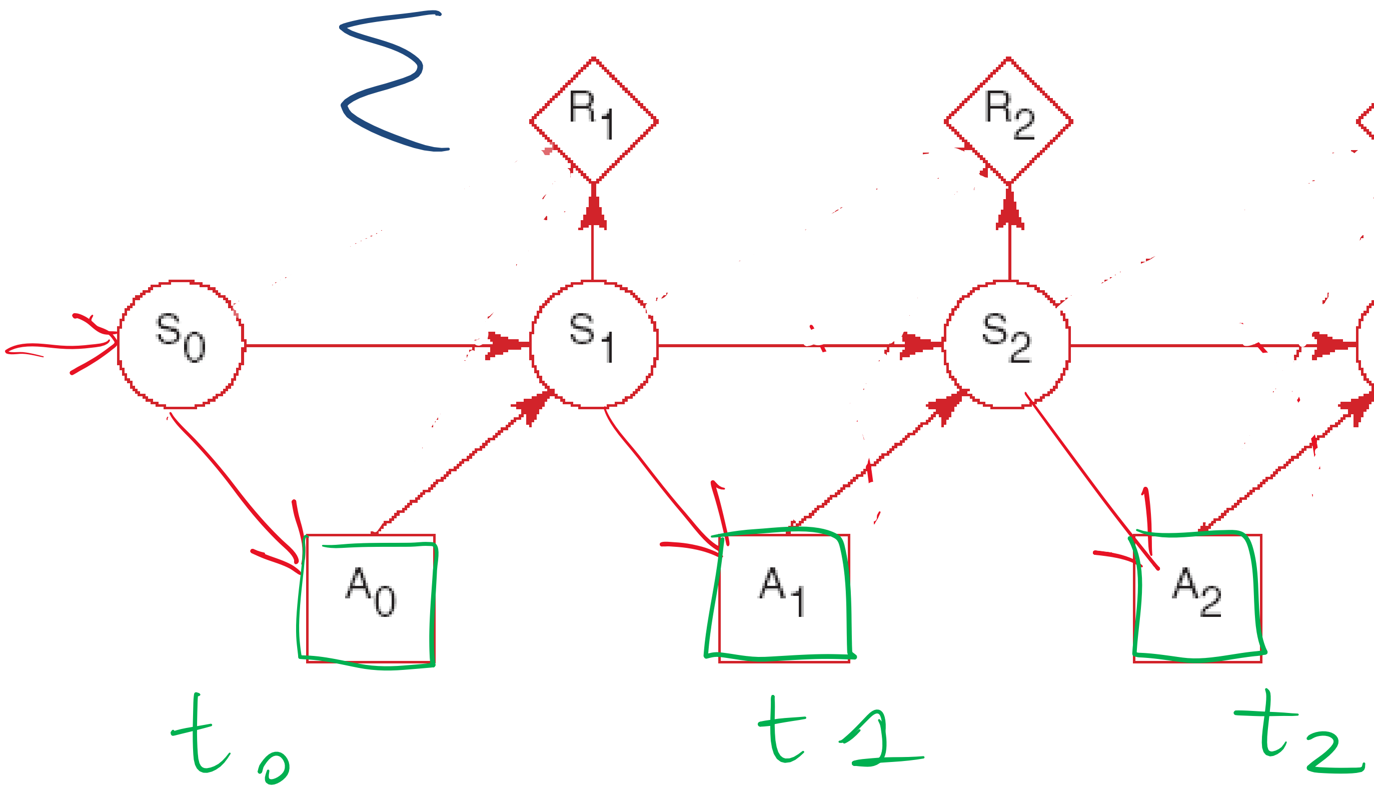
- Defined based on a reward/punishment that the agent receives in each time slice
- Typically summing them up

