Probability and Time: Markov Models

Computer Science cpsc322, Lecture 31

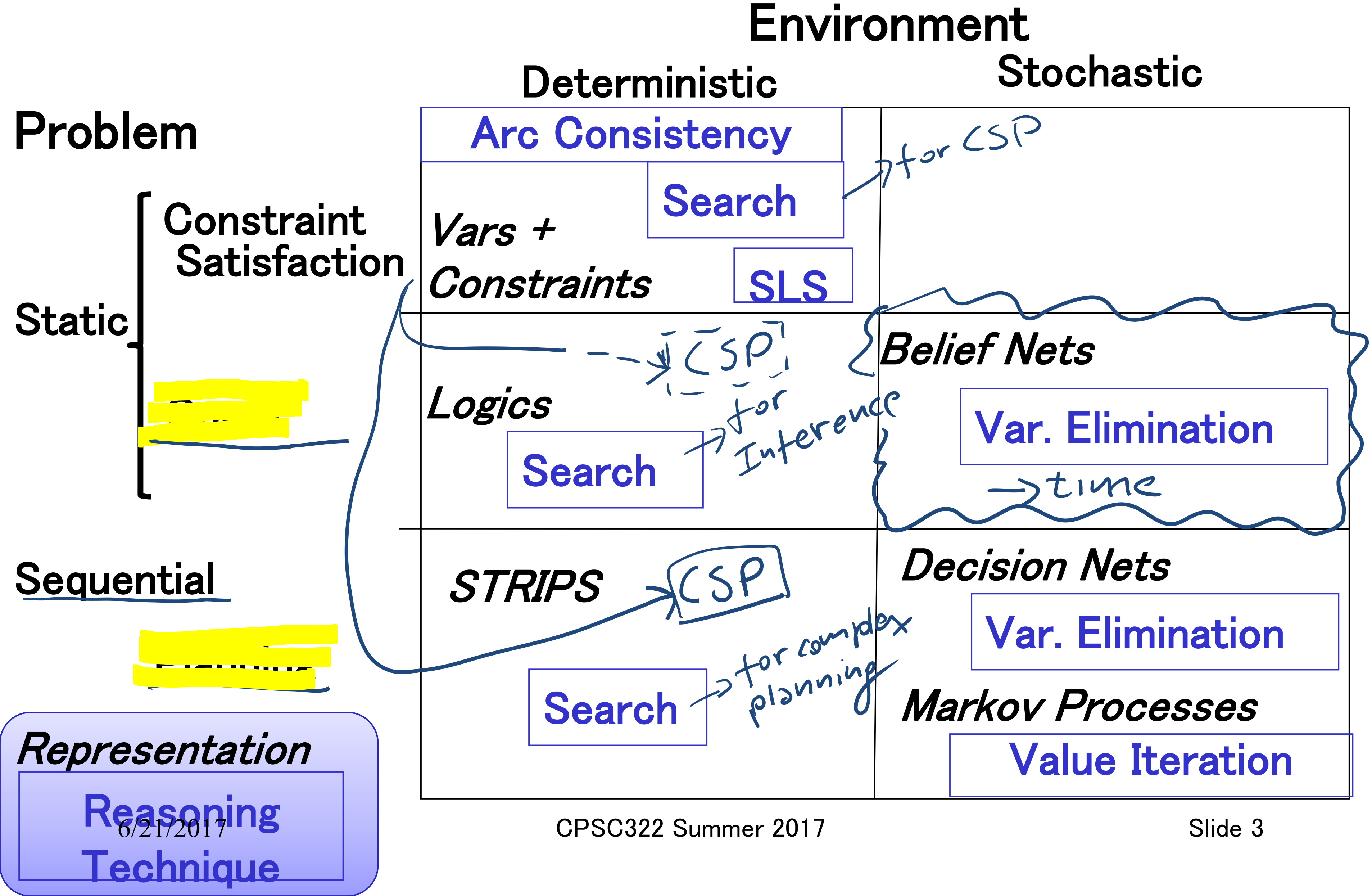
(Textbook Chpt 6.5.1)

