Intelligent Systems (AI-2)

Computer Science cpsc422, Lecture 22

Mar, 10, 2021

Slide credit: some from Prof. Carla P. Gomes (Cornell)
some slides adapted from Stuart Russell (Berkeley), some from Prof. Jim Martin (Univ. of Colorado)

Relationships between different Logics (better with colors)

First Order Logic

$\forall X \exists Y p(X,Y) \Leftrightarrow \neg q(Y)$

$p(a_1, a_2)$

$\neg q(a_5)$

...

Datalog

$p(X) \leftarrow q(X) \land r(X,Y)$

$r(X,Y) \leftarrow s(Y)$

$s(a_1), q(a_2)$

Propositional Logic

$\neg(p \lor q) \rightarrow (r \land s \land f),$

p, r

PDCL

$p \leftarrow s \land f$

$r \leftarrow s \land q \land p$

r

p

Lecture Overview

- **SAT : example**
- **First Order Logics**
  - Language and Semantics
  - Inference

Satisfiability problems (SAT)

Consider a CNF sentence, e.g.,

$(\neg D \lor \neg B \lor C) \land (B \lor \neg A \lor \neg C) \land (\neg C \lor \neg B \lor E)$
$\land (E \lor \neg D \lor B) \land (B \lor E \lor \neg C)$

Is there an interpretation in which this sentence is true (i.e., that is a model of this sentence)?

Many combinatorial problems can be reduced to checking the satisfiability of propositional sentences ……and returning a model

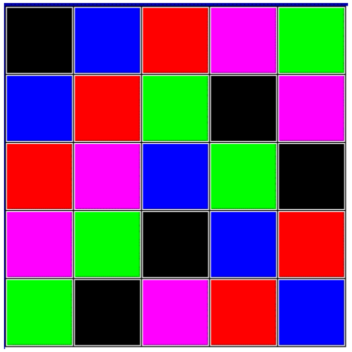
Encoding the Latin Square Problem in Propositional Logic

In **combinatorics** and in experimental design, a **Latin square** is

- an ***n* × *n* array**
- filled with ***n* different sym**bols,
- each occurring exactly once in each row and exactly once in each column.
- Here is an example:

A	B	C
C	A	B
B	C	A

Here is another one:

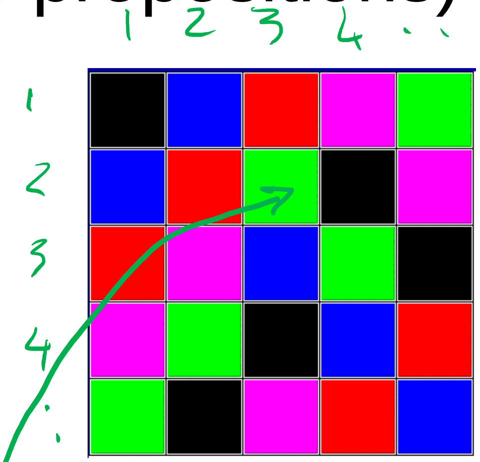


Encoding Latin Square in Propositional Logic: Propositions

Variables must be binary! (They must be propositions)

Each variables represents a color assigned to a cell ***i j***.

Assume colors are encoded as an integer ***k***



$$x_{ijk} \in \{0,1\}$$

Assuming colors are encoded as follows

(black, 1) (red, 2) (blue, 3) (green, 4) (purple, 5)

$x_{234} = 1$

x_{233} $= 0$ True or false, ie. 1 or 0 with respect to the interpretation represented by the picture?

Example $5^3 = 125$

How many vars/propositions overall? n^3

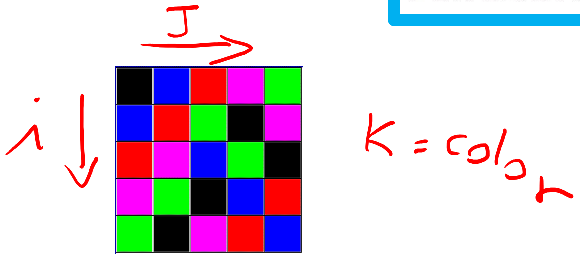
X_{111} T/F

⋮

X_{555}

Encoding Latin Square in Propositional Logic: Clauses

- Some color must be assigned to each cell (clause of length 5);