MDP graphical specification

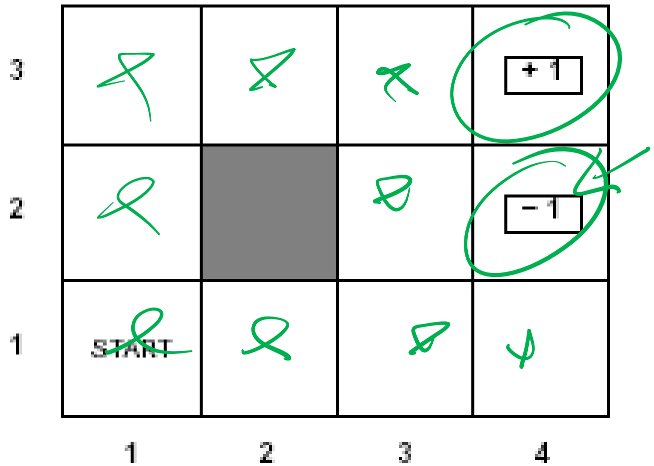
Basically a MDP is a Markov Chain augmented with actions and rewards/values



When Rewards only depend on the state



Example MDP: Rewards

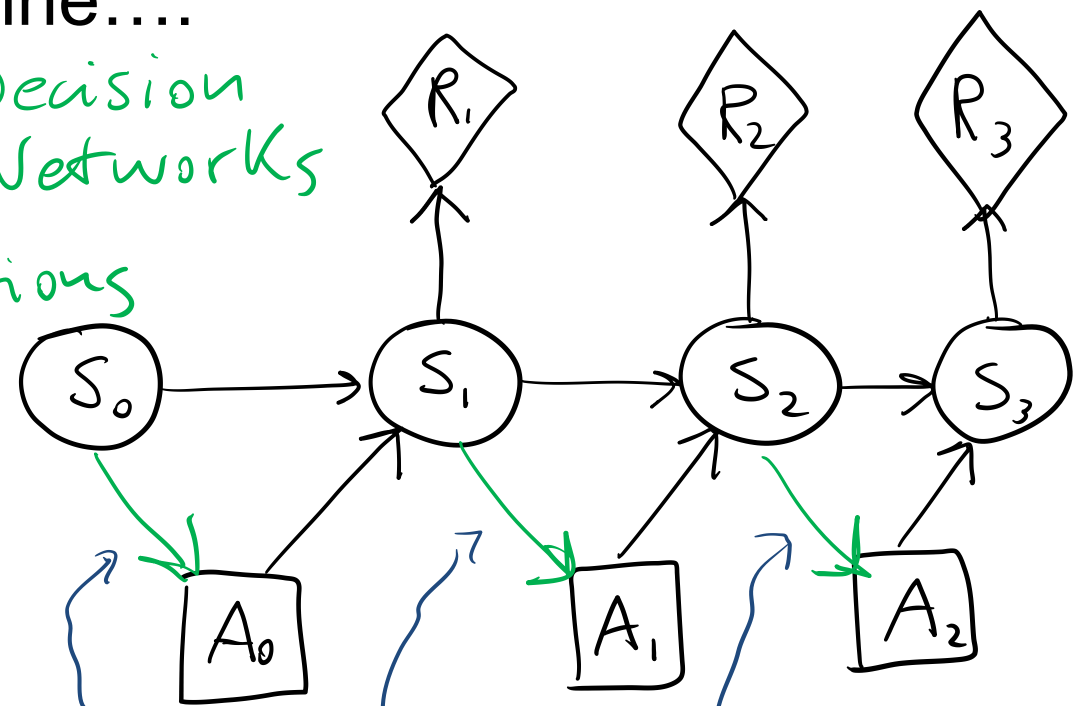


$$R(s) = \begin{cases} -0.04 & \text{(small penalty) for nonterminal states} \\ \pm 1 & \text{for terminal states} \end{cases}$$

Summary Decision Processes: MDPs

To manage an ongoing (indefinite… infinite) decision process, we combine….

Markov Chains & Decision Networks



Markovian

Stationary

Assumptions

Utility not just at the end

BUT

Sequence of rewards

Fully Observable

MDP: formal specification

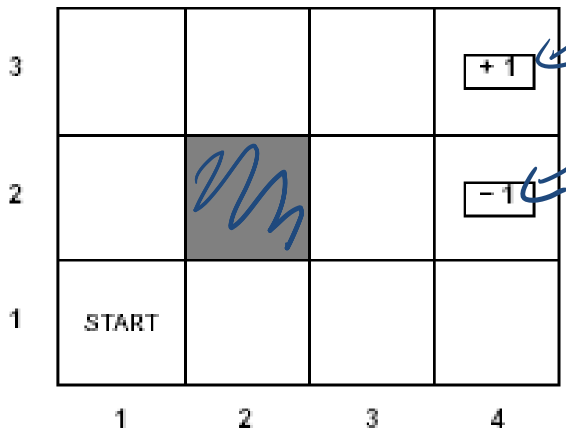
For an MDP you specify:

