Intelligent Systems (AI-2)

Computer Science cpsc422, Lecture 20

Mar, 3, 2021

Slide credit: some slides adapted from Stuart Russell (Berkeley),
some from Padhraic Smyth (UCIrvine)

422 big picture

StarAI (statistical relational AI)

Hybrid: Det +Sto

Prob CFG
Prob Relational Models
Markov Logics

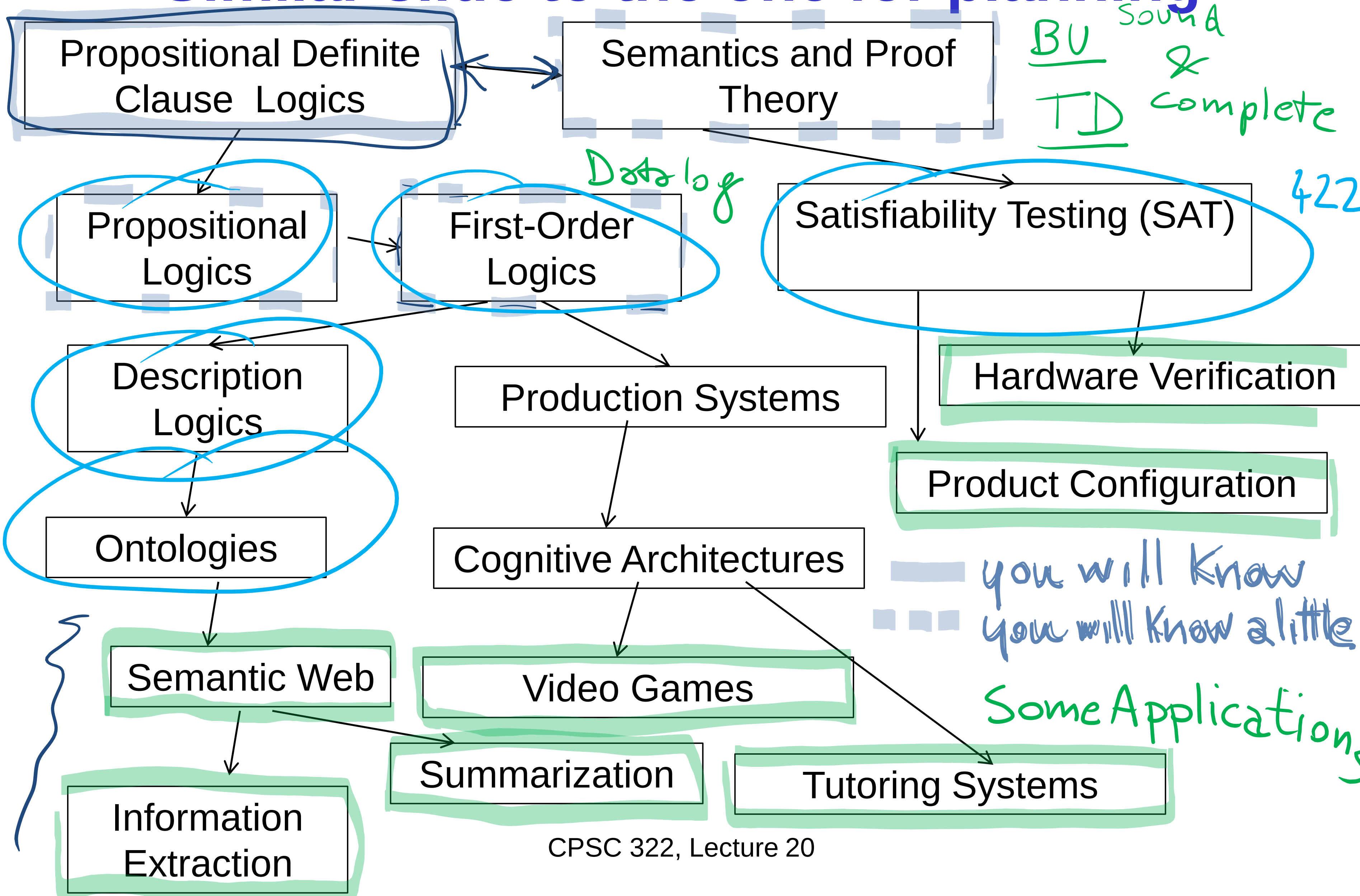
	Deterministic	**Stochastic**
Query	*Logics* *First Order Logics* *Ontologies* • Full Resolution • SAT	*Belief Nets* Approx. : Gibbs *Markov Chains and HMMs* Forward, Viterbi…. Approx. : Particle Filtering *Undirected Graphical Models* *Markov Networks* *Conditional Random Fields*
Planning		*Markov Decision Processes and Partially Observable MDP* • Value Iteration • Approx. Inference *Reinforcement Learning*