$\forall_{ij}\ (x_{ij1} \vee x_{ij2} \ \dots \ x_{ij5})$

A.

i	j	
1	1	$x_{111} \vee x_{112} \vee \dots \vee x_{115}$
1	2	...
2	2	...
⋮		
5	5	

$\forall_{ik}\ (x_{i1k} \vee x_{i2k} \ \dots \ x_{i5k})$

B.

i	k	
1	1	$x_{111} \vee x_{121} \vee \dots \vee x_{151}$
1	2	...
2	2	...
⋮		
5	5	

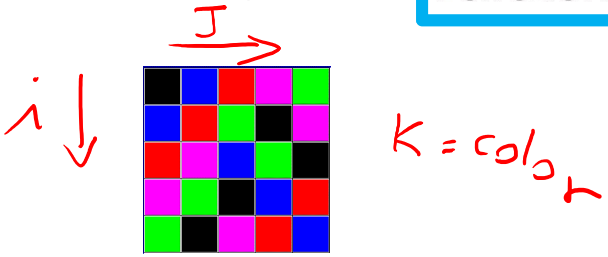
How many clauses?

Encoding Latin Square in Propositional Logic: Clauses

- Some color must be assigned to each cell (clause of length n);

$$\forall_{ij}(x_{ij1} \vee x_{ij2} \ldots x_{ijn}) \qquad \forall_{ik}(x_{i1k} \vee x_{i2k} \ldots x_{ink})$$

A. B.



- No color is repeated in the same row (sets of negative binary clauses);

$$\forall_{ik}(\neg x_{i1k} \vee \neg x_{i2k}) \wedge (\neg x_{i1k} \vee \neg x_{i3k}) \ldots (\neg x_{i1k} \vee \neg x_{ink}) \cdots (\neg x_{i(n-1)k} \vee \neg x_{ink})$$

How many clauses?

Encoding Latin Square in Propositional Logic: Clauses

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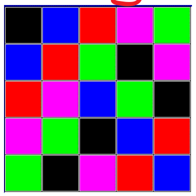
$$\forall_{ij}(x_{ij1} \vee x_{ij2} \ldots x_{ijn})$$



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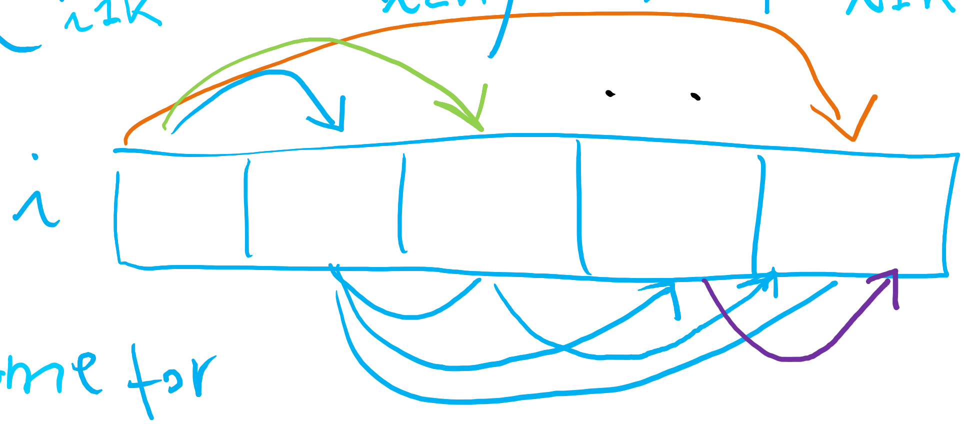
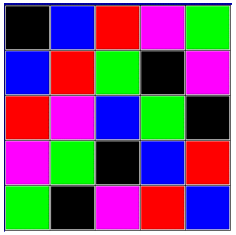


How many clauses?

$O(n^4)$

Encoding Latin Square Problems in Propositional Logic: FULL MODEL



n^3

Variables: x_{ijk} *cell i, j has color k*; $i, j, k = 1, 2, \ldots, n.$ $\quad x_{ijk} \in \{0,1\}$

Each variables represents a color assigned to a cell.