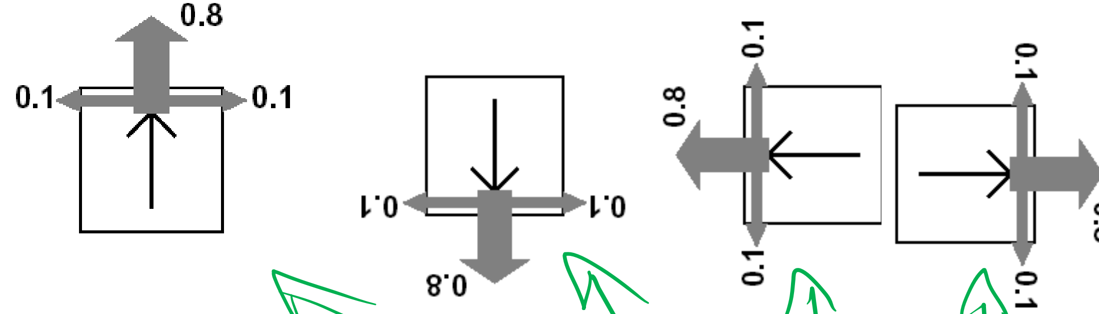
- set S of states and set A of actions
- the process' dynamics (or ***transition model***)

  $P(S_{t+1}|S_t, A_t)$
- The **reward function**
  - $R(s)$ is used when the reward depends only on the state **s** and not on how the agent got there
  - More complex $R(s, a, s')$ describing the reward that the agent receives when it performs action a in state s and ends up in state s'
- **Absorbing/stopping/terminal state** S_{ab}

for all action $P(S_{ab} | a, S_{ab}) = 1$ $R(S_{ab}, a, S_{ab}) = 0$

Example MDP: Scenario and Actions





Agent moves in the above grid via actions *Up, Down, Left, Right*

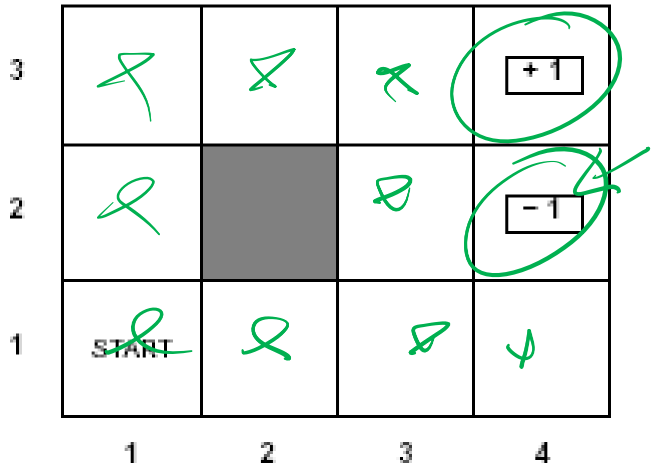
Each action has:

- 0.8 probability to reach its intended effect
- 0.1 probability to move at right angles of the intended direction
- If the agents bumps into a wall, it says there

How many states? 11 {(11), (12), ..., (24), (34)}

There are two terminal states (3,4) and (2,4)

Example MDP: Rewards

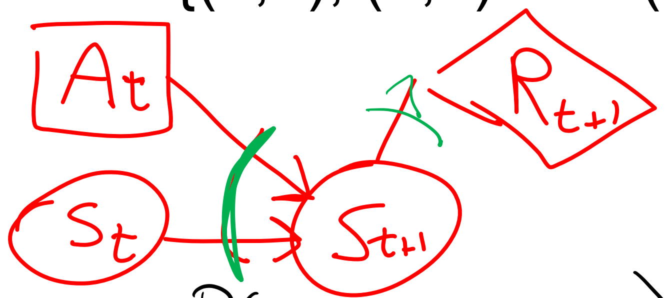


$$R(s) = \begin{cases} -0.04 & \text{(small penalty) for nonterminal states} \\ \pm 1 & \text{for terminal states} \end{cases}$$