June, 20, 2017

Lecture Overview

- **Recap**
- Temporal Probabilistic Models
- Start Markov Models
  - Markov Chain
  - Markov Chains in Natural Language Processing

Big Picture: R&R systems



Environment

Deterministic

Stochastic

Problem

Static

Constraint Satisfaction

Vars + Constraints

Arc Consistency

Search → for CSP

SLS

[illegible]

Logics → CSP

Search → for Inference

Belief Nets

Var. Elimination

→ time

Sequential

[illegible]

STRIPS → CSP

Search → for complex planning

Decision Nets

Var. Elimination

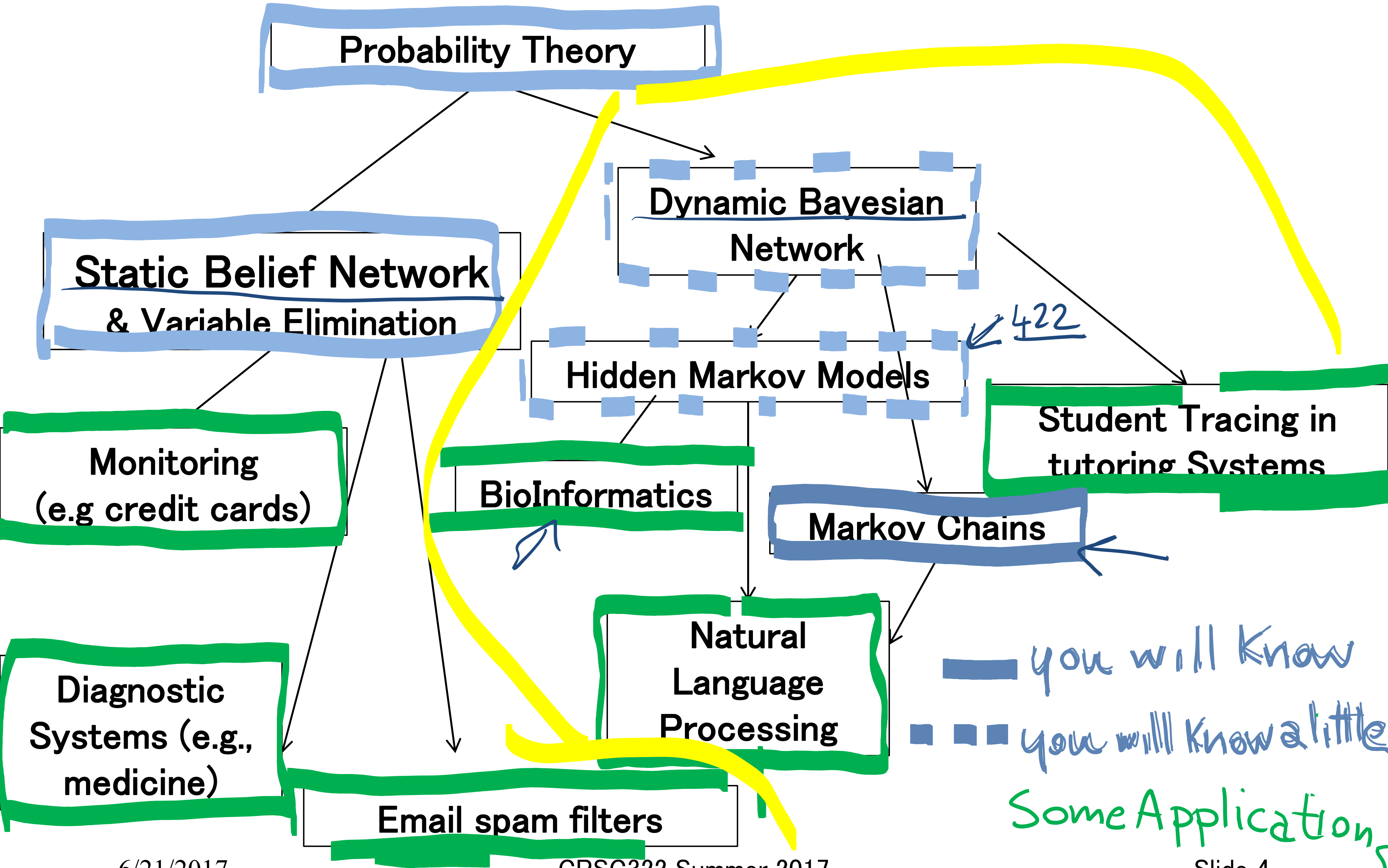
Markov Processes

Value Iteration

Representation

Reasoning Technique

Answering Query under Uncertainty



Probability Theory

Static Belief Network & Variable Elimination

Dynamic Bayesian Network