Clauses: $O(n^4)$

- Some color must be assigned to each cell (clause of length n);

$$\forall_{ij} (x_{ij1} \vee x_{ij2} \ldots x_{ijn})$$

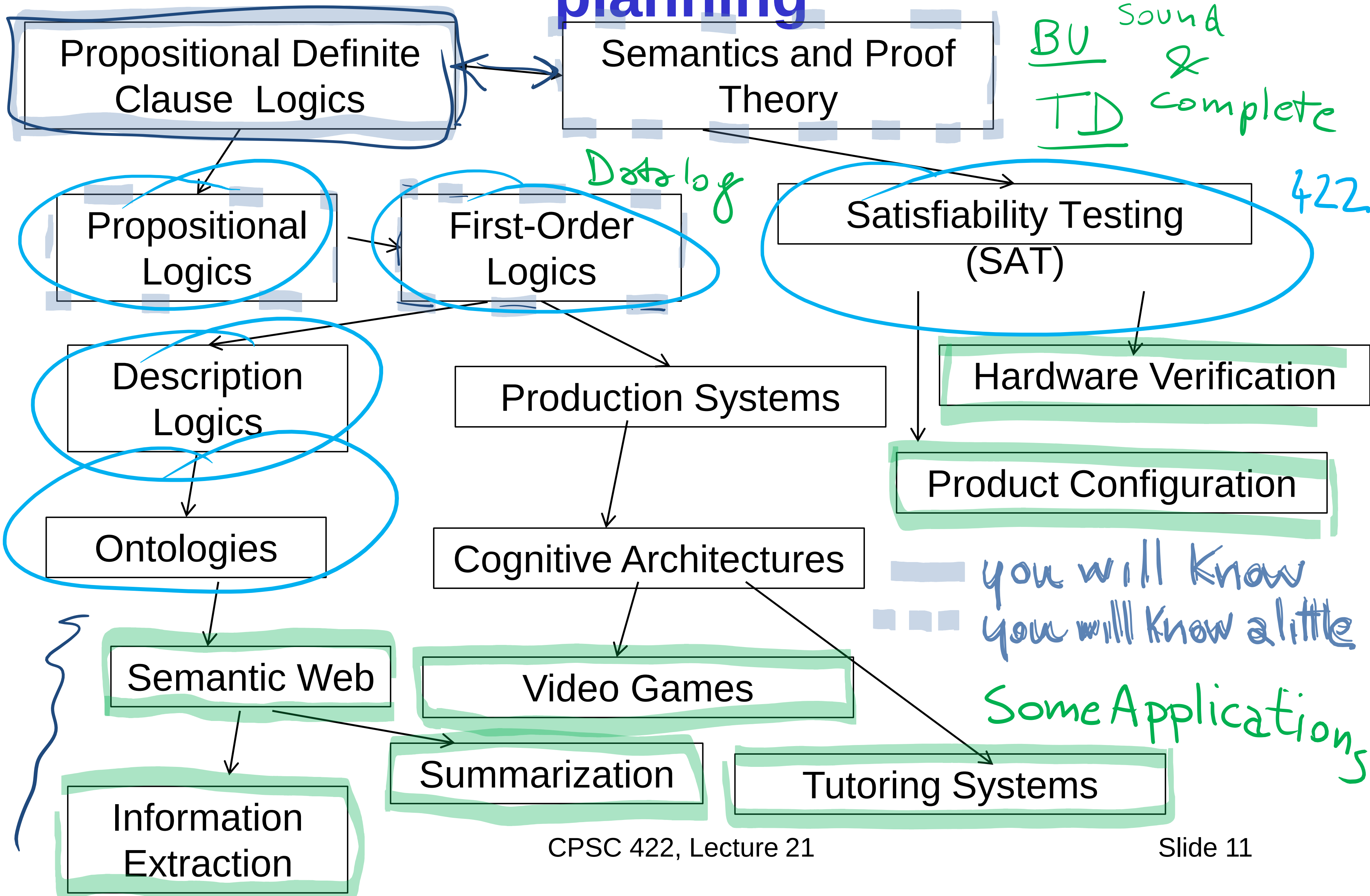
- No color repeated in the same row (sets of negative binary clauses);

$$\forall_{ik} (\neg x_{i1k} \vee \neg x_{i2k}) \wedge (\neg x_{i1k} \vee \neg x_{i3k}) \ldots (\neg x_{i1k} \vee \neg x_{ink}) \cdots (\neg x_{i(n-1)k} \vee \neg x_{ink})$$