Applications of AI

Representation

Reasoning Technique

Logics in AI (322): Similar slide to the one for planning



Relationships between different Logics (better with colors)

First Order Logic

$\forall X \exists Y p(X,Y) \Longleftrightarrow \neg q(Y)$

$p(a_1, a_2)$

$-q(a_5)$

...

Propositional Logic

$\neg(p \vee q) \rightarrow (r \wedge s \wedge f)$,

p, r

Datalog

$p(X) \leftarrow q(X) \wedge r(X,Y)$

$r(X,Y) \leftarrow s(Y)$

$s(a_1), q(a_2)$

PDCL

$p \leftarrow s \wedge f$

$r \leftarrow s \wedge q \wedge p$

r

p

Lecture Overview

- **Basics Recap: Interpretation / Model /..**
- Propositional Logics
- Satisfiability, Validity
- Resolution in Propositional logics

Basic definitions from 322 (Semantics)

Definition (interpretation)
An interpretation I assigns a truth value to each atom.

Definition (truth values of statements cont'**):** A knowledge base *KB* is true in I if and only if every clause in *KB* is true in I.

PDC Semantics: Knowledge Base (KB)

- A **knowledge base KB** is true in **I** if and only if every clause in KB is true in I.

	p	q	r	s
$\mathbf{I_1}$	*true*	*true*	*false*	*false*

Which of the three KB below is *true* in I_1 ?

A

p
r
$s \leftarrow q \wedge p$

B

p
q
$s \leftarrow q$

C

p
$q \leftarrow r \wedge s$

PDC Semantics: Knowledge Base (KB)

- A **knowledge base KB** is true in I if and only if every clause in KB is true in I.

	p	q	r	s
I_1	*true*	*true*	*false*	*false*

KB_1

p
r
$s \leftarrow q \wedge p$

KB_2

p
q
$s \leftarrow q$

KB_3

p
$q \leftarrow r \wedge s$

Which of the three KB above is True in I_1 ? ***KB_3***

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Definition (interpretation)
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Definition (truth values of statements cont'**):** A knowledge base *KB* is true in I if and only if every clause in *KB* is true in I.

Definition (model)
A model of a set of clauses (a KB) is an interpretation in which all the clauses are *true*.

Example: Models

$$KB = \begin{cases} p \leftarrow q. \\ q. \\ r \leftarrow s. \end{cases}$$

	p	q	r	s	
I_1	*true*	*true*	*true*	*true*	M
I_2	*false*	*false*	*false*	*false*	
I_3	*true*	*true*	*false*	*false*	M
I_4	*true*	*true*	*true*	*false*	M
I_5	*true*	*true*	*false*	*true*	

Which interpretations are models?

Basic definitions from 322 (Semantics)

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Definition (model)
A model of a set of clauses (a KB) is an interpretation in which all the clauses are *true*.

Definition (logical consequence)
If *KB* is a set of clauses and *G* is a conjunction of atoms, *G* is a logical consequence of *KB*, written $KB \models G$, if *G* is *true* in every model of *KB*.

Example: Logical Consequences

	p	q	r	s
I_1	true	true	true	true
I_2	true	true	true	false
I_3	true	true	false	false
I_4	true	true	false	true
I_5	false	true	true	true
I_6	false	true	true	false
I_7	false	true	false	false
I_8	false	true	false	true
...	....	...	...	...