Hidden Markov Models

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Monitoring (e.g credit cards)

BioInformatics

Markov Chains

Student Tracing in tutoring Systems

Natural Language Processing

Diagnostic Systems (e.g., medicine)

Email spam filters

▬ you will know

▪ ▪ ▪ you will know a little

Some Applications

Lecture Overview

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Modelling static Environments

So far we have used Bnets to perform inference in **static environments**

- For instance, the system keeps collecting evidence to diagnose the cause of a fault in a system (e.g., a car).

AIspace

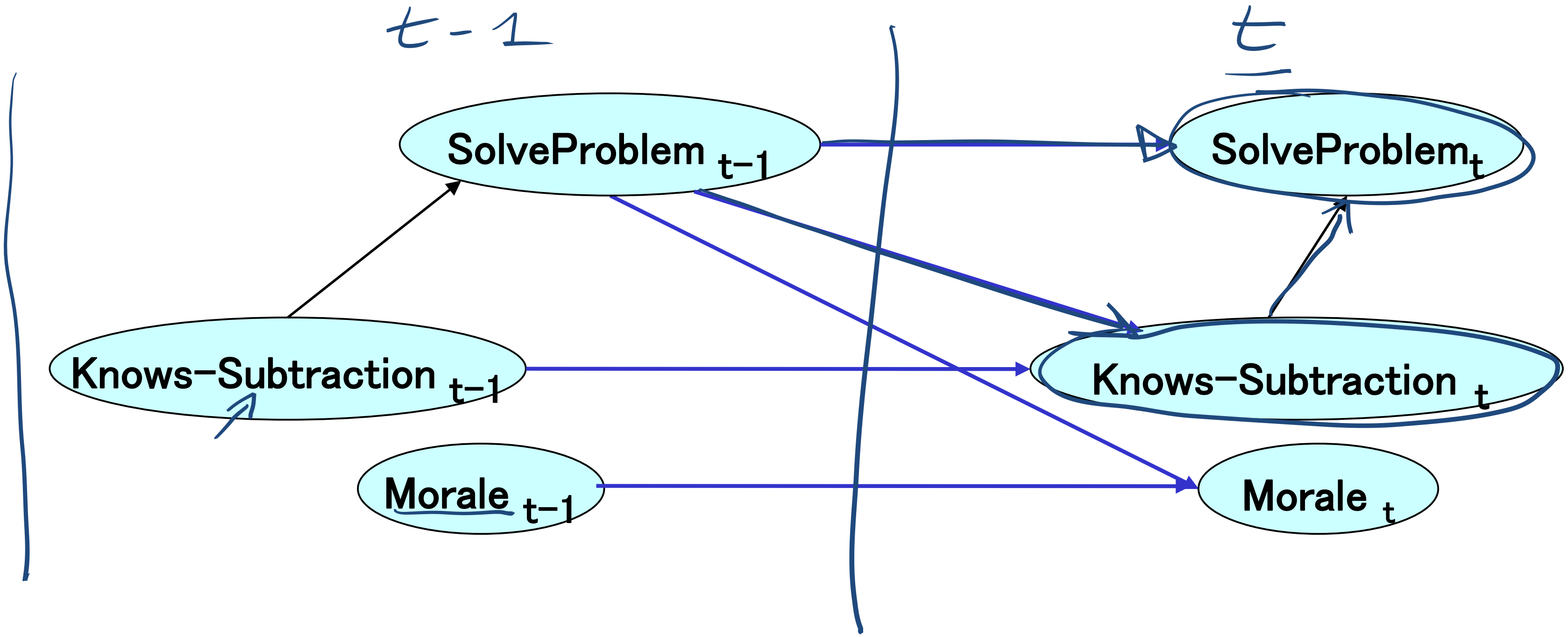
- The environment (values of the evidence, the true cause) does not change as I gather new evidence