- No color repeated in the same column (sets of negative binary clauses);

$$\forall_{jk} (\neg x_{1jk} \vee \neg x_{2jk}) \wedge (\neg x_{1jk} \vee \neg x_{3jk}) \ldots (\neg x_{1jk} \vee \neg x_{njk}) \cdots (\neg x_{(n-1)jk} \vee \neg x_{njk})$$

Logics in AI: Similar slide to the one for planning



Relationships between different Logics (better with colors)

First Order Logic

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$p(a_1, a_2)$

$-q(a_5)$

...

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$r(X,Y) \leftarrow s(Y)$

$s(a_1), q(a_2)$

Propositional Logic

$\neg(p \vee q) \rightarrow (r \wedge s \wedge f),$

p, r

PDCL

$p \leftarrow s \wedge f$

$r \leftarrow s \wedge q \wedge p$

r

p

Lecture Overview

- Finish SAT (example)
- **First Order Logics**
  - Language and Semantics
  - Inference

Representation and Reasoning in Complex domains (from 322)

- In complex domains expressing knowledge with **propositions** can be quite limiting

up_s_2
up_s_3
ok_cb_1
ok_cb_2
$live\_w_1$
$connected\_w_1\_w_2$

- It is often natural to consider **individuals** and their **properties**

$up(\ s_2\ )$
$up(\ s_3\ )$
$ok(\ cb_1\ )$
$ok(\ cb_2\ )$
$live(\ w_1)$
$connected(\ w_1\ ,\ w_2\ )$

There is no notion that

up_s_2
up_s_3

$live\_w_1$
$connected\_w_1\_w_2$

(from 322) What do we gain….

By breaking propositions into relations applied to individuals?

- Express **knowledge** that **holds for set of individuals** (by introducing variables)

  live(W) <- connected_to(W,W1) ∧ live(W1) ∧ wire(W) ∧ wire(W1).

- We can **ask generic queries** (i.e., containing vars variables)

  ? connected_to(W, w_1)

“Full” First Order Logics (FOL)

LIKE DATALOG: Whereas propositional logic assumes the world contains facts, FOL (like natural language) assumes the world contains

- Objects: people, houses, numbers, colors, baseball games, wars, …
- Relations: red, round, prime, brother of, bigger than, part of, comes between, …
- Functions: father of, best friend, one more than, plus, …

FURTHERMORE WE HAVE

- More Logical Operators:….
- Equality: coreference (two terms refer to the same object)
- Quantifiers
  - ✓ Statements about unknown objects
  - ✓ Statements about classes of objects

Syntax of FOL

Constants	KingJohn, 2, ,...
Predicates	Brother, >,...
Functions	Sqrt, LeftLegOf,...
Variables	x, y, a, b,...
Connectives	$\neg$, $\Rightarrow$, $\wedge$, $\vee$, $\Leftrightarrow$
Equality	=
Quantifiers	$\forall$, $\exists$

Atomic sentences

Term is a *function* (*$term_1$,...,$term_n$*) or *constant* or *variable*

Atomic sentence is *predicate* (*$term_1$,...,$term_n$*) or *$term_1$ = $term_2$*

E.g.,

predicate(constant, constant)

- *Brother(KingJohn, RichardTheLionheart)*

predicate(function(function(constant)), (function(function(constant)))

- *> (Length(LeftLegOf(Richard)), Length(LeftLegOf(KingJohn)))*

Complex sentences

Complex sentences are made from atomic sentences using connectives

$$\neg S, \quad S_1 \wedge S_2, \quad S_1 \vee S_2, \quad S_1 \Rightarrow S_2, \quad S_1 \Leftrightarrow S_2,$$

E.g.