Example MDP: Underlying info structures

Four actions *Up, Down, Left, Right*

Eleven States: {(1,1), (1,2)…… (3,4)}



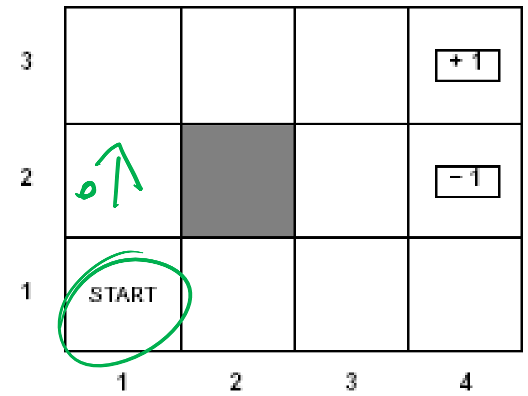


Table $4 \times 11 \times 11$ $P(S_{t+1} \mid S_t, A_t)$

Up

	1,1	2,1	1,2	3,1	…
1,1	.1	.8	.1	0	0 0 0 0
2,1	0	.2	0	.8	
⋮					

Down

L

R

$P(S_0)$

1,1	1
⋮	0
	0
	0
	0
	0
	0

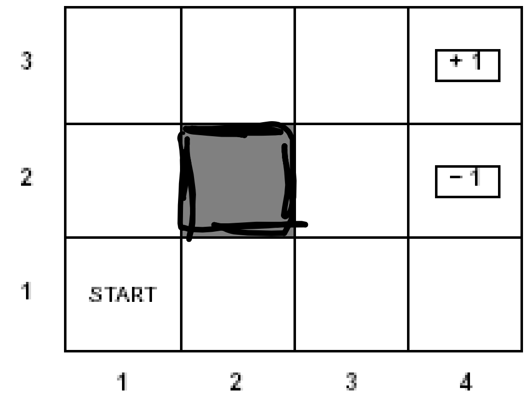
$R(s)$

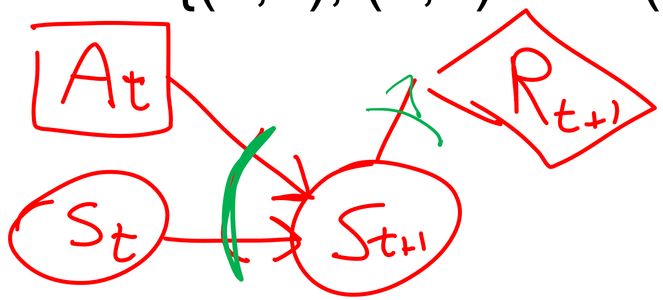
11	-.04
⋮	⋮
(2 4)	-1
(3 4)	+1

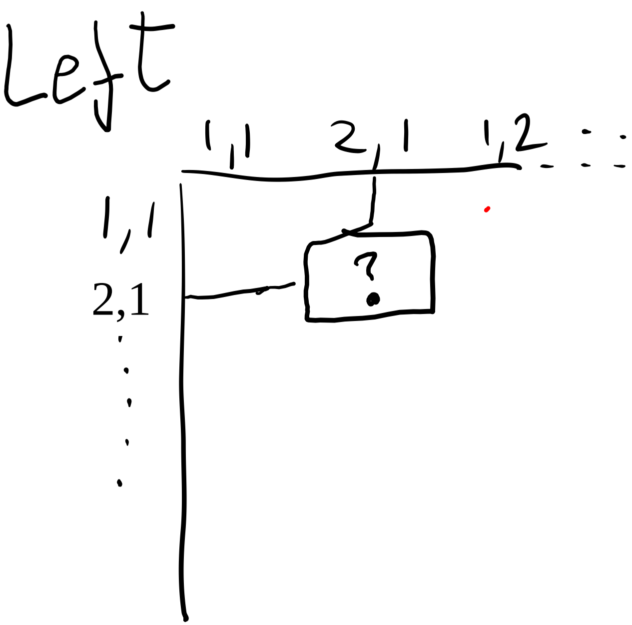