- What does change?

The system's beliefs over possible causes

Modeling Evolving Environments

- Often we need to make inferences about evolving environments.
- Represent the state of the world at each specific point in time via a series of snapshots, or ***time slices***,



Tutoring system tracing student *knowledge* and *morale*

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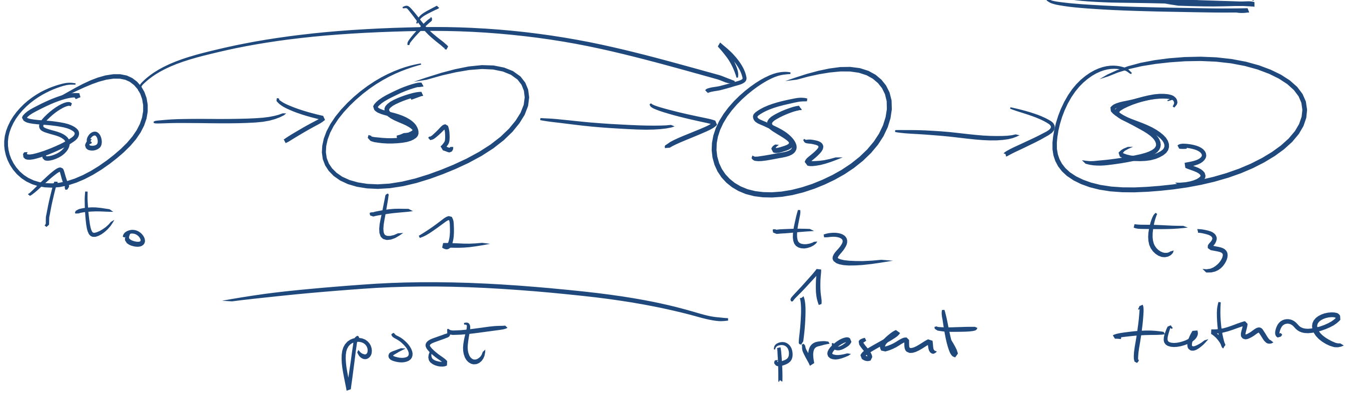
Simplest Possible DBN

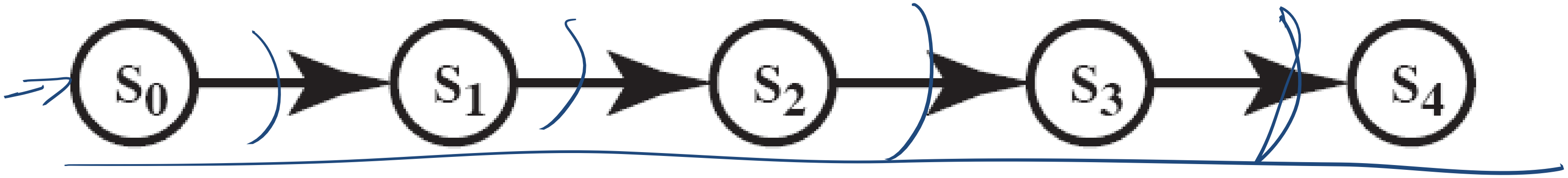
- **One random variable** for each time slice: let's assume S_t represents the state at time t. with domain $\{\underline{\boxed{v_1 \cdots v_n}}\}$



- **Each random variable depends only on the previous one**
- Thus $P(S_{t+1} \mid S_0 \cdots S_t) = P(\underline{S_{t+1}} \mid \underline{S_t})$
- Intuitively S_t conveys all of the information about the history that can affect the future states.
- "The future is independent of the past given the present."