Sibling(KingJohn, Richard) $\Rightarrow$ *Sibling(Richard, KingJohn)*

$\forall$x *P(x)* is true in an interpretation ***I*** iff *P* is true with *x* being each possible object in ***I***

$\exists$x P(x) is true in an interpretation I iff P is true with x being some possible object in I

Truth in first-order logic

Like in Prop. Logic a sentences is true with respect to an interpretation

$\neg A \wedge (B \Rightarrow C)$,

2^3

A	B	C	
T	F	F	×
F	T	T	✓

In FOL interpretations are **much more complex** but still same idea possible configuration of the world

symbols { CONSTANTS → objects; Predicates → relations; Functions → functions }

2 objects △ □

2 CONSTANT SYMBOLS $\{C_1, C_2\}$ → △ □ (C_1 → △, C_2 → □)

1 unary Predicate P → {△}

1 binary Predicate Q → {{△, △}}

IS $\forall x\, P(x)$ TRUE?

A. yes
B. no

Truth in first-order logic

Like in Prop. Logic a sentences is true with respect to an interpretation

$$\neg A \wedge B \Rightarrow C,$$

2^3

A	B	C	
T	F	F	×
T	T	T	✓

In FOL interpretations are much more complex but still same idea: possible configuration of the world

symbols { CONSTANTS → objects; Predicates → relations; Functions → functions }

2 objects △ □

2 CONSTANT SYMBOLS $\{c_1, c_2\}$ → c_1 → △, c_2 → □

1 unary Predicate P → {△}

but if P → {△, □}

1 binary Predicate Q → {{△, △}}

IS $\forall x\, P(x)$ TRUE? NO Yes!