Models

$$KB = \begin{cases} p \leftarrow q. \\ q. \\ r \leftarrow s. \end{cases}$$

$2^4 = 16$ interpretations in total, only 3 are models

remaining 8 cannot be models because q is false

F

Which of the following is true?

- $KB \models q$, $KB \models p$, $KB \not\models s$, $KB \not\models r$

Is it true that if

$M(KB)$ is the set of all models of KB

$M(\alpha)$ is the set of all models of α

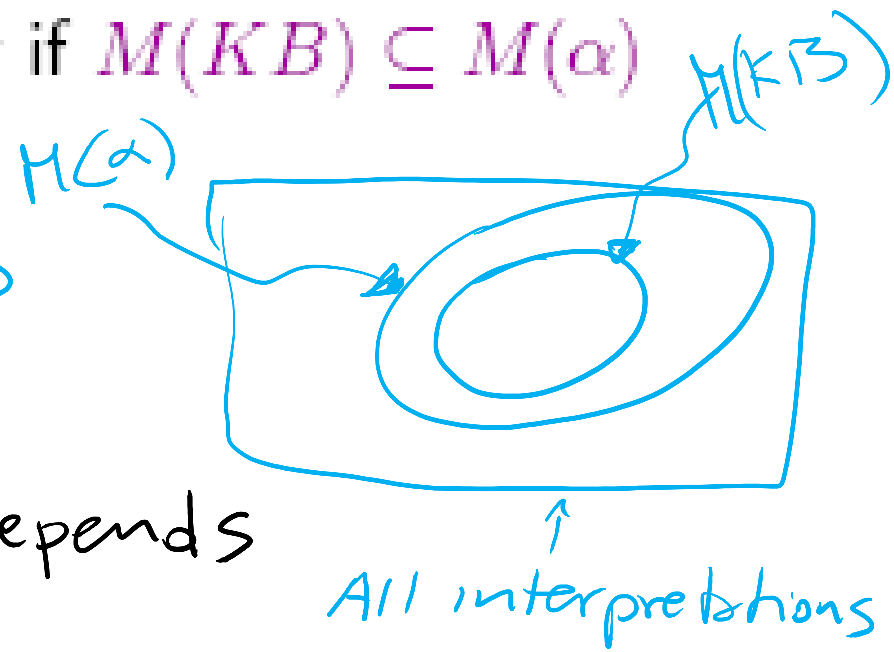
Then $KB \models \alpha$ if and only if $M(KB) \subseteq M(\alpha)$

α true in all the models of KB

A. yes

B. no

C. It depends



Basic definitions from 322 (Proof Theory)

Definition (soundness)

A proof procedure is sound if $KB \vdash G$ implies $KB \models G$.

Definition (completeness)

A proof procedure is complete if $KB \models G$ implies $KB \vdash G$.

Lecture Overview

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- **Propositional Logics**
- Satisfiability, Validity
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r

p

Propositional logic: Syntax

Atomic sentences = single proposition symbols

- E.g., P, Q, R
- Special cases: True = always true, False = always false

Complex sentences:

- If S is a sentence, $\neg$S is a sentence (negation)
- If S_1 and S_2 are sentences, $S_1 \wedge S_2$ is a sentence (conjunction)
- If S_1 and S_2 are sentences, $S_1 \vee S_2$ is a sentence (disjunction)
- If S_1 and S_2 are sentences, $S_1 \Rightarrow S_2$ is a sentence (implication)
- If S_1 and S_2 are sentences, $S_1 \Leftrightarrow S_2$ is a sentence (biconditional)

Propositional logic: Semantics

Each interpretation specifies true or false for each proposition symbol

E.g.	**p**	**q**	**r**
	false	*true*	*false*

Rules for evaluating truth with respect to an interpretation I :

$\neg S$ is true iff S is false

$S_1 \wedge S_2$ is true iff S_1 is true and S_2 is true

$S_1 \vee S_2$ is true iff S_1 is true or S_2 is true

$S_1 \Rightarrow S_2$ is true iff S_1 is false or S_2 is true
i.e., is false iff S_1 is true and S_2 is false