Simplest Possible DBN (cont’)

P(S4 | S3)



- How many CPTs do we need to specify?

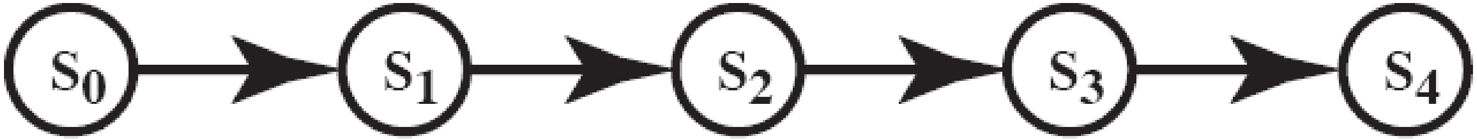
4 P(S1 | S0) P(S2 | S1) etc.

A. *1*   C . *2*   D. *3*   [✓ B. *4*]

- *Stationary process assumption:* the mechanism that regulates how state variables change overtime is **stationary**, that is it can be described by a single transition model

- $P(S_t|S_{t-1})$ is the same for all t

Stationary Markov Chain (SMC)



Astationary Markov Chain : for all t >0

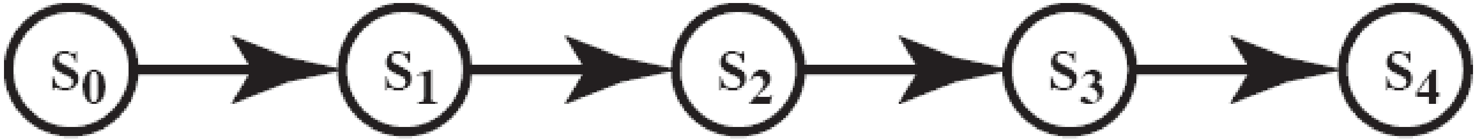
- $\underline{P(S_{t+1}|\ S_0,\cdots,S_t) = P(S_{t+1}|S_t)}$ and Markov assumption
- $\underline{P(S_{t+1}|S_t)}$ is the same stationary

We only need to specify $P(S_0)$ and $P(S_{t+1}|S_t)$

- Simple Model, easy to specify ←
- Often the natural model ←
- The network can extend indefinitely ←
- **Variations of SMC are at the core of many Natural Language Processing (NLP) applications!**

also used in the PageRank algo (used by Google to rank web pages)

Stationary Markov Chain (SMC)



A stationary Markov Chain : for all t >0

- $P(S_{t+1}| S_0,\cdots,S_t) = P(S_{t+1}|S_t)$ and Markov assumption
- $P(S_{t+1}|S_t)$ is the same stationary

So we only need to specify?

[✓ A. $P(S_{t+1}|S_t)$ and $P(S_0)$]

B. $P(S_0)$

C . $P(S_{t+1}|S_t)$

D. $P(S_t|S_{t+1})$

Stationary Markov-Chain: Example

Domain of variable S_i is {t , q, p, a, h, e}

t	.6
q	.4
p	0
a	0
h	0
e	

Probability of initial state $P(S_0)$

Stochastic Transition Matrix $P(S_{t+1}|S_t)$

Which of these two is a possible STM?