Example MDP: Underlying info structures

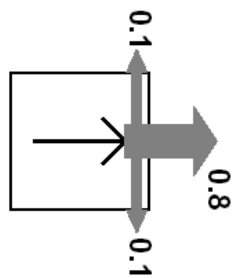
Four actions *Up, Down, Left, Right*

Eleven States: {(1,1), (1,2)…… (3,4)}









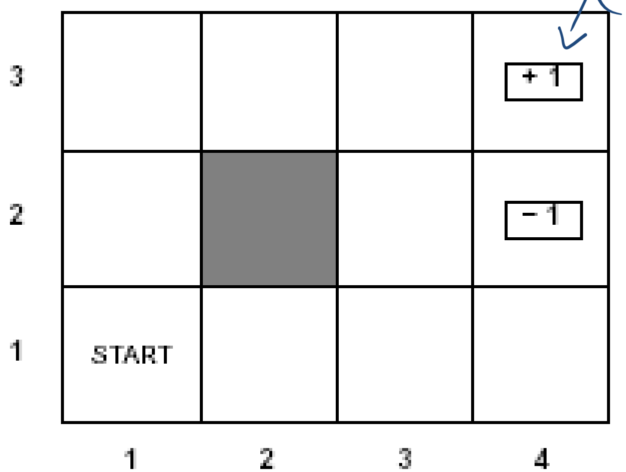
A. *0.1*

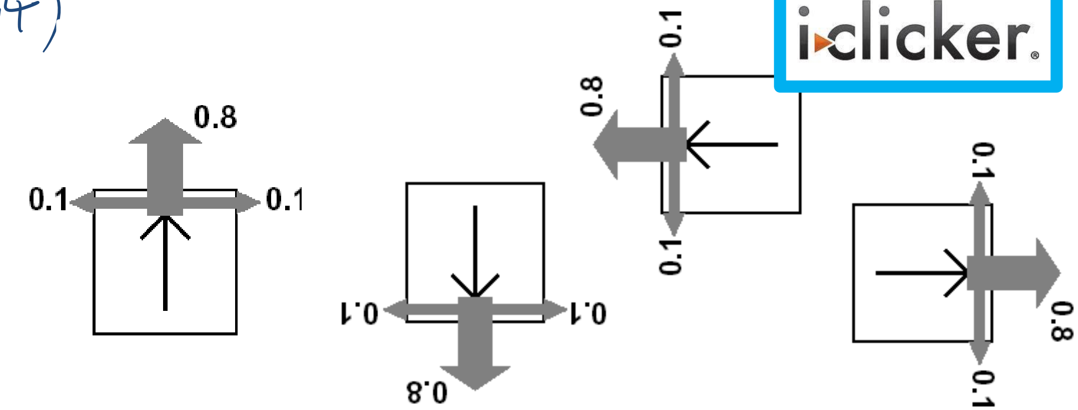
B. *0.8*

C. *0.2*

Example MDP: Sequence of actions

(3,4)





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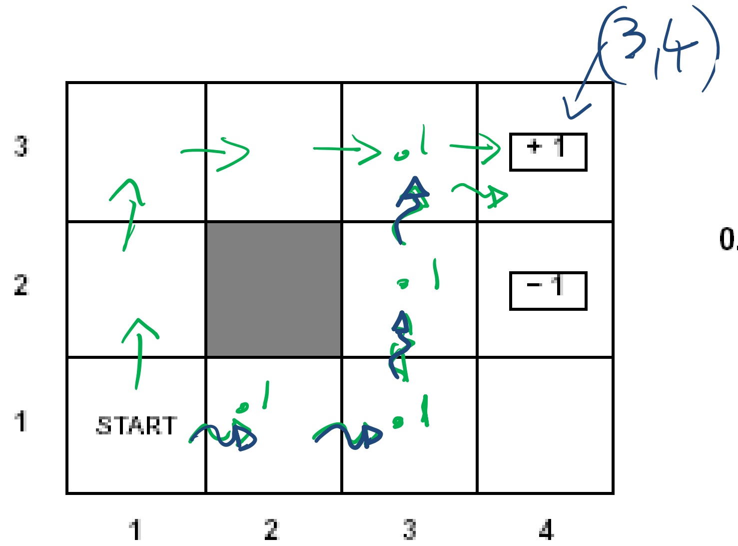
The sequence of actions [*Up, Up, Right, Right, Right*] will take the agent in terminal state (3,4)…