$S_1 \Leftrightarrow S_2$ is true iff $S_1 \Rightarrow S_2$ is true and $S_2 \Rightarrow S_1$ is true

Simple recursive process evaluates an arbitrary sentence, e.g.,

$\underbrace{(\neg p \wedge (q \vee r)) \Leftrightarrow \neg p}_{T}$ = $(\neg F \wedge (T \vee F)) \Leftrightarrow \neg F$

$(T \wedge T) \Leftrightarrow T$

$T \Leftrightarrow T$ → $(T \Rightarrow T) \wedge (T \Rightarrow T)$

$T \wedge T$

T

Logical equivalence

Two sentences are logically equivalent iff true in same interpretations

$\alpha \equiv \beta$ if and only if $\alpha \models \beta$ and $\beta \models \alpha$ they have the same models

$(\alpha \wedge \beta)$	$\equiv$	$(\beta \wedge \alpha)$	commutativity of $\wedge$
$(\alpha \vee \beta)$	$\equiv$	$(\beta \vee \alpha)$	commutativity of $\vee$
$((\alpha \wedge \beta) \wedge \gamma)$	$\equiv$	$(\alpha \wedge (\beta \wedge \gamma))$	associativity of $\wedge$
$((\alpha \vee \beta) \vee \gamma)$	$\equiv$	$(\alpha \vee (\beta \vee \gamma))$	associativity of $\vee$
$\neg(\neg\alpha)$	$\equiv$	α	double-negation elimination
$(\alpha \Rightarrow \beta)$	$\equiv$	$(\neg\beta \Rightarrow \neg\alpha)$	contraposition
$(\alpha \Rightarrow \beta)$	$\equiv$	$(\neg\alpha \vee \beta)$	implication elimination
$(\alpha \Leftrightarrow \beta)$	$\equiv$	$((\alpha \Rightarrow \beta) \wedge (\beta \Rightarrow \alpha))$	biconditional elimination
$\neg(\alpha \wedge \beta)$	$\equiv$	$(\neg\alpha \vee \neg\beta)$	De Morgan
$\neg(\alpha \vee \beta)$	$\equiv$	$(\neg\alpha \wedge \neg\beta)$	De Morgan
$(\alpha \wedge (\beta \vee \gamma))$	$\equiv$	$((\alpha \wedge \beta) \vee (\alpha \wedge \gamma))$	distributivity of $\wedge$ over $\vee$
$(\alpha \vee (\beta \wedge \gamma))$	$\equiv$	$((\alpha \vee \beta) \wedge (\alpha \vee \gamma))$	distributivity of $\vee$ over $\wedge$

$$
\begin{array}{rcll}
(\alpha \wedge \beta) & \equiv & (\beta \wedge \alpha) & \text{commutativity of } \wedge \\
(\alpha \vee \beta) & \equiv & (\beta \vee \alpha) & \text{commutativity of } \vee \\
((\alpha \wedge \beta) \wedge \gamma) & \equiv & (\alpha \wedge (\beta \wedge \gamma)) & \text{associativity of } \wedge \\
((\alpha \vee \beta) \vee \gamma) & \equiv & (\alpha \vee (\beta \vee \gamma)) & \text{associativity of } \vee \\
\neg(\neg\alpha) & \equiv & \alpha & \text{double-negation elimination} \\
(\alpha \Rightarrow \beta) & \equiv & (\neg\beta \Rightarrow \neg\alpha) & \text{contraposition} \\
(\alpha \Rightarrow \beta) & \equiv & (\neg\alpha \vee \beta) & \text{implication elimination} \\
(\alpha \Leftrightarrow \beta) & \equiv & ((\alpha \Rightarrow \beta) \wedge (\beta \Rightarrow \alpha)) & \text{biconditional elimination} \\
\neg(\alpha \wedge \beta) & \equiv & (\neg\alpha \vee \neg\beta) & \text{De Morgan} \\
\neg(\alpha \vee \beta) & \equiv & (\neg\alpha \wedge \neg\beta) & \text{De Morgan} \\
(\alpha \wedge (\beta \vee \gamma)) & \equiv & ((\alpha \wedge \beta) \vee (\alpha \wedge \gamma)) & \text{distributivity of } \wedge \text{ over } \vee \\
(\alpha \vee (\beta \wedge \gamma)) & \equiv & ((\alpha \vee \beta) \wedge (\alpha \vee \gamma)) & \text{distributivity of } \vee \text{ over } \wedge
\end{array}
$$

Can be used to rewrite formulas….