S_{t+1}

S_t

	t	q	p	a	h	e
t	0	.3	0	.3	.4	0
q	.4	0	.6	0	0	0
p	0	0	1	0	0	0
a	0	0	.4	.6	0	0
h	0	0	0	0	0	1
e	1	0	0	0	0	0

S_{t+1}

S_t

	t	q	p	a	h	e
t	1	0	0	0	0	0
q	0	1	0	0	0	0
p	.3	0	1	0	0	0
a	0	0	0	1	0	0
h	0	0	0	0	0	1
e	0	0	0	.2	0	1

$\Sigma > 1$

[✓ A. Left one only]

B. *Right one only*

C . *Both*

D. *None*

Stationary Markov-Chain: Example

Domain of variable S_i is {t , q, p, a, h, e}

six possible values

We only need to specify…

$P(S_0)$

Probability of initial state

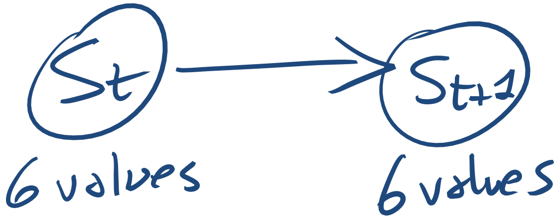
t	.6
q	.4
p	0
a	0
h	0
e	0

Stochastic Transition Matrix

$P(S_{t+1}|S_t)$

S_{t+1}

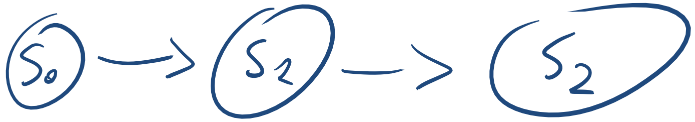
S_t	t	q	p	a	h	e	
t	0	.3	0	.3	.4	0	…
q	.4	0	.6	0	0	0	← $P(S_{t+1}\|S_t = q)$
p	0	0	1	0	0	0	← $P(S_{t+1}\|S_t = p)$
a	0	0	.4	.6	0	0	…
h	0	0	0	0	0	1	…
e	1	0	0	0	0	0	…



Markov-Chain: Inference

Probability of a sequence of states $S_0 \ldots S_T$

$$P(S_0,\ldots,S_T) = P(s_0)\, P(s_1|s_0)\, P(s_2|s_1) \ldots\ldots$$



P(u,e,e)

$P(S_0)$

t	.6
q	.4
p	0
a	0
h	0
e	0

$P(S_{t+1}|S_t)$

	t	q	p	a	h	e
t	0	.3	0	.3	.4	0
q	.4	0	.6	0	0	0
p	0	0	1	0	0	0
a	0	0	.4	.6	0	0
h	0	0	0	0	0	1
e	1	0	0	0	0	0

Example:

$$P(t,q,p) =$$

$$P(t) * P(q|t) * P(p|q)$$

$$.6 \times .3 \times .6 = .108$$

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Key problems in NLP

Noun Verb

"Book me a room near UBC"

w_1 w_2 w_3 w_4 w_5 w_6

$$\boxed{P(w_1,..,w_n)?}$$

Assign a probability to a sentence (a sequence of words)

- Part-of-speech tagging
- Word-sense disambiguation,
- Probabilistic Parsing

Summarization, Machine Translation……..

Predict the next word

$$P(w_n | w_1 \ldots \ldots w_{n-1}) =$$

$$= P(w_1 \ldots w_n) / P(w_1 \ldots w_{n-1})$$

- Speech recognition
- Hand-writing recognition
- Augmentative communication for the disabled

$$\boxed{P(w_1,..,w_n)?}$$

Impossible to estimate ☹

$\boxed{P(w_1, .., w_n)?}$

Impossible to estimate!

Assuming 10^5 words and average sentence contains 10 words …….

$(10^5)^{10} = 10^{50}$

would contain ↑ probabilities

collected from the whole web

Google language repository (22 Sept. 2006) contained "only": 95,119,665,584 sentences

$\sim 10^{11}$

Most sentences will not appear or appear only once ☹

What can we do?

Make a strong simplifying assumption!

Sentences are generated by a Markov Chain

w_1 at the beginning of a sentence

$$P(w_1,..,w_n) = P(w_1 |< S >) \prod_{k=2}^{n} P(w_k \mid w_{k-1})$$

$$= P(w_1|<S>)\,P(w_2|w_1)\,P(w_3|w_2)\,..\,P(w_k|w_{k-1})$$

P(The big red dog barks)=

**P(The|<S>) *** P(big | the) * P(red | big) * ...

* P(dog | red) * P(barks | dog)

These probs can be assessed in practice!