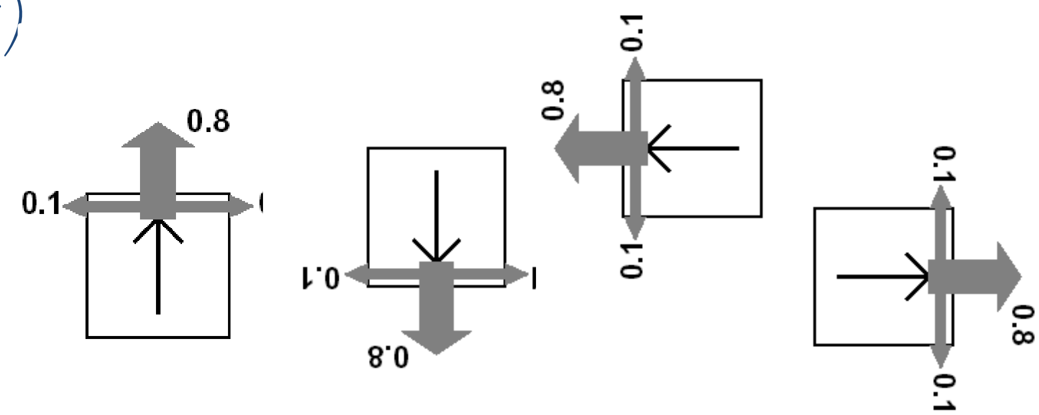
A. ***always*** **B.** ***never*** **C.** ***Only sometimes***

With what probability?

A. $(0.8)^5$ **B.** $(0.8)^5 + ((0.1)^4 \times 0.8)$ **C.** $((0.1)^4 \times 0.8)$

Example MDP: Sequence of actions





Can the sequence [*Up, Up, Right, Right, Right*] take the agent in terminal state (3,4)?

$(.8)^5$

Can the sequence reach the goal in any other way?

$(.1)^4 \cdot .8$ ← with prob   yes

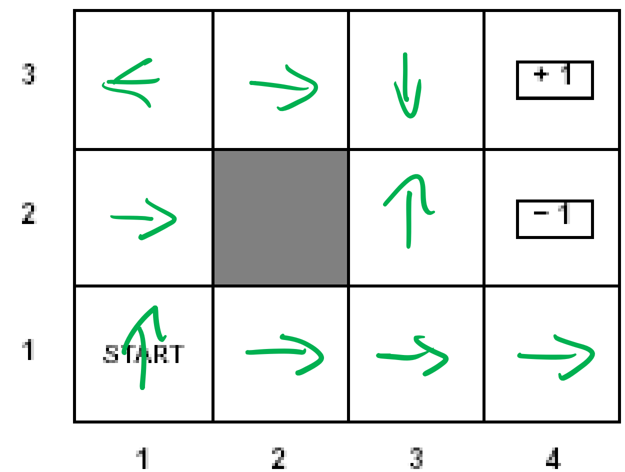
MDPs: Policy

- The robot needs to know what to do as the decision process unfolds…
- It starts in a state, selects an action, ends up in another state selects another action….

- Needs to make the same decision over and over: Given the current state what should I do?