$(p \Rightarrow \neg(q \wedge r))$

$\neg p \vee \neg(q \wedge r) \rightarrow \neg p \vee \neg q \vee \neg r$

$(\alpha \wedge \beta)$	$\equiv$	$(\beta \wedge \alpha)$	commutativity of $\wedge$
$(\alpha \vee \beta)$	$\equiv$	$(\beta \vee \alpha)$	commutativity of $\vee$
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$(p \Rightarrow \neg(q \wedge r$

$\neg p \vee \neg(q \wedge r$

Can be used to rewrite formulas….

$(p \Rightarrow \neg(q \wedge r))$

$(q \wedge r) \Rightarrow \neg p$

$\neg(q \wedge r) \vee \neg p$

$\neg q \vee \neg r \vee \neg p$

Validity and satisfiability

A sentence is valid if it is true in **all** interpretations

e.g., $True$, $A \vee \neg A$, $A \Rightarrow A$, $(A \wedge (A \Rightarrow B)) \Rightarrow B$

Validity is connected to inference via the Deduction Theorem:

$KB \models \alpha$ if and only if $(KB \Rightarrow \alpha)$ is valid

There is no I where KB is false and α is true

A sentence is satisfiable if it is true in **some** interpretation

e.g., $A \vee B$, C

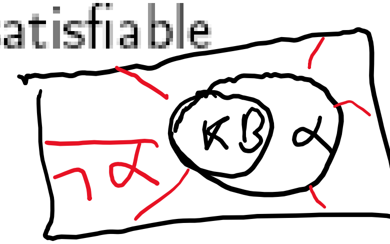
A sentence is unsatisfiable if it is true in **no** interpretations

e.g., $A \wedge \neg A$

Satisfiability is connected to inference via the following:

$KB \models \alpha$ if and only if $(KB \wedge \neg\alpha)$ is unsatisfiable

i.e., prove α by *reductio ad absurdum*



Validity and satisfiability

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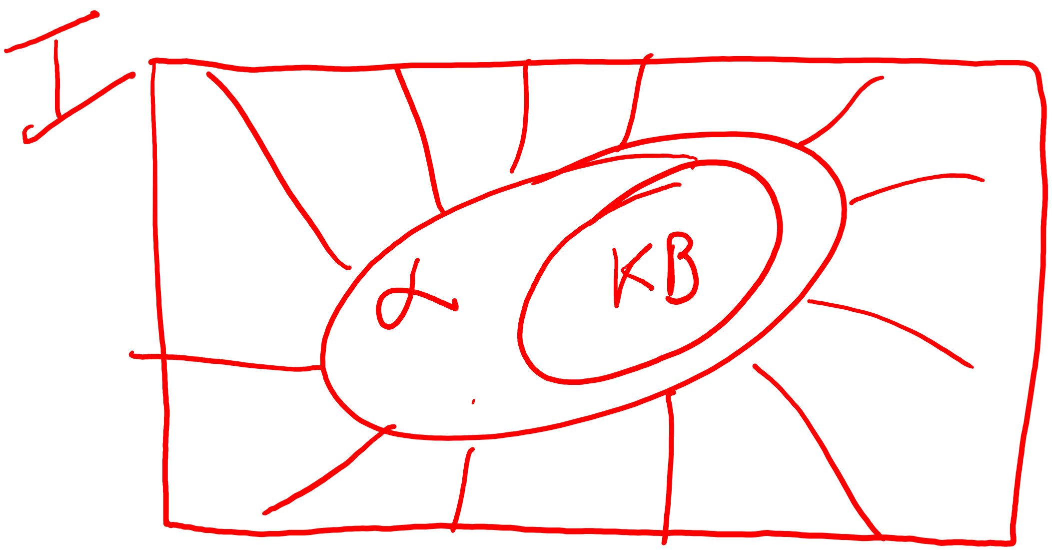
e.g., $True$, $A \vee \neg A$, $A \Rightarrow A$, $(A \wedge (A \Rightarrow B)) \Rightarrow B$