☺

Estimates for Bigrams

$P(w_i \mid w_{i-1})$

Silly language repositories with **only two sentences**:

"<S> *The big red dog barks against the big pink dog*"

"<S> *The big pink dog is much smaller*"

Count how many times in your documents you have "big red" and "big"

count

$$P(red \mid big) = \frac{P(big,red)}{P(big)} = \frac{\dfrac{C(big,red)}{\cancel{N_{pairs}}}}{\dfrac{C(big)}{\cancel{N_{words}}}} = \frac{C(big,red)}{C(big)} = \frac{1}{3}$$

$P(w_i \mid w_{i-1})$

$10^5 * 10^5$ matrix

$\cancel{10^5}$

$$\boxed{P(w_i \mid w_{i-1}, w_{i-2})}$$

some models use two preceeding words

Bigrams in practice…

If you have 10^5 words in your dictionary $P(w_i \mid w_{i-1})$

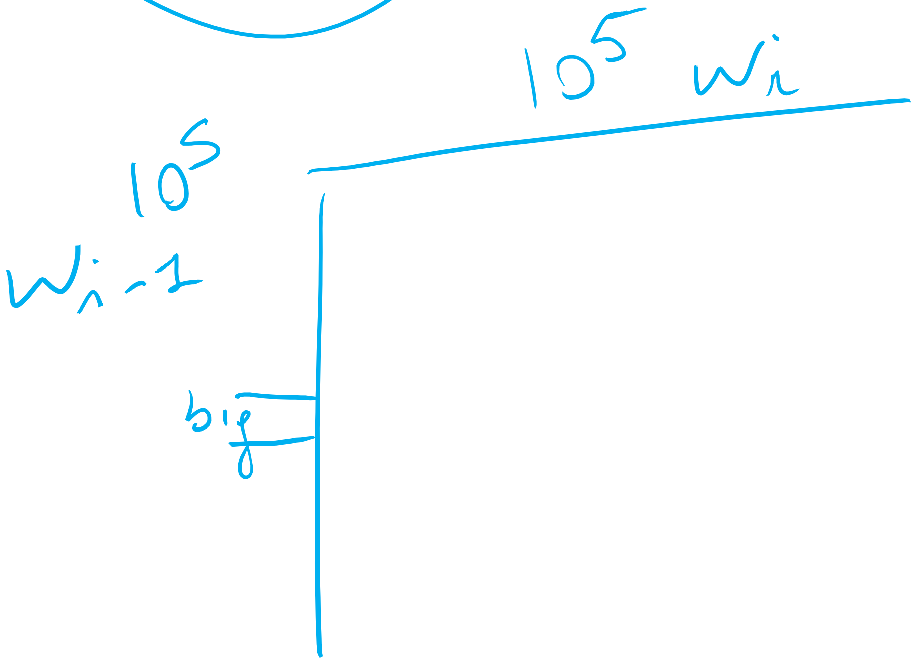
will contain this many numbers.. ??

A. $2 * 10^5$

[✓ B. 10^{10}]