policy

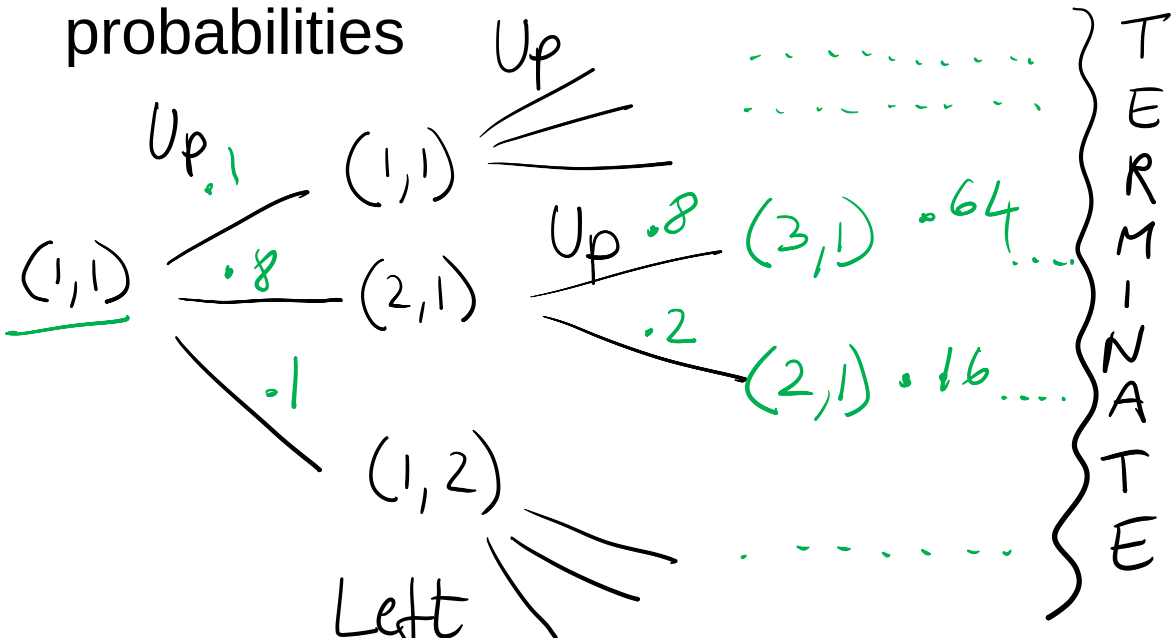
  - So a policy for an MDP is a single decision function *π(s)* that specifies what the agent should do for each state *s*

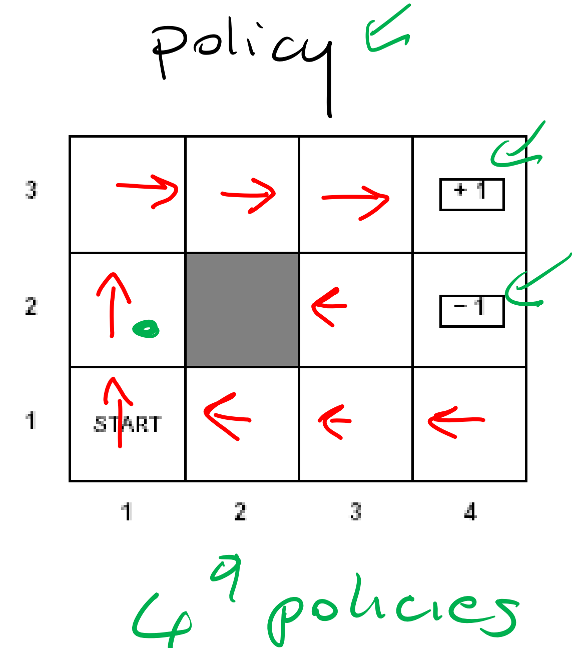


How to evaluate a policy

(in essence how to compute $V^{\pi}(s)$ brute force)

A policy can generate a set of state sequences with different probabilities





Each state sequence has a corresponding reward. Typically the (*discounted*) sum of the rewards for each state in the sequence

$-.04 \quad -.04 \quad \ldots \quad +1$

$\sum (1,1) \rightarrow (1,1) \rightarrow (2,1) \rightarrow (3,1) \rightarrow (3,1) \rightarrow (3,2) \rightarrow (3,3) \rightarrow (3,4)$

$+.72$

MDPs: expected value/total reward of a policy and optimal policy

Each sequence of states (environment history) associated with a policy has

- a certain **probability** of occurring
- a given amount of total **reward** as a function of the rewards of its individual states

Expected value /total reward

$$\sum \overbrace{P(s_0, s_1, \ldots, s_{TERMINAL})}^{\text{probability}} * \overbrace{\sum R(s_0) \ldots R(s_T)}^{\text{rewards}}$$

For all the sequences of states generated by the policy

we sum the product of its probability times its reward

Optimal policy is the policy that maximizes *expected total reward*

Lecture Overview

Markov Decision Processes

- Formal Specification and example
- Policies and Optimal Policy
- **Intro to Value Iteration**