Validity is connected to inference via the Deduction Theorem:

$\boxed{KB \models \alpha}$ if and only if $\boxed{(KB \Rightarrow \alpha)}$ is valid

i.e There is no interpretations where KB is true and α is false



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e.g., $A \vee B$, C

A sentence is unsatisfiable if it is true in no interpretations

e.g., $A \wedge \neg A$

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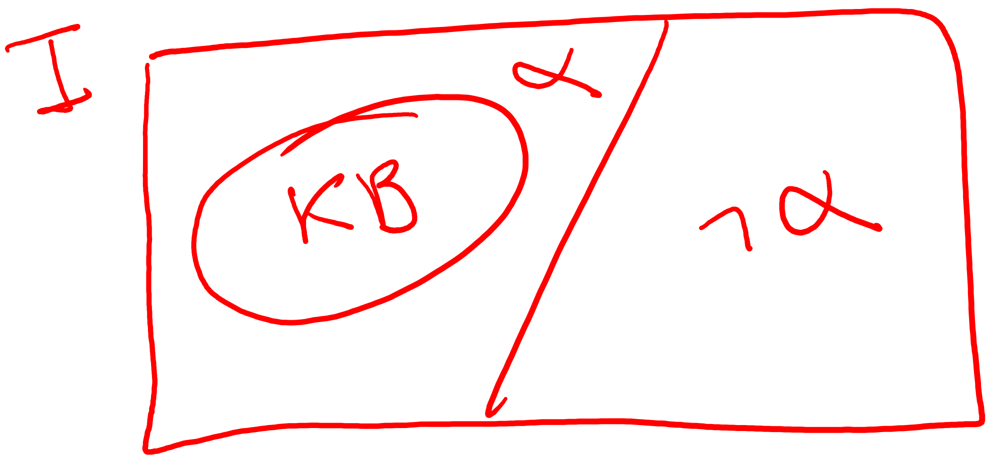


Validity and Satisfiability

iclicker.

$\langle \alpha$ is valid iff $\neg\alpha$ unsatisfiable$)$

$\langle \alpha$ is satisfiable iff $\neg\alpha$ is valid$\rangle$

The statements above are:

A: All false

B: Some true Some false

C: All true

Validity and Satisfiability

cannot be true in any model

true in all models

< α is valid iff ¬α unsatisfiable > T

< α is satisfiable iff ¬α is valid > F

true in some models

true in all models

The statements above are:

A: All false

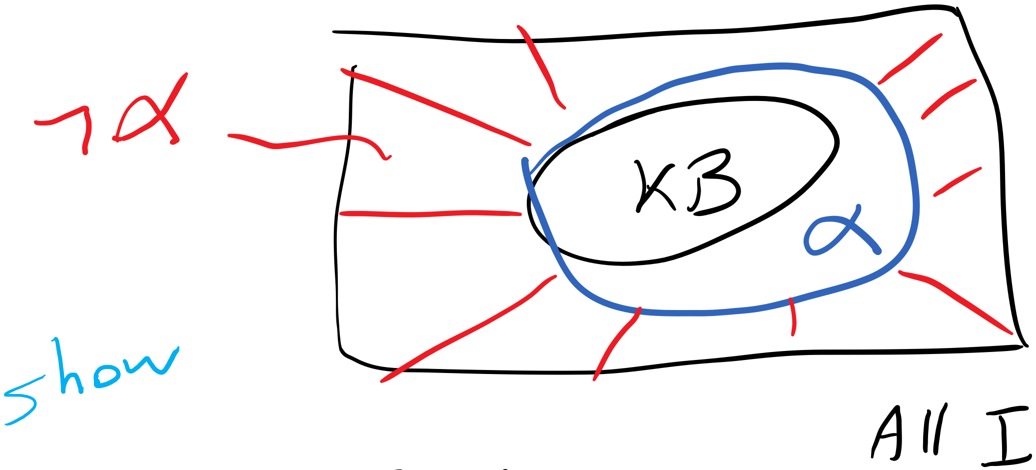
B: Some true Some false

C: All True

Lecture Overview

- Basics Recap: Interpretation / Model /
- Propositional Logics
- Satisfiability, Validity
- **Resolution in Propositional logics**