C. $5 * 10^5$

D. $2 * 10^{10}$



$10^5 * 10^5$

Learning Goals for today’s class

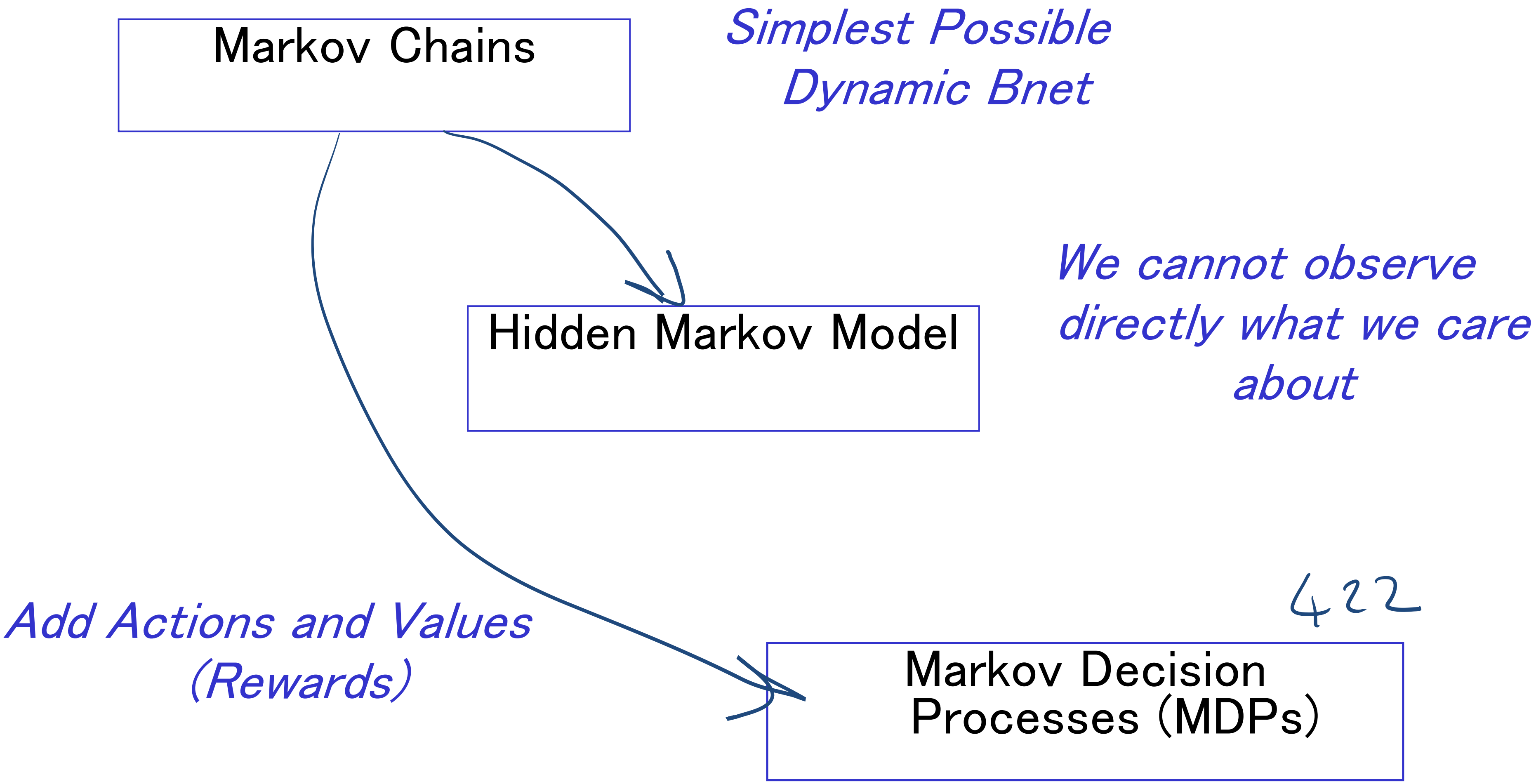
You can:

- Specify a Markov Chain and compute the probability of a sequence of states ✓

- Justify and apply Markov Chains to compute the probability of a Natural Language sentence

(NOT to compute the conditional probabilities - slide 18)

Markov Models



Markov Chains

Simplest Possible Dynamic Bnet

Hidden Markov Model

We cannot observe directly what we care about

Add Actions and Values (Rewards)

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Markov Decision Processes (MDPs)

Next Class

- **Finish Probability and Time:** Hidden Markov Models (HMM) *(TextBook 6.5.2)*
- **Start Decision networks** *(TextBook chpt 9)*

Course Elements

- **Assignment 4 is available on Connect. Due Sunday, June 25th @ 11:59 pm. Late submissions will not be accepted, and late days may not be used. This is due to next week being exam week, and we want to be able to release the solutions immediately.**