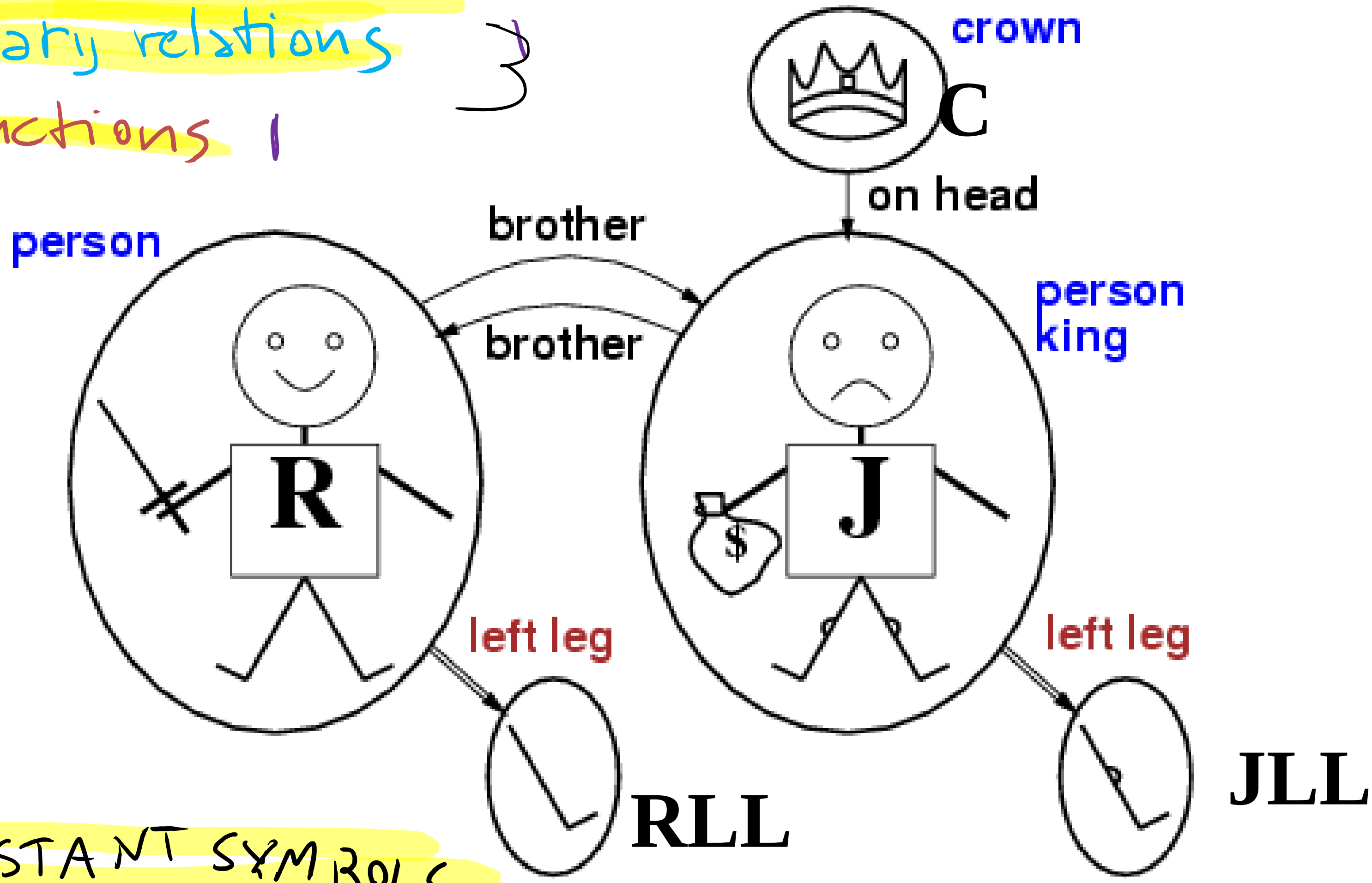
Interpretations for FOL: Example

5 objects

binary relations 2

unary relations 3

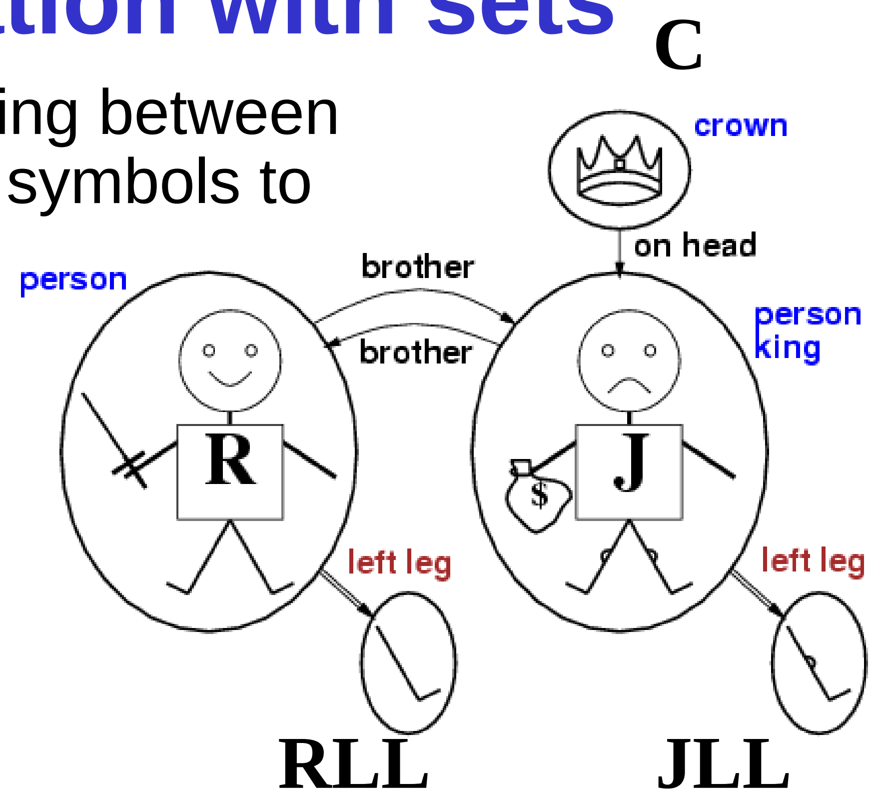
functions 1



CONSTANT SYMBOLS 5

Same interpretation with sets

Since we have a one to one mapping between symbols and object we can use symbols to refer to objects



- {R, J, RLL, JLL, C}

Property Predicates

- Person = {R, J}
- Crown = {C}
- King = {J}

Relational Predicates

- Brother = { <R,J>, <J,R>}
- OnHead = {<C,J>}

Functions

- LeftLeg = {<R, RLL>, <J, JLL>}

How many Interpretations with….