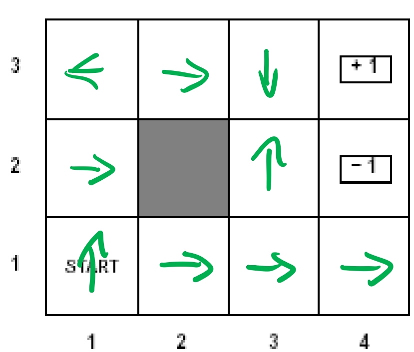
Sketch of ideas to find the optimal policy for a MDP (Value Iteration)

We first need a couple of definitions

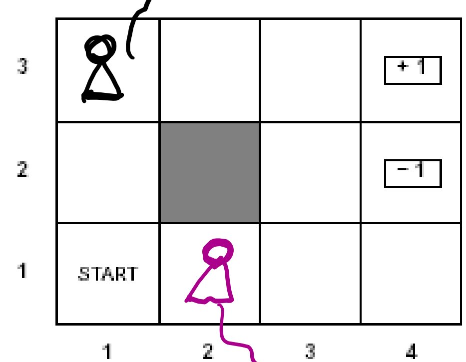
- **V^{π}(s)**: the expected value of following policy ***π*** in state **s**
- **Q^{π} (s, a)**, where **a** is an action: expected value of performing ***a*** in ***s***, and then following policy ***π***.

Example $V^{\hat{\pi}}((3,1))$ $Q^{\hat{\pi}}((1,2), Up)$

policy $\hat{\pi}$



follows $\hat{\pi}$



perform Up, then follows $\hat{\pi}$

Sketch of ideas to find the optimal policy for a MDP (Value Iteration)

We first need a couple of definitions

- **V^π(s)**: the expected value of following policy ***π*** in state **s**
- **Q^π (s, a)**, where **a** is an action: expected value of performing ***a*** in ***s***, and then following policy ***π***.

Can we express **Q^π (s, a)** in terms of **V^π (s)** ?

$$Q^\pi(s, a) = V^\pi(s) + R(s)$$

A.

$$Q^\pi(s, a) = R(s) + \sum_{s' \in X} P(s' \mid s, a) * V^\pi(s')$$

B.

$$Q^\pi(s, a) = R(s) + \sum_{s' \in X} V^\pi(s')$$

C.

D. None of the above

X: set of states reachable from *s* by doing *a*

Discounted Reward Function

- Suppose the agent goes through states s_1, s_2,…,s_k and receives rewards r_1, r_2,…,r_k

- We will look at ***discounted reward*** to define the reward for this sequence, i.e. its *utility* for the agent

$$\gamma \ \textbf{\textit{discount factor}}, \ 0 \le \gamma \le 1$$

$$U[s_1, s_2, s_3,..] \ = \ r_1 + \gamma r_2 + \gamma^2 r_3 + ...$$

Sketch of ideas to find the optimal policy for a MDP (Value Iteration)

We first need a couple of definitions

- $V^{\pi}(s)$: the expected value of following policy π in state s
- $Q^{\pi}(s, a)$, where *a* is an action: expected value of performing *a* in *s*, and then following policy π.

We have, by definition

$$Q^{\pi}(s, a) = R(s) + \gamma \sum_{s'} P(s' \mid s, a) V^{\pi}(s')$$

reward obtained in *s*

Discount factor

states reachable from *s* by doing *a*

Probability of getting to *s'* from *s* via *a*

expected value of following policy π in *s'*

Learning Goals for today's class

You can:

- Compute the probability distribution on states given a sequence of actions in an MDP
- Define a policy for an MDP
- Define and Justify a discounted reward function
- Derive the Bellman equations on which Value Iteration is based (we will likely finish this in the next lecture)

TODO for Mon

Read textbook

- **9.5.3 Value Iteration**

CPSC 322 Review "Exam"

https://forms.gle/SpQwrXfonTZrVf4P7

Based on CPSC 322 material

- Logic
- Uncertainty
- Decision Theory

Review material (e.g., 322 slides from 2017):

https://www.cs.ubc.ca/~carenini/TEACHING/CPSC322-17S/index.html