# Q2

1. $$P(x_1, x_2, y)$$

| Y = night | | | Y = day | | |
|---|---|---|---|---|---|
| | $X_1$ = hot | $X_1$ = cold | | $X_1$ = hot | $X_1$ = cold |
| $X_2$ = dry | 0.04 | 0.16 | $X_2$ = dry | 0.3375 | 0.0375 |
| $X_2$ = rain | 0.06 | 0.24 | $X_2$ = rain | 0.1125 | 0.0125 |

2. $$P(x_1, x_2) = \sum_y P(x_1, x_2, y)$$

| | $X_1$ = hot | $X_1$ = cold |
|---|---|---|
| $X_2$ = dry | 0.3775 | 0.1975 |
| $X_2$ = rain | 0.1725 | 0.2525 |

3. $$P(y|x_1, x_2) = \frac{P(x_1, x_2, y)}{P(x_1, x_2)}$$

| Y = night | | | Y = day | | |
|---|---|---|---|---|---|
| | $X_1$ = hot | $X_1$ = cold | | $X_1$ = hot | $X_1$ = cold |
| $X_2$ = dry | 0.10596 | 0.810127 | $X_2$ = dry | 0.89404 | 0.189873 |
| $X_2$ = rain | 0.347826 | 0.950495 | $X_2$ = rain | 0.652174 | 0.049505 |

4. $$P(x_1) = \sum_{x_2} P(x_1, x_2)$$

| $X_1$ = hot | $X_1$ = cold |
|---|---|
| 0.55 | 0.45 |

5. $$P(x_2) = \sum_{x_1} P(x_1, x_2)$$

| $X_2$ = dry | $X_2$ = rain |
|---|---|
| 0.575 | 0.425 |

6. $$P(x_1|x_2) = \frac{P(x_1, x_2)}{P(x_2)}$$

| | $X_1$ = hot | $X_1$ = cold |
|---|---|---|
| $X_2$ = dry | 0.656522 | 0.343478 |
| $X_2$ = rain | 0.405882 | 0.594118 |

7.

$$P(x_2|x_1) = \frac{P(x_1, x_2)}{P(x_1)}$$

| | $X_1$ = hot | $X_1$ = cold |
|---|---|---|
| $X_2$ = dry | 0.686364 | 0.438889 |
| $X_2$ = rain | 0.313636 | 0.561111 |

8.

$$P(x_1|x_2, y) = \frac{P(x_1, x_2, y)}{P(x_2, y)} = \frac{P(y)P(x_1|y)P(x_2|y)}{P(y)P(x_2|y)} = P(x_1|y)$$

| Y = night | | | Y = day | | |
|---|---|---|---|---|---|
| | $X_1$ = hot | $X_1$ = cold | | $X_1$ = hot | $X_1$ = cold |
| $X_2$ = dry | 0.2 | 0.8 | $X_2$ = dry | 0.9 | 0.1 |
| $X_2$ = rain | 0.2 | 0.8 | $X_2$ = rain | 0.9 | 0.1 |

9.

$$P(x_2|x_1, y) = \frac{P(x_1, x_2, y)}{P(x_1, y)} = \frac{P(y)P(x_1|y)P(x_2|y)}{P(y)P(x_1|y)} = P(x_2|y)$$

| Y = night | | | Y = day | | |
|---|---|---|---|---|---|
| | $X_1$ = hot | $X_1$ = cold | | $X_1$ = hot | $X_1$ = cold |
| $X_2$ = dry | 0.4 | 0.4 | $X_2$ = dry | 0.75 | 0.75 |
| $X_2$ = rain | 0.6 | 0.6 | $X_2$ = rain | 0.25 | 0.25 |

The variables $X_1$ and $X_2$ are conditionally independent given $Y$. Indeed,

$$P(x_1, x_2|y) = \frac{P(x_1, x_2, y)}{P(y)} = \frac{P(y)P(x_1|y)P(x_2|y)}{P(y)} = P(x_1|y)P(x_2|y)$$

But they are not marginally independent integrating over $Y$.

$$P(X_1 = \text{hot}, X_2 = \text{dry}) = 0.6864 \neq 0.31625 = P(X_1 = \text{hot})P(X_2 = \text{dry})$$