assuming unique names

5 Objects and 5 symbols

- {R, J, RLL, JLL, C}

5!   J   R   C   RLL   JLL

3 Property Predicates (Unary Relations)

- Person
- Crown
- King

R   J   RLL   JLL   C 2^5

0/1   0/1   0/1   0/1   0/1

A. 2^5 B. 2^{25} C. 25^2

2 Relational Predicates

- Brother
- OnHead

25 possibilities; each one can be 0/1 so 2^{25}

1 Function

- LeftLeg

5^5

TOTAL $5! * (2^5)^3 * (2^{25})^2 * 5^5$

To summarize: Truth in first-order logic

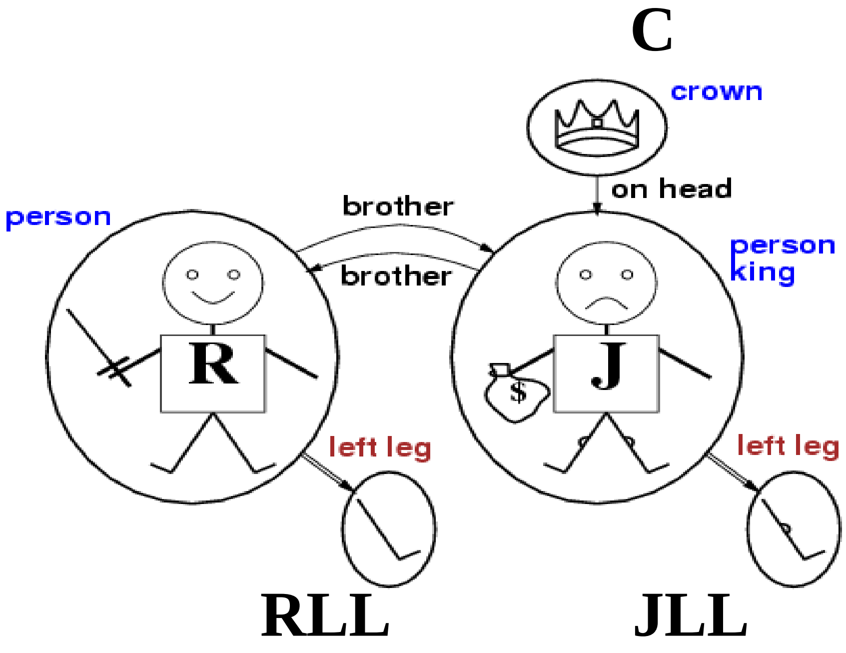
- Sentences are true with respect to an **interpretation**
- World contains objects (**domain elements**)
- Interpretation specifies referents for

constant symbols	→	**objects**
predicate symbols	→	**relations**
function symbols	→	**functional relations**

- An atomic sentence ***predicate(term$_1$,...,term$_n$)*** is true

  iff the **objects** referred to by ***term$_1$,...,term$_n$***

  are in the **relation** referred to by ***predicate***



Quantifiers

Allows us to express

- **Properties of collections of objects** instead of enumerating objects by name
- **Properties of an unspecified object**

Universal: “for all” $\forall$

Existential: “there exists” $\exists$

Universal quantification

∀*<variables>* *<sentence>*

Everyone at UBC is smart:

$\forall x\ At(x, UBC) \Rightarrow Smart(x)$

∀x *P* is true in an interpretation ***I*** iff *P* is true with *x* being each possible object in ***I***

Equivalent to the conjunction of instantiations of *P*

$At(KingJohn, UBC) \Rightarrow Smart(KingJohn)$
$\wedge At(Richard, UBC) \Rightarrow Smart(Richard)$
$\wedge At(Ralphie, UBC) \Rightarrow Smart(Ralphie)$
$\wedge \ldots$

Existential quantification

∃<*variables*> <*sentence*>

Someone at UBC is smart:

$\exists x$ At(x, UBC) $\wedge$ Smart(x)

$\exists x$ *P* is true in an interpretation ***I*** iff *P* is true with *x* being some possible object in ***I***

Equivalent to the disjunction of instantiations of *P*

At(KingJohn, UBC) $\wedge$ Smart(KingJohn)

$\vee$ At(Richard, UBC) $\wedge$ Smart(Richard)

$\vee$ At(Ralphie, UBC) $\wedge$ Smart(Ralphie)

$\vee$ ...