Proof by resolution



Key ideas

proof

$KB \models \alpha$

show

equivalent to: $KB \wedge \neg\alpha$ *unsatisfiable*

(there is no I in which both are true)

- Simple Representation for Conjunctive Normal Form
- Simple Rule of Derivation

Resolution

Conjunctive Normal Form (CNF)

Rewrite $KB \wedge \neg\alpha$ into **conjunction of disjunctions**

literals

$$\underbrace{(A \vee \neg B)}_{\text{Clause}} \wedge \underbrace{(B \vee \neg C \vee \neg D)}_{\text{Clause}}$$

- Any KB can be converted into CNF !

Example: Conversion to CNF

$A \Leftrightarrow (B \vee C)$

1. Eliminate $\Leftrightarrow$, replacing $\alpha \Leftrightarrow \beta$ with $(\alpha \Rightarrow \beta) \wedge (\beta \Rightarrow \alpha)$.
   $(A \Rightarrow (B \vee C)) \wedge ((B \vee C) \Rightarrow A)$

2. Eliminate $\Rightarrow$, replacing $\alpha \Rightarrow \beta$ with $\neg\alpha \vee \beta$.
   $(\neg A \vee B \vee C) \wedge (\neg(B \vee C) \vee A)$

3. Using de Morgan's rule replace $\neg(\alpha \vee \beta)$ with $(\neg\alpha \wedge \neg \beta)$:
   $(\neg A \vee B \vee C) \wedge ((\neg B \wedge \neg C) \vee A)$

4. Apply distributive law ($\vee$ over $\wedge$) and flatten:
   $(\neg A \vee B \vee C) \wedge (\neg B \vee A) \wedge (\neg C \vee A)$

Example: Conversion to CNF

$A \Leftrightarrow (B \vee C)$

5. KB is the conjunction of all of its sentences (all are true), so write each clause (disjunct) as a sentence in KB:

…
$(\neg A \vee B \vee C)$
$(\neg B \vee A)$
$(\neg C \vee A)$
…

Resolution Deduction step

Resolution: inference rule for CNF: sound and complete! *

$(A \vee B \vee C)$

$(\neg A)$ — "If A or B or C is true, but not A, then B or C must be true."

– – – – – – – – – – – – –

$\therefore (B \vee C)$

$(A \vee B \vee C)$

$(\neg A \vee D \vee E)$

– – – – – – – – – – – – –

$\therefore (B \vee C \vee D \vee E)$

"If A is false then B or C must be true,
or if A is true then D or E must be true,

hence since A is either true or false,
B or C or D or E must be true."

$(A \vee B)$

$(\neg A \vee B)$

– – – – – – – – – –

$\therefore (B \vee B) \equiv B$

Simplification

Learning Goals for today's class

You can:

- Describe relationships between different logics
- Apply the definitions of Interpretation, model, logical entailment, soundness and completeness
- Define and apply satisfiability and validity
- Convert any formula to CNF
- Justify and apply the resolution step

Next Class Fri

- Finish Resolution

- Another proof method for Prop. Logic

  Model checking - Searching through truth assignments. Walksat.

- Start First Order Logics

Midterm, Mon, March 8

Will be a Canvas Quiz

We will start at 4pm sharp

55 minutes

How to prepare…

- Go to **Office Hours**
- **Learning Goals** (look at the end of the slides for each lecture – complete list has been posted)
- Revise all the **clicker questions** and **practice exercises**
- Practice material has been posted
- Check questions and answers on Piazza