Properties of quantifiers

$\exists x\ \forall y$ is not the same as $\forall y\ \exists x$

$\exists x\ \forall y$ Loves(x,y)

- "There is a person who loves everyone in the world"

$\forall y\ \exists x$ Loves(x,y)

- "Everyone in the world is loved by at least one person"

Quantifier duality: each can be expressed using the other

$\forall x$ Likes(x,IceCream) $\neg\exists x\ \neg$Likes(x,IceCream)

$\exists x$ Likes(x,Broccoli) $\neg\forall x\ \neg$Likes(x,Broccoli)

Lecture Overview

- Finish SAT (example)
- **First Order Logics**
  - Language and Semantics
  - **Inference**

FOL: Inference

Resolution Procedure can be generalized to FOL

- **Every formula** can be rewritten in **logically equivalent CNF**
  - Additional rewriting rules for quantifiers
- **Similar Resolution step,** but variables need to be unified (like in DATALOG)

$In(x, y) \lor \neg Charged(x)$

$\neg In(z, v) \lor Connected(z)$

$\neg Charged(x) \lor Connected(x)$

$\theta = \{z/x, v/y\}$

NLP Practical Goal for FOL: the ultimate Web question-answering system?

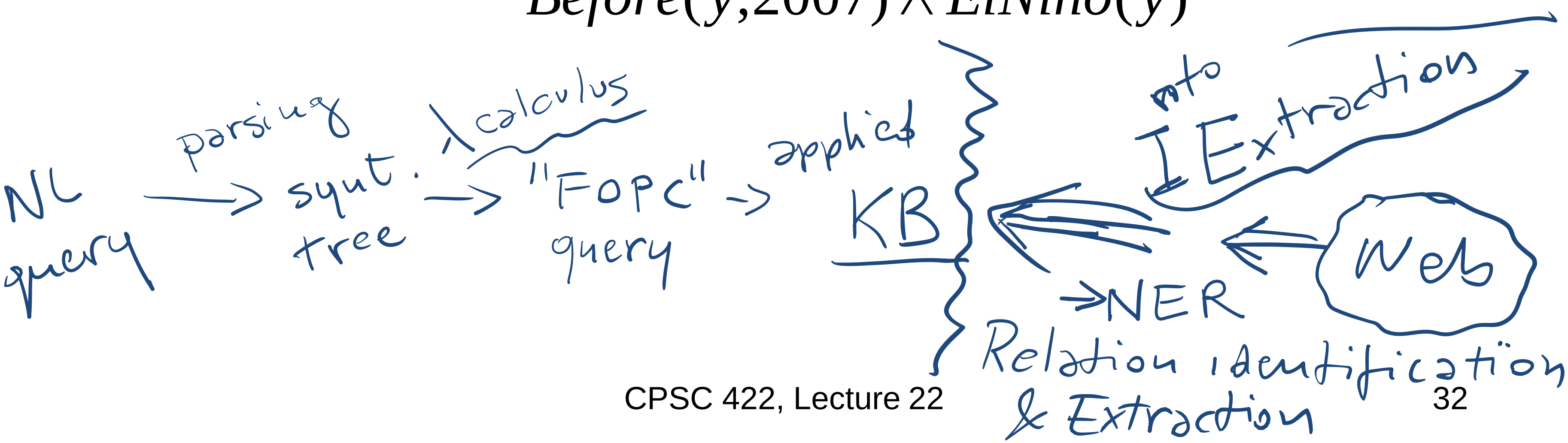
Map NL queries into FOPC so that answers can be effectively computed

What African countries are not on the Mediterranean Sea?

$$\exists c\ Country(c) \wedge \neg Borders(c, Med.Sea) \wedge In(c, Africa)$$

- *Was 2007 the first El Nino year after 2001?*

$$ElNino(2007) \wedge \neg \exists y\ Year(y) \wedge After(y, 2001) \wedge Before(y, 2007) \wedge ElNino(y)$$



Learning Goals for today's class

You can:

- Explain differences between Proposition Logic and First Order Logic
- Compute number of interpretations for FOL
- Explain the meaning of quantifiers
- Describe application of FOL to NLP: Web question answering

Next class Fri

- Ontologies (e.g., Wordnet, Probase), Description Logics…
- Midterm will be likely returned next week

Assignment-3 will be out soon (tonight or tomorrow at the